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Position-dependent mass Schrödinger equation for the q-deformed Woods-Saxson plus hyperbolic tangent potential

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ABSTRACT

In this work, we propose a new potential called the "q-deformed Woods-Saxon plus hyperbolic tangent potential." We derive the generalized Schrödinger equation for quantum mechanical systems with position-dependent masses under these potentials using the Nikiforov-Uvarov method, with the mass relationship defined as $m(x) = m_1/(1 + qe^{-2\lambda x})$. The solutions to this equation, expressed in terms of hypergeometric functions and Jacobi polynomials, offer insights into the quantum behavior of particles. The energy eigenvalues depend on system parameters such as the deformation parameter q, potential parameters, and quantum numbers. We analyzed the effect of the deformation parameter q numerically and visually using different values of these parameters.

1. Introduction

The Woods-Saxon potential (Woods et al., 1954), also known as the Woods-Saxon potential well or nuclear potential well, is a mathematical model used in nuclear physics to describe the potential energy experienced by nucleons (protons and neutrons) inside an atomic nucleus. It is essential to the nuclear shell concept, which aims to describe the behavior of nucleons within nuclei. The Woods-Saxon potential is thought to be the most useful short-range potential in nuclear physics. It is extensively used in the research of the nuclear structure within the shell model. The interaction between a nucleon and a heavy nucleus is explained by the nuclear shell concept. Furthermore, other extended versions of this potential have been developed to study elastic and quasi-elastic nuclear particle scattering (Wang and Scheid, 2008). For this reason, the Woods–Saxon potential, whether in its spherical or deformed form, has seen a rise in application in nuclear numerical simulations (Bespalova et al., 2003; Goldberg et al., 2004; Khounfais et al., 2004; Guo and Sheng, 2005). It was also applied in other branches of physics, such as the study of valence electron behavior in metallic systems and the helium model (Dudek et al., 2003). It is also applicable to the nonlinear scalar theory of mesons (Erkol and Demiralp, 2007). Within the context of Several researchers (Arda et al., 2010; Berkdemir et al., 2006; Badalov et al., 2010; Ikhdair and Sever, 2007; 2008; Arda and Sever, 2009; Ikhdair and Sever, 2010; Guo and Sheng, 2005; Hagino and Tanimura, 2010; Panella et al., 2010; Abadi et al., 2019; Romaniega et al., 2020) have used a variety of approaches to work with the Woods-Saxon potential, particularly in relativistic quantum mechanics. A method for obtaining the bound state solution of the one-dimensional Dirac equation for the Woods-Saxon potential was devised by Roja and Villalba (2005). The Nikiforov-Uvarov approach is used by Ikot et al. (2015) to solve the radial Schrödinger equation for the More General Woods-Saxon Potential (MGWSP) (Okon et al., 2014). Additionally, a hierarchy of Hamiltonians for the Spherical Woods-Saxon potential was developed by Sadeghi and Pahlavani (2004).

The aim of this paper is to use a wave function transformation to study the position-dependent effective mass Schrödinger equation. The position-dependent effective mass Schrödinger equation was solved using the Nikiforov-Uvarov method in the presence of the q-deformed Woods-Saxon plus hyperbolic tangent potential. The remainder of the paper is structured as follows: In Section 2, the Nikiforov-Uvarov method is discussed. Section 3 is focused on the position-dependent mass Schrödinger equation. Section 4 is devoted to the solution of the position-dependent mass Schrödinger

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equation for Woods-Saxon plus hyperbolic tangent potential using the Nikiforov-Uvarov method.

2. Nikiforov-Uvarov method

The Nikiforov-Uvarov method (Nikiforov and Uvarov, 1988) is a method used in mathematics to solve linear differential equations of second order. Schrödinger-type equations in quantum mechanics can be solved efficiently with this approach (Al-Hawamdeh et al., 2023; Jaradat et al., 2019; Yazdankish, 2021; Gu et al., 2022), especially when dealing with quantum systems that have unique potential forms.

Using this method, the Schrödinger equation in one dimension is reduced to a generalized hypergeometric equation with the required coordinate transformation, x=x(s), for a given potential, and the following is one way to write it:

$$\psi''(s) + \frac{\tilde{\tau}(s)}{\sigma(s)}\psi'(s) + \frac{\tilde{\sigma}(s)}{\sigma^2(s)}\psi(s) = 0 \tag{1}$$

where, $\sigma(s)$ and $\tilde{\sigma}(s)$ are polynomials of degree at most two, $\tilde{\tau}(s)$ is first-degree polynomial, and $\psi(s)$ is a function of hypergeometric type.

