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Strongly D_α^p −closed graphs in bitopological spaces

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ABSTRACT

In this paper, we introduce the concepts of D_α^p −open, D_α^p −closed subsets, pairwise−α −closed, pairwise−g −closed subsets, pairwise−strongly α −closed graph $G(f)$ and strongly D_α^p −closed graph of bitopological spaces. We showed that each closed graph is D_α^p −closed. In addition, the concepts of D_α^p −continuous, open, and closed functions are defined, and the relations between $\tau_p - \alpha, \tau_p - g$, and D_α^p −continuous functions are clarified. The fact that strongly D_α^p −closed graph is D_α^p −closed is illustrated. We studied when the graph $G(f)$ is p −strongly closed and $p - D_\alpha$ −closed subsets of the bitopological space (X, τ_1, τ_2). Moreover, the notions of D_α^i −interior of a subset of X and D_α^i −closure are defined.

1. Introduction

Several sorts of generalized open sets in topological spaces like semi-open, pre-open, and β −open sets were studied by lots of mathematicians (Al-Saadi and Al-Malki, 2024). Sarsak (2013, 2022) covered certain features of generalized open sets in generalized topological spaces (GTSs), which are an essential part of general topology. A key problem in real analysis and general topology is the study of variously modified versions of continuity, separation axioms, and other ideas utilizing extended open sets. The most well-known and inspiring ideas are those of α −open sets, introduced by Njåstad (1965), and generalized closed or (g −closed) subsets, introduced by Levine (1970). Both ideas have been thoroughly studied in the literature. Since then, many mathematicians have concentrated on generalizing many topological concepts through the usage of α −open sets and generalized closed sets.

Kelly (1963) published a paper named "Bitopological Spaces," which marked the beginning of the study of bitopological spaces. Since then, several articles have been submitted that attempt to extend topological concepts to bitopological ones. A non-empty set X with the two topologies τ_1 and τ_2 is called a bitopological space, as is the triple (X, τ_1, τ_2), or just X. The concept of generalized topological spaces was introduced by Cs'asz'ar in the 20th century, and other mathematicians worldwide have studied it. Consequently, mathematicians took a different tack and tried to apply several topological ideas to this new field.

Dunham (1982) defined a new topological space (X, τ^*) by using g −closed subsets of X to define a new closure operator. He did this by transferring regularity conditions from a topological space (X, τ) to separation conditions in the new topological space (X, τ^*). The concept of an operation on topological spaces was introduced, and α −closed graphs of an operation were introduced by Kasahara (1979). Ogata (1991) established the concept of τ_γ, which is the set of all γ −open sets, and introduced the operation α as γ −operation.

In section 2, we study D_α − Sets in bitopological space (X, τ_1, τ_2), features and properties such as the class of all D_α −open sets are bounded between g −open sets and the class of all α −open sets, introduce D_α^p −continuous function, and illustrate the relation between both of $\tau_p - \alpha$ −continuous and $\tau_p - g$ −continuous functions and D_α^p −continuous function.

2. D_α − Sets in bitopological space (X, τ_1, τ_2)

D_α −open sets are a novel class of sets that were developed and investigated in topological spaces by

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Sayed and Khalil (2006). Almuhur and Al-Labadi (2021) studied D_α −open and D_α −closed functions in bitopological spaces. They investigated whether subsets of the bitopological space (X,τ_1,τ_2) are pairwise−D_α −closed and when the graph $G(f)$ is pairwise−strongly closed. Furthermore, they define D_α^i −closure and D_α^i −interior of subsets of X.

The class of all D_α −open sets is bounded between g −open sets and the class of all α −open sets. In addition, they presented and examined D_α −continuous, D_α −open, and D_α −closed functions between topological spaces as applications. A subset U of a bitopological space (X,τ_1,τ_2) is called $\tau_1 - g$ −closed (Almuhur et al., 2023) if $Cl_{\tau_1}(U) \subset O_{\tau_2}(U) \subset \tau_2$ and U is $\tau_2 - g$ −closed if $Cl_{\tau_2}(U) \subset O_{\tau_1}(U) \subset \tau_1$.

If U is $\tau_1 - g$ −closed and $\tau_2 - g$ −closed, then it will be a pairwise−g −closed subset and hence, $X - U$ is pairwise−g −open.

Theorem 2.1: In a bitopological space (X,τ_1,τ_2), if $\{F_i: i \in \mathbb{N}\}$ is a family of $\tau_p - g$ −closed sets, then $\bigcup_{i\in\mathbb{N}} F_i$ is τ_p −closed (Sarsak, 2013; 2022).

Definition 2.2: For the bitopological space (X,τ_1,τ_2), if $U \subseteq X$, then:

(i) If $U \subset int\ (cl\ (int\ (U))$, then U is a $\tau_p - \alpha$ −open subset of X.
(ii) If $U \subset cl\ (int(cl(U)))$, then U is $\tau_p - \alpha$ −closed.
(iii) If $cl\ (U) \subset V$ for some τ_p −open ($\tau_p - g$ −closed) subset V of (X,τ_i), V is τ_p −generalized closed ($\tau_p - g$ −closed)
(iv) $GO(X) = \{U: U\ is\ \tau_p - g - open\}$
(v) $GC(X) = \{F: F\ is\ \tau_p - g - closed\}$
(vi) If $\tilde{U} = \cap \{U: U\ is\ \tau_p - \alpha - open, U \subseteq O\}$ for some O a $\tau_p - \alpha - open$ in (X,τ_i), then $\tilde{U} = int_{\alpha_p}(U)$.
(vii) If $\tilde{F} = \cap \{F: F\ is\ \tau_p - \alpha - closed, K \subset F\}$ for some K a $\tau_p - \alpha - closed$ in (X,τ_i), then $\tilde{F} = cl_{\alpha_p}(K)$.

Definition 2.3: If f is a function from (X,τ_1,τ_2) to (Y,σ_1,σ_2), then:

i) The graph of f (denoted by $G(f)$) is the subspace $\{(x,f(x)): x \in X\}\ of\ X \times Y$
ii) The function f is pairwise−closed if $G(f)$ is a (τ_i,σ_i) −closed subset of $(X,\tau_i) \times (Y,\sigma_i)\ \forall i = 1,2$.
iii) The function f is pairwise−strongly closed (pairwise−strongly α −closed) graph if $\forall\,(x,y) \in G(f)$ $(\forall\,(x,y) \in X \times Y - G(f)), \exists U_1$ and U_2 such that $x \in U_1$ and $\left(U_1 \times cl_j(U_2)\right) \cap G(f)$ is empty for some U_1 a τ_p −open subset of X and U_2 a σ_p −open subset of Y.

Definition 2.4: If F is a subset of (X,τ_1,τ_2), then F is D_α^p −closed if $cl^*\left(int\left(cl(F)\right)\right) \subseteq F$.

If F is $\tau_p - D_\alpha$ −closed, then $X - F$ is $\tau_p - D_\alpha$ −open. F is D_α^p −closed if it is $\tau_p - D_\alpha$ −closed, and the set of all D_α^p −closed subsets is denoted by $D_\alpha^p - C(X)$.

Theorem 2.5: The graph $G(f)$ is pairwise−strongly α −closed if and only if $\forall (a,b) \in X \times Y - G(f)$ for some U_1 a τ_p −open subset of X and U_2 a σ_p −open subset of Y containing a and b, respectively such that $f(U_1) \cap cl(U_2)$ is empty.

Theorem 2.6: If K is a subset of (X,τ_1,τ_2), then K is D_α^p −closed if it is $\tau_p - \alpha$ −closed.

Proof: Suppose that K is $\tau_p - \alpha$ −closed subset of X, then $cl^*\,(K) \subset cl(K)$.
So, $cl(int(cl(K))) \subset K$, hence $cl^*\left(int\left(cl(K)\right)\right) \subset cl^*\left(int\left(cl^*\,(K)\right)\right) \subset K$.

Thus, F is $\tau_p - D_\alpha$ −closed and so it is D_α^p −closed.

Theorem 2.7: If A is a D_α^p −closed subset of (X,τ_1,τ_2), then it is pairwise−g −closed.

Proof: Suppose that A is $\tau_p - g$ −closed, then, $cl^*(A) = A$. Hence, $int\left(cl^*(A)\right) \subset cl^*(A)$. Thus, $cl^*\left(int\left(cl^*(A)\right)\right) \subset cl^*(cl^*(A) \subset cl^*(A) = A$. Therefore, A is D_α^p −closed.