Schrödinger equation is written for any potentials in the general form as:

$$[\frac{d^2}{ds^2} + \frac{\alpha_1-\alpha_2 s}{s(1-\alpha_3 s)}\frac{d}{ds} + \frac{-\xi_1 s^2+\xi_2 s-\xi_3}{s^2(1-s)^2}]\psi = 0. \tag{2}$$

By separating the variables, a specific solution to Eq. 1 can be obtained by multiplying two distinct parts of the wave function, which are as follows:

$$\psi(s) = \varphi(s)y(s) \tag{3}$$

If one deals with the above transformation, Eq. 1 is reduced to a hypergeometric equation:

$$\sigma(s)y_n''(s) + \tau(s)\, y_n'(s) + \lambda y_n(s) = 0 \tag{4}$$

where

$$\tau(s) = 2\pi(s) + \tilde{\tau}(s). \tag{5}$$

Its derivative is negative $\tau'(s) < 0$, this condition helps to generate physical solutions. And φ(s) is described as a derivative of a logarithm:

$$\frac{\varphi'(s)}{\varphi(s)} = \frac{\pi(s)}{\sigma(s)}. \tag{6}$$

Here, $\pi(s)$ is a polynomial with one degree or less. The solution of the hypergeometric-type differential Eq. 3 is a, is given by Rodrigues relation.

$$y_n(s) = \frac{B_n}{\rho(s)}\frac{d^n}{ds^n}[\sigma^n(s)\rho(s)] \tag{7}$$

where, B_n is the normalization constant, $\rho(s)$ is the weight function, and n is a fixed given number.

The weight function satisfies the following differential equation.

$$\frac{d}{ds}[\sigma(s)\rho(s)] = \tau(s)\rho(s) \tag{8}$$

or

$$\frac{\rho'(s)}{\rho(s)} = \frac{\tau(s)-\sigma'(s)}{\sigma(s)}. \tag{9}$$

The function of $\pi(s)$ is given by.

$$\pi(s) = \frac{\sigma' - \tilde{\tau}(s)}{2} \pm \sqrt{(\frac{\sigma' - \tilde{\tau}(s)}{2})^2 - \tilde{\sigma}(s) + k\sigma(s)} \tag{10}$$

and

$$k = \lambda - \pi'(s) \tag{11}$$

Consequently, the most important step is to find k by setting the discriminant of the square root in Eq. 10 to zero for the computation of $\pi(x)$. Furthermore, the eigenvalue equation given in Eq. 11 now takes on the following new form:

$$\lambda = \lambda_n = -n\,\tau' - \frac{n(n-1)}{2}\sigma''(s)\,, (n = 0, 1, 2, \dots). \tag{12}$$

Here, prime factors represent the first-degree differentials.

3. Position-dependent mass Schrödinger equation

Von Roos's (1983) general position-dependent effective mass Hamiltonian is

$$H_{eff} = -\frac{1}{2}\Big(M^{\alpha}(x)\frac{d}{dx}M^{\beta}(x)\frac{d}{dx}M^{\gamma}(x) + M^{\gamma}(x)\frac{d}{dx}M^{\beta}(x)\frac{d}{dx}M^{\alpha}(x)\Big) + V_{eff}(x). \tag{13}$$

With $\hbar = 2m_0 = 1$, $M(x)$ is the dimensionless form of the function $m(x) = m_0 M(x)$, and $\alpha + \beta + \gamma = -1$, by Von Roos's (1983) introduced restriction $\alpha = \gamma = 0 \; and \; \beta = -1$ these parameters are called the ambiguity parameters. Eq. 13 becomes:

$$H_{eff} = -\frac{1}{2}\Big(\frac{d}{dx}M^{-1}(x)\frac{d}{dx} + \frac{d}{dx}M^{-1}(x)\frac{d}{dx}\Big)) + V_{eff}(x). \tag{14}$$

Now the one-dimensional effective mass Hamiltonian of the Schrödinger equation reads as

$$H_{eff} = -\frac{d}{dx}\Big(\frac{1}{m(x)}\Big)\frac{d}{dx} + V_{eff}(x). \tag{15}$$

Schrödinger equation is given by

$$\Big[-\frac{d}{dx}\Big(\frac{1}{m(x)}\Big)\frac{d}{dx} + V_{eff}(x) - E\Big]\varphi(x) = 0 \tag{16}$$

After expanding Eq. 16, the Schrödinger equation takes form

$$\Big[-\frac{1}{m(x)}\frac{d^2}{dx^2} + \frac{m'(x)}{m^2(x)}\frac{d}{dx} + V_{eff}(x) - E\Big]\varphi(x) = 0. \tag{17}$$

The effective potential is given by:

$$V_{eff}(x) = V(x) + \frac{1}{2}(\beta+1)\frac{m''(x)}{m^2(x)} - [\alpha(\alpha+\beta+1) + (\beta+1)]\frac{m'^2(x)}{m^3(x)} \quad (18)$$

where, α, β are ambiguity parameters. Applying the following transformation (Tezcan et al., 2008) on Eq. 17:

$$\varphi(x) = m^{\eta}(x)\psi(x) \quad (19)$$

where,

$$\varphi'(x) = m^{\eta-1}(x)[m(x)\psi'(x) + \eta m'(x)\psi(x)] \quad (20)$$

and

$$\varphi''(x) = m^{\eta-2}(x)[\eta(\eta-1)\, m'^2(x)\psi(x) + \eta\, m(x)(2m'(x)\psi'(x) + \psi(x)m''(x)) + m^2(x)\psi''(x). \quad (21)$$

Putting Eqs. 19, 20, and 21 into Eq. 17. we get

$$\left[-\frac{d^2}{dx^2} - \eta(\eta-1)\frac{m'^2(x)}{m^2(x)} + (-2\eta+1)\frac{m'(x)}{m(x)}\frac{d}{dx} - \eta\frac{m''(x)}{m(x)} + \eta\frac{m'^2(x)}{m^2(x)} + \left(V_{eff}(x) - E\right)\right]\psi = 0. \quad (22)$$

Now, substituting the effective potential in Eq. 22, we get

$$\left[-\frac{d^2}{dx^2} - (2\eta-1)\frac{m'(x)}{m(x)}\frac{d}{dx} + \left(\frac{1}{2}(\beta+1) - \eta\right)\frac{m''(x)}{m(x)} - \left(\eta(\eta-2) + \alpha(\alpha+\beta+1) + (\beta+1)\right)\frac{m'^2(x)}{m^2(x)} + m(x)(V(x) - E)\right]\psi = 0. \quad (23)$$

This last equation is called the position-dependent mass Schrödinger equation. In the next section, we are going to generalize these expressions to the q-deformed Woods-Saxon plus hyperbolic tangent potential case.

4. Developing the q-deformed Woods-Saxon plus hyperbolic tangent potential in Schrödinger equation with position-dependent mass

The q-deformed Woods-Saxon (Chabab et al., 2012; Falaye et al., 2013; Okon et al., 2014) plus hyperbolic tangent potential is given by

$$V(x) = -V_0\frac{1}{1+qe^{-2\lambda x}} + V_1 tanh_q(\lambda x) \quad (24)$$

where, V_0 represents the potential well's depth, and the deformation parameter is q, which measures the system's level of non-extensivity. It reduces to the ordinary Woods-Saxon potential when $q = 1$.

Fig. 1 shows the plot of the Woods-Saxon plus hyperbolic tangent potential as a function of x for different values of λ. The Woods-Saxon plus hyperbolic tangent potential values for given values of λ is increase when moving to the right.

Now, assuming the mass relation:

$$m(x) = \frac{m_1}{(1+qe^{-2\lambda x})}. \quad (25)$$

Fig. 2 shows the mass function as a sigmoid-like curve with smooth transitions between 0 and 1. When $\lambda = 1$ we see that the shift between 0 and 1 is gradual. As parameter values increase, the sigmoid function becomes steeper. Larger values of λ cause the curve to approach 0 and 1 faster as it moves away.

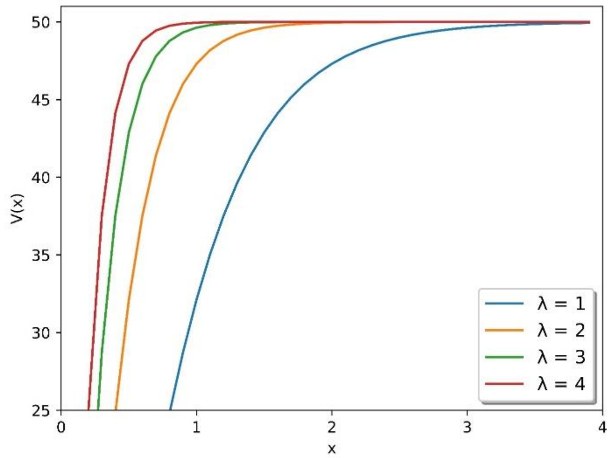

Fig. 1: Woods-Saxon plus hyperbolic tangent potential for $q = 1, V_0 = 50$, and $V_1 = 100$

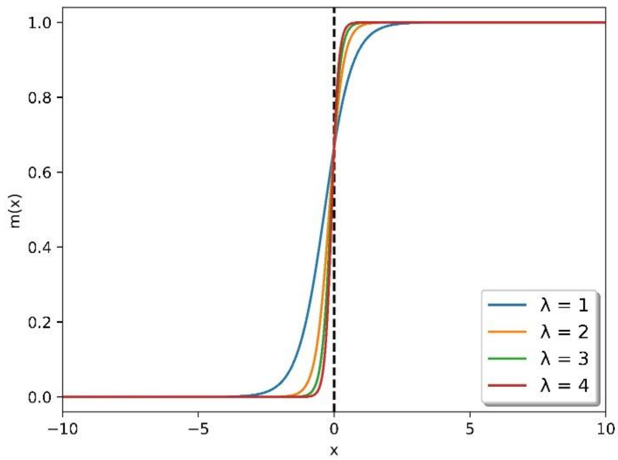

Fig. 2: Plot of mass function $m(x)$ for $q = 0.5$, $m_1 = 1$

Where,

$$m'(x) = \frac{2q\lambda e^{-2\lambda x}}{(1+qe^{-2\lambda x})^2} \quad (26)$$

and

$$m''(x) = 2q\lambda^2\left[\frac{4qe^{-4\lambda x}}{(1+qe^{-2\lambda x})^3} - \frac{2e^{-2\lambda x}}{(1+qe^{-2\lambda x})^2}\right] \quad (27)$$