Corollary 2.8: If K is a pairwise−g −closed subset of (X,τ_1,τ_2) such that $int(cl^*(K)) \subset F \subset K$ for some $F \subset X$, then K is D_α^p −closed.

Proof: Since K is pairwise−closed subset of $X, cl^*(K) = K$. So, $cl^*\left(int\left(cl^*(F)\right)\right) \subset cl^*\left(int^*(F)\right) \subset K$ for some $F \subset X$. Thus, K is D_α^p −closed.

Theorem 2.9: Arbitrary intersection of D_α^p −closed sets is D_α^p −closed.

Proof: Let $\tilde{F} = \{F_\gamma: \gamma \in \Gamma\}$ be a family of D_α^p −closed subsets of the topological space (X,τ^1,τ^2), then, $cl^*\left(int\left(cl^*(F_\gamma)\right)\right) \subset F_\gamma\ \forall\gamma \in \Gamma$. Now, $\cap_{\gamma\in\Gamma} F_\gamma \subset \gamma \in \Gamma$ $\forall\gamma \in \Gamma$. Hence, $cl^*\left(int\left(cl^*(F_\gamma)\right)\right) \subset \cap_{\gamma\in\Gamma} cl(F_\gamma)$. Thus, $cl^*\left(int\left(cl^*(F_\gamma)\right)\right) \subset \cap_{\gamma\in\Gamma} cl(F_\gamma) \subset cl^*\left(int\left(cl^*(F_\gamma)\right)\right) \subset \cap_{\gamma\in\Gamma} cl(F_\gamma)\ \ \forall\gamma \in \Gamma$. Therefore, $\cap_{\gamma\in\Gamma} F_\gamma$ is D_α^p −closed.

Theorem 2.10: If A_1 and A_2 are two subsets of (X,τ_1,τ_2) such that A_1 is D_α^p −closed and A_2 is pairwise−α −closed, then $A_1 \cap A_2$ is D_α^p −closed.

Corollary 2.11: If B_1 and B_2 are two subsets of (X,τ_1,τ_2) such that B_1 is D_α^p −closed and B_2 is pairwise−g −closed, then $F_1 \cap F_2$ is D_α^p −closed.

Lemma 2.12: In the bitopological space (X,τ_1,τ_2), if A is a subset of X, then:

(i) $cl(A) = X - int(X - A)$.

(ii) $int(A) = X - cl(X - A)$.

Theorem 2.13: In (X, τ_1, τ_2), a subset U is D_α^p −open if and only if $U \subset int(cl^*(int(U)))$.

Proof: Let U be a D_α^p −open subset of X , then, $X - U$ is D_α^p −closed and $cl^*(int(cl^*(U)) \subset X - U$. Hence, we have $U \subset int^*\left(cl(int^*(U))\right)$. Now, if $U \subset int^*\left(cl(int^*(U))\right)$. Then, $X - int^*\left(cl(int^*(U))\right) \subset X - U$. Therefore, $int^*\left(cl(int^*(U))\right) \subset X - U$. Thus, $X - U$ is D_α^p −closed and U is D_α^p −open.

Corollary 2.13: In the bitopological space (X, τ_1, τ_2), a subset U is D_α^p −open if $\exists$ W a pairwise−g −open subset such that $W \subset U \subset int^*(cl(W))$, then U is D_α^p −open.

Proof: Let W be a pairwise−g −open subset of X, hence $X - W$ is pairwise−g −closed and $X - int^*\left(cl_j(X - W)\right) \subset X - U \subset X - W$. So, $cl^*(int(X - W)) \subset X - U \subset X - W$ and $X - U$ is D_α^p −closed. Therefore, U is D_α^p −open.

Corollary 2.14: Every pairwise−α −open (pairwise−g −open) is D_α^p −open.

Corollary 2.15: An arbitrary union of the D_α^p −open set is D_α^p −open.

Corollary 2.16: The union of D_α^p −open set and pairwise−α −open set is D_α^p −open.

Corollary 2.17: The union of the D_α^p −open set and the pairwise−g −open set is D_α^p −open.