then, the parameters

$$\frac{m'(x)}{m(x)} = \frac{2q\lambda e^{-2\lambda x}}{(1+qe^{-2\lambda x})} \quad (28)$$

and

$$\frac{m''(x)}{m(x)} = 2q\lambda^2\left[\frac{4qe^{-4\lambda x}}{(1+qe^{-2\lambda x})^2} - \frac{2e^{-2\lambda x}}{(1+qe^{-2\lambda x})}\right]. \quad (29)$$

Now, putting Eqs. 24, 25, 28 and 29 into Eq. 23

$$\Big[-\frac{d^2}{dx^2}-(2\eta-1)\frac{2q\lambda e^{-2\lambda x}}{(1+qe^{-2\lambda x})}\frac{d}{dx}+\left(\frac{1}{2}(\beta+1)-\eta\right)2q\lambda^2\Big[\frac{4qe^{-4\lambda x}}{(1+qe^{-2\lambda x})^2}-\frac{2e^{-2\lambda x}}{(1+qe^{-2\lambda x})}\Big]-\big(\eta(\eta-2)+\alpha(\alpha+\beta+1)+(\beta+1)\big)\frac{4q^2\lambda^2e^{-4\lambda x}}{(1+qe^{-2\lambda x})^2}+\frac{m_1}{(1+qe^{-2\lambda x})}\Big(-V_0\frac{1}{1+qe^{-2\lambda x}}+V_1 tanh_q(\lambda x)-E\Big)\Big]\psi=0. \quad (30)$$

We do the variable changing, so this variable changing to convert Eq. 23 into Nikiforov-Uvarov equation.

$$s=m(x)=\frac{1}{(1+qe^{-2\lambda x})},(0\le s\le 1) \quad (31)$$

then

$$s+sqe^{-2\lambda x}=1 \quad (32)$$

so that

$$(1-s)=sqe^{-2\lambda x} \quad (33)$$

using Eq. 32 into Eq. 28

$$\frac{m'(x)}{m(x)}=2\lambda(1-s). \quad (34)$$

Now, using Eq. 33 into Eq. 29, we get

$$\frac{m''(x)}{m(x)}=2\lambda^2[4(1-2s+s^2)-2s+s] \quad (35)$$

then

$$\frac{m''(x)}{m(x)}=4\lambda^2(1-3s+2s^2) \quad (36)$$

so we have

$$\frac{m''(x)}{m(x)}=4\lambda^2(1-s)(1-2s) \quad (37)$$

setting that

$$A=\big(\eta(\eta-2)+\alpha(\alpha+\beta+1)+(\beta+1)\big)$$
$$B=\left(\frac{1}{2}(\beta+1)-\eta\right) \quad (38)$$
$$\zeta=\frac{1}{2}-\eta$$

After using the above parameters, Eqs. 34, 37, and 38, Eq. 30 becomes.

$$\frac{d^2\psi}{ds^2}+\frac{2\eta-(2\eta+1)s}{s(1-s)}\frac{d\psi}{ds}+\frac{1}{s^2(1-s)^2}[A(1-s)^2-B(1-s)(1-2s)-m(x)(V(x)-E)]\psi=0. \quad (39)$$

Putting Eq. 24 and Eq. 31 and expanding

$$\frac{d^2\psi}{ds^2}+\frac{2\eta-(2\eta+1)s}{s(1-s)}\frac{d\psi}{ds}+\frac{1}{s^2(1-s)^2}\Big[A-2As+As^2-B+3Bs-2Bs^2-\frac{1}{(1+qe^{-2\lambda x})}\Big(-V_0\frac{1}{1+qe^{-2\lambda x}}+V_1 tanh_q(\lambda x)-E\Big)\Big]\psi=0. \quad (40)$$

By rearranging the terms inside the brackets in Eq. 40, we obtain the generalized hypergeometric-type equation, which represents the parametric generalization of the NU approach.

$$\frac{d^2\psi}{ds^2}+\frac{2\eta-(2\eta+1)s}{s(1-s)}\frac{d\psi}{ds}+\frac{1}{s^2(1-s)^2}\Big[\Big(A-2B+\frac{V_0}{4q\lambda^2}-\frac{2V_1}{4\lambda^2}\Big)s^2+\Big(-2A+3B+\frac{V_0}{4q\lambda^2}-\frac{V_1}{4\lambda^2}+\frac{E}{4\lambda^2}\Big)s+(A-B)\Big]\psi(x)=0 \quad (41)$$

where,

$$\xi_1=-A+2B-\frac{V_0}{4q\lambda^2}+\frac{2V_1}{4\lambda^2}$$
$$\xi_2=-2A+3B+\frac{V_0}{4q\lambda^2}-\frac{V_1}{4\lambda^2}+\frac{E}{4\lambda^2} \quad (42)$$
$$\xi_3=-A+B$$

using these parameters, Eq. 41 becomes

$$\frac{d^2\psi}{ds^2}+\frac{2\eta-(2\eta+1)s}{s(1-s)}\frac{d\psi}{ds}+\frac{1}{s^2(1-s)^2}[-\xi_1 s^2+\xi_2 s-\xi_3]\psi(x)=0. \quad (43)$$

Comparing Eq. 43 with Eq. 2, we get

$$\tilde{\tau}(s)=2\eta-(2\eta+1)s,\quad \sigma(s)=s(1-s),\quad \tilde{\sigma}(s)=-\xi_1 s^2+\xi_2 s-\xi_3. \quad (44)$$

After using the above polynomials into Eq. 10, we get

$$\pi(s)=\left(\frac{1}{2}-\eta\right)(1-s)\pm\begin{cases}(\nu-\varepsilon)s+\varepsilon\ ; k=\xi_2-2\xi_3+2\nu\varepsilon\\(\nu+\varepsilon)s-\varepsilon\ ;\ k=\xi_2-2\xi_3-2\nu\varepsilon\end{cases} \quad (45)$$

where,

$$\nu^2=\frac{V_1}{4\lambda^2}+\frac{E}{4\lambda^2} \quad (46)$$

and

$$\varepsilon^2=-\frac{1}{2}\left(\beta+\frac{1}{2}\right)-\alpha(\alpha+\beta+1) \quad (47)$$

we take

$$k=\xi_2-2\xi_3-2\nu\varepsilon \quad (48)$$

and using

$$\pi(s)=\left(\frac{1}{2}-\eta\right)(1-s)-(\nu+\varepsilon)s+\varepsilon. \quad (49)$$

Now, calculating the polynomial $\tau(s)$ from Eq. 45 by using Eq. 5

$$\tau(s)=2\left(\frac{1}{2}-\eta\right)(1-s)-2((\nu+\varepsilon)s+\varepsilon)+2\eta-(2\eta+1)s \quad (50)$$

expanding the above equation

$$\tau(s)=1-s-2\eta+2\eta s-2\big((\nu+\varepsilon)s-\varepsilon\big)+2\eta-2\eta s-s \quad (51)$$

we get

$$\tau(s)=1-2s-2\big((\nu+\varepsilon)s-\varepsilon\big), \quad (52)$$

and its derivative

$$\tau'(s)=-(2+2\nu+2\varepsilon). \quad (53)$$

Now, from Eqs. 11 and 12

$$\lambda = \lambda_n = -n\,(-2 - 2(\nu + \varepsilon)) + n(n-1) \quad (54)$$

and

$$\lambda = \xi_1 - (\nu + \varepsilon)^2 - (\nu + \varepsilon) + \eta^2 - \frac{1}{4} \quad (55)$$

after comparing Eq. 54 with Eq. 55, we have

$$(\nu + \varepsilon) = -\left(n + \frac{1}{2}\right) \pm \sqrt{\xi_1 + \eta^2}. \quad (56)$$

Now using Eqs. 42, 46, and 47, then two energy levels become

$$E_n = -4\lambda^2 \left[\left(n + \frac{1}{2}\right) \pm \sqrt{\frac{2qV_1 - V_0}{4q\lambda^2} - \alpha(\alpha + \beta + 1)} - \sqrt{-\frac{1}{2}\left(\beta + \frac{1}{2}\right) - \alpha(\alpha + \beta + 1)}\right]^2, (0 \le n < \infty) \quad (57)$$

Table 1 and Table 2 represent the numerical results for the values of the energy levels at different values of the parameter λ.

Table 1: Numerical outcomes of energy spectra for $q = 0.5$, $\alpha = 0$, $\beta = -1$, $V_0 = 50$, and $V_1 = 100$

Energy	E_0	E_1	E_2	E_3
$\lambda = 1$	−100	−144	−196	−256
$\lambda = 2$	−100	−196	−324	−484
$\lambda = 3$	−100	−256	−484	−784
$\lambda = 4$	−100	−324	−676	−1156

Table 2: Numerical outcomes of energy spectra for $q = 1$, $\alpha = 0$, $\beta = -1$, $V_0 = 50$, and $V_1 = 100$

Energy	E_0	E_1	E_2	E_3
$\lambda = 1$	−150	−202.989	−263.979	−332.969
$\lambda = 2$	−150	−263.979	−409.959	−587.938
$\lambda = 3$	−150	−332.969	−587.938	−914.908
$\lambda = 4$	−150	−409.959	−797.91	−1313.877

To find $\rho(s)$ the weight function, we use Eqs. 8, 44, and 52, so we get.

$$\frac{d}{ds}[(s - s^2)\rho(s)] = [1 - 2s - 2((\nu + \varepsilon)s - \varepsilon)]\,\rho(s) \quad (58)$$

expanding the left-hand term

$$(1 - 2s)\rho(s) + (s - s^2)\rho'(s) = \rho(s) - [2s - 2((\nu + \varepsilon)s - \varepsilon)]\rho(s) \quad (59)$$

thus

$$(s - s^2)\rho'(s) + [2((\nu + \varepsilon)s - 2\varepsilon)]\rho(s) = 0 \quad (60)$$

and then

$$\rho'(s) + \frac{[2((\nu+\varepsilon)s - 2\varepsilon)]}{(s - s^2)}\rho(s) = 0. \quad (61)$$

The above equation is a first-order differential equation; after solving it, we get the weight function.

$$\rho(s) = s^{2\varepsilon}(1 - s)^{2\nu}. \quad (62)$$

Using the weight function yields in Eq. 62, we defined the solution of y from Eq. 7

$$y_n(s) = \frac{C_n}{s^{2\varepsilon}(1-s)^{2\nu}}\frac{d^n}{ds^n}[(s(1-s))^n s^{2\varepsilon}(1-s)^{2\nu}] \quad (63)$$

thus

$$y_n(s) = C_n s^{-2\varepsilon}(1-s)^{-2\nu}\frac{d^n}{ds^n}[s^{n+2\varepsilon}(1-s)^{n+2\nu}] \quad (64)$$

and from Eqs. 44, 45, and 62, we get

$$\varphi(s) = s^{\left(\frac{1}{2}-\eta\right)+\varepsilon}(1-s)^{\nu}. \quad (65)$$

By using the Jacobi polynomial's properties (Von Roos, 1983)

$$P_n^{(\zeta,\xi)}(x) = \frac{(-1)^n(1-x)^{-\zeta}(1+x)^{-\xi}}{2^n n!}\frac{d^n}{dx^n}[(1-x)^{n+\zeta}(1+x)^{n+\xi}] \quad (66)$$

and

$$P_n^{(2\zeta,2\xi)}(1-2s) = \frac{(-2)^n(s)^{-2\zeta}(1-s)^{-2\xi}}{2^n n!}\frac{d^n}{dx^n}[s^{n+2\zeta}(1-s)^{n+2\xi}] \quad (67)$$

where, $P_n^{(a,b)}(x), (a > -1, b > -1)$ represents the Jacobi polynomial. Then we have

$$y_n(s) = P_n^{(2\varepsilon,2\nu)}(1-2s). \quad (68)$$

The wave functions $\psi_n(s)$ are obtained from Eqs. 3, 7, 62, and 65 has the following form

$$\psi_n(s) = N_n\, s^{\left(\frac{1}{2}-\eta\right)+\varepsilon}(1-s)^{\nu}P_n^{(2\varepsilon,2\nu)}(1-2s). \quad (69)$$

The normalization constant is denoted by N_n.One can find it by examining the normalizing condition.

$$\int_{-\infty}^{\infty}|\psi_n(x)|^2\,dx = 1 = \int_0^1|\psi_n(s)|^2\,ds \quad (70)$$

5. Results and discussion

The theoretical framework for analyzing quantum systems with complex potentials is extended by deriving the position-dependent mass Schrödinger equation and solving it using the Nikiforov-Uvarov method. This framework makes it possible to study systems with non-standard potentials and position-dependent masses, which are important in many branches of physics. The solutions that were obtained are represented in Figs. 3 and 4 provide a physical description of the particle's quantum behavior when the q-deformed Woods-Saxon plus hyperbolic tangent potential is present. The system's fundamental features and behavior are clarified by the energy eigenvalues and wave functions, which provide insights into the bound states and scattering aspects of the system. Potential uses for the findings in nuclear and atomic physics, as well as other areas of theoretical physics, are indicated by the paper's results. Understanding the behavior of particles in complex potentials advances understanding in these domains and could have consequences for practical quantum technology applications. This research provides new opportunities for investigating quantum systems with non-standard potentials and position-dependent masses. To improve prediction accuracy and expand the reach of the theoretical framework, future research could investigate extending the existing model to higher dimensions, different kinds of potentials, or include more quantum effects.

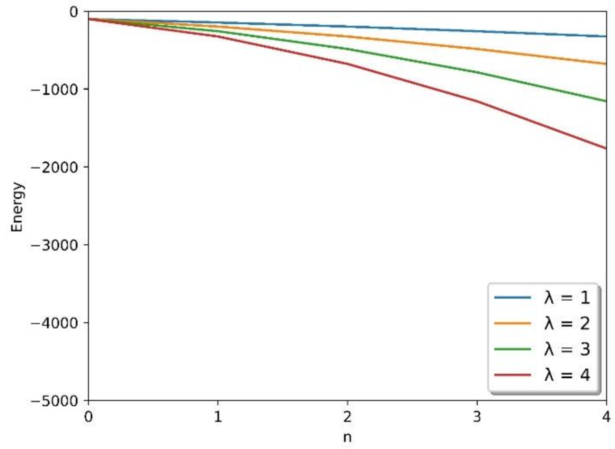

Fig. 3: Woods-Saxon plus hyperbolic tangent energy for $q = 0.5, \alpha = 0, \beta = -1, V_0 = 50$, and $V_1 = 100$

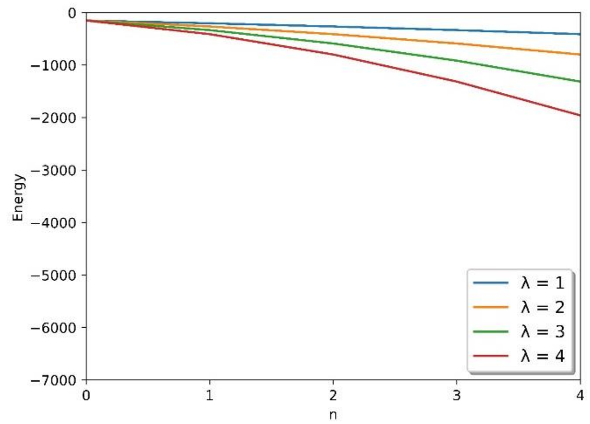

Fig. 4: Woods-Saxon plus hyperbolic tangent energy for $q = 1, \alpha = 0, \beta = -1, V_0 = 50$, and $V_1 = 100$

6. Conclusions

The solution of the mass variable Schrödinger equation with a Woods-Saxon plus hyperbolic tangent potential is obtained in this study by applying the Nikiforov-Uvarov method. The Jacobi polynomials are used to represent the wave function, and the energy eigenvalues are determined analytically. One of the toy models that can be used in various branches of physics, including quantum field theories and solid-states physics, is the hyperbolic tangent potential.

Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

References

Abadi VMM, Ranjbar AH, Mohammadi J, and Kharame RK (2019). Numerical solution of the Schrodinger equation for types of Woods-Saxon potential. Arxiv Preprint Arxiv:1910.03808. https://doi.org/10.48550/arXiv.1910.03808

Al-Hawamdeh MA, Akour AN, Jaradat EK, and Jaradat OK (2023). Involving Nikiforov-Uvarov method in Schrodinger equation obtaining Hartmann potential. East European Journal of Physics, 2: 117-123. https://doi.org/10.26565/2312-4334-2023-2-10

Arda A and Sever R (2009). Approximate ℓ-state solutions to the Klein–Gordon equation for modified Woods–Saxon potential with position dependent mass. International Journal of Modern Physics A, 24: 3985-3994. https://doi.org/10.1142/S0217751X0904600X

Arda A, Aydoğdu O, and Sever R (2010). Scattering of the Woods–Saxon potential in the Schrödinger equation. Journal of Physics A: Mathematical and Theoretical, 43: 425204. https://doi.org/10.1088/1751-8113/43/42/425204

Badalov VH, Ahmadov HI, and Badalov SV (2010). Any l-state analytical solutions of the Klein–Gordon equation for the Woods–Saxon potential. International Journal of Modern Physics E, 19: 1463-1475. https://doi.org/10.1142/S0218301310015862

Berkdemir A, Berkdemir C, and Sever R (2006). Eigenvalues and eigenfunctions of Woods–Saxon potential in PT-symmetric quantum mechanics. Modern Physics Letters A, 21: 2087-2097. https://doi.org/10.1142/S0217732306019906

Bespalova OV, Romanovsky EA, and Spasskaya TI (2003). Nucleon–nucleus real potential of Woods–Saxon shape between– 60 and +60 MeV for the 40⩽ A⩽ 208 nuclei. Journal of Physics G: Nuclear and Particle Physics, 29: 1193. https://doi.org/10.1088/0954-3899/29/6/318

Chabab M, Lahbas A, and Oulne M (2012). Analytic l-state solutions of the Klein-Gordon equation for q-deformed Woods-Saxon plus generalized ring shape potential. Arxiv Preprint Arxiv:1203.5039. https://arxiv.org/abs/1203.5039

Dudek J, Pomorski K, Schunck N, and Dubray N (2003). Hyperdeformed and megadeformed nuclei: Lessons from the slow progress and emerging new strategies. The European Physical Journal A-Hadrons and Nuclei, 20: 15-29. https://doi.org/10.1140/epja/i2002-10313-4

Erkol H and Demiralp E (2007). The Woods–Saxon potential with point interactions. Physics Letters A, 365: 55-63. https://doi.org/10.1016/j.physleta.2006.12.050

Falaye BJ, Oyewumi KJ, and Abbas M (2013). Exact solution of Schrödinger equation with q-deformed quantum potentials using Nikiforov—Uvarov method. Chinese Physics B, 22: 110301. https://doi.org/10.1088/1674-1056/22/11/110301

Goldberg VZ, Chubarian GG, Tabacaru G, Trache L, Tribble RE, Aprahamian A, Rogachev GV, Skorodumov BB, and Tang XD (2004). Low-lying levels in F 15 and the shell model potential for drip-line nuclei. Physical Review C, 69: 031302. https://doi.org/10.1103/PhysRevC.69.031302

Gu Y, Chen B, Ye F, and Aminakbari N (2022). Soliton solutions of nonlinear Schrödinger equation with the variable coefficients under the influence of Woods–Saxon potential. Results in Physics, 42: 105979. https://doi.org/10.1016/j.rinp.2022.105979

Guo JY and Sheng ZQ (2005). Solution of the Dirac equation for the Woods–Saxon potential with spin and pseudospin symmetry. Physics Letters A, 338: 90-96. https://doi.org/10.1016/j.physleta.2005.02.026

Hagino K and Tanimura Y (2010). Iterative solution of a Dirac equation with an inverse Hamiltonian method. Physical Review C, 82(5): 057301. https://doi.org/10.1103/PhysRevC.82.057301

Ikhdair S and Sever R (2010). Approximate analytical solutions of the generalized Woods-Saxon potentials including the spin-orbit coupling term and spin symmetry. Open Physics, 8(4): 652-666. https://doi.org/10.2478/s11534-009-0118-5

Ikhdair SM and Sever R (2007). Exact solution of the Klein-Gordon equation for the PT-symmetric generalized Woods-Saxon potential by the Nikiforov-Uvarov method. Annalen der Physik, 519(3): 218-232. https://doi.org/10.1002/andp.20075190303

Ikhdair SM and Sever R (2008). Solutions of Dirac equation for symmetric generalized Woods-Saxon potential by the hypergeometric method. Arxiv Preprint Arxiv:0808.1002. https://arxiv.org/abs/0808.1002

Ikot AN, Hassanabadi H, and Abbey TM (2015). Spin and pseudospin symmetries of Hellmann potential with three tensor interactions using Nikiforov–Uvarov method. Communications in Theoretical Physics, 64(6): 637–643. https://doi.org/10.1088/0253-6102/64/6/637

Jaradat EK, Tarawneh SR, Akour NA, and Jaradat OK (2019). Demonstrating Shrodenger equation involving harmonic oscillator potential with a position dependent mass in an external electric field. Advanced Physics Research, 6(1): 15-28. https://doi.org/10.62476/apr61.28

Khounfais K, Boudjedaa T, and Chetouani L (2004). Scattering matrix for Feshbach-Villars equation for spin 0 and 1/2: Woods-Saxon potential. Czechoslovak Journal of Physics, 54: 697-710. https://doi.org/10.1023/B:CJOP.0000038524.36986.19

Nikiforov AF and Uvarov VB (1988). Special functions of mathematical physics. Volume 205, Birkhäuser, Basel, Switzerland. https://doi.org/10.1007/978-1-4757-1595-8

Okon IB, Popoola O, and Isonguyo CN (2014). Exact bound state solution of q-deformed Woods-Saxon plus modified coulomb potential using conventional Nikiforov-Uvarov method. International Journal of Recent Advances in Physics, 3(4): 29-38. https://doi.org/10.14810/ijrap.2014.3402

Panella O, Biondini S, and Arda ALTU Ğ (2010). New exact solution of the one-dimensional Dirac equation for the Woods–Saxon potential within the effective mass case. Journal of Physics A: Mathematical and Theoretical, 43(32): 325302. https://doi.org/10.1088/1751-8113/43/32/325302

Rojas C and Villalba VM (2005). Scattering of a Klein-Gordon particle by a Woods-Saxon potential. Physical Review A, 71(5): 052101. https://doi.org/10.1103/PhysRevA.71.052101

Romaniega C, Gadella M, Id Betan RM, and Nieto LM (2020). An approximation to the Woods–Saxon potential based on a contact interaction. The European Physical Journal Plus, 135(4): 1-27. https://doi.org/10.1140/epjp/s13360-020-00388-7

Sadeghi J and Pahlavani MR (2004). The hierachy of Hamiltonian for spherical Woods-Saxon potential. African Journal of Mathematical Physics, 1(2): 195-199.

Tezcan C, Sever R, and Yeşiltaş Ö (2008). A new approach to the exact solutions of the effective mass Schrödinger equation. International Journal of Theoretical Physics, 47: 1713-1721. https://doi.org/10.1007/s10773-007-9613-x

Von Roos O (1983). Position-dependent effective masses in semiconductor theory. Physical Review B, 27(12): 7547–7552. https://doi.org/10.1103/PhysRevB.27.7547

Wang N and Scheid W (2008). Quasi-elastic scattering and fusion with a modified Woods-Saxon potential. Physical Review C, 78(1): 014607. https://doi.org/10.1103/PhysRevC.78.014607

Yazdankish E (2021). Bound state solution of the Schrodinger equation for the Woods–Saxon potential plus coulomb interaction by Nikiforov–Uvarov and supersymmetric quantum mechanics methods. International Journal of Modern Physics E, 30: 2150023. https://doi.org/10.1142/S0218301321500233