Definition 2.18: In the bitopological space (X, τ_1, τ_2), the D_α^p −interior of a subset B of X is denoted by $D_\alpha^p - int_p(B)$ and $D_\alpha^p - int_p(B) = \cup_{\gamma \in \Gamma}\{V_\gamma : V_\gamma \in pairwise - D_\alpha O(X),\ V_\gamma \subset B\}$.

Definition 2.19: The D_α^p −closure of a subset B of the bitopological space (X, τ_1, τ_2) is denoted by $D_\alpha^p - cl(B)$ such that $D_\alpha^p - cl(B) = \cap_{\gamma \in \Gamma}\{K_\gamma : K_\gamma \in pairwise - D_\alpha C(X), B \subset K_\gamma\}$.

Lemma 2.20: In (X, τ_1, τ_2), if $B \subset X$, then $X - (D_\alpha^p - int(B)) = D_\alpha^p - cl(B)$ and $X - (D_\alpha^p - cl(B)) = D_\alpha^p - int(B)$.

Theorem 2.21: In (X, τ_1, τ_2), if U is a subset of X, then:

(i) $D_\alpha^p - int(\phi) = \phi$ and $D_\alpha^p - int(X) = X$.
(ii) U is $p - D_\alpha$ open iff $D_\alpha^p - int(U) = U$ and $D_\alpha^p - int(U) = U$.
(iii) $p - \alpha - int(U) \subset D_\alpha^p - int(U) \subset U$.
(iv) $int^*(U) \subset D_\alpha^p - int(U)$
(v) $D_\alpha^p - int\left(D_\alpha^p - int(U)\right) = D_\alpha^p - int(U)$.

Theorem 2.22: In (X, τ_1, τ_2), if A, B are two subsets of X, then:

(i) If $A \subset B$, then $D_\alpha^p - int(A) \subset D_\alpha^p - int(B)$
(ii) $(D_\alpha^p - int(A)) \cup (D_\alpha^p - int(B)) \subset D_\alpha^p - int(A \cup B)$
(iii) $D_\alpha^p - int\ (A \cap B) \subset (D_\alpha^p - int(A)) \cap (D_\alpha^p - int(B))$.

Theorem 2.23: In (X, τ_1, τ_2), if $U \subset X$, then:

(i) $D_\alpha^p - int(U) = U \cap int^*\left(cl(int^*(U))\right)$.
(ii) $D_\alpha^p - cl(U) = A \cup int^*\left(cl(int^*(A))\right)$.

Proof:
(i) $D_\alpha^p - int(U)$ is D_α^p −open and $D_\alpha^p - int(U) \subset U$. So, $D_\alpha^p - int(U) \subset int^*\left(cl(D_\alpha^p - int(U))\right) \subset int^*(cl(int^*(U))$.
$D_\alpha^p - int(U) \subset U \cup int^*\left(cl(int^*(U))\right)$. So, $U \cup int^*\left(cl(int^*(U))\right)$ is $D_\alpha^p -$ open $U \cup int^*\left(cl(int^*(U))\right) \subset D_\alpha^p - int(U)$. Thus, $D_\alpha^p - int(U) = U \cap int^*\left(cl(int^*(U))\right)$.
(ii) $D_\alpha^p - cl(U) = X - int(X - U) = X - (X - U) = X - (X - U) \cup (X - int^*\left(cl(int^*(X - U))\right)) = U \cup cl^*\left(int(cl^*(U))\right)$

Definition 2.24: In the bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2), the function $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be D_α^p −continuous if the inverse image of each σ_p −open set in Y is D_α^p −open in X.

Lemma 2.25: Each $\tau_p - \alpha$ −continuous function is D_α^p −continuous.

Lemma 2.26: Each $\tau_p - g$ −continuous function is D_α^p −continuous.

Theorem 2.27: If $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, and $\forall U \subset X, V \subset Y$, then the following are equivalent:

(1) f is $\tau_p - D_\alpha$ −continuous.
(2) For each V a τ_p −open subset of Y and $\forall x \in X$ such that $f(x) \in V$, $\exists$ U a $\tau_j - D_\alpha$ −open subset of X containing $f^{-1}(V)$ such that $f(U) \subset V$.
(3) The inverse image of a σ_p −closed subset of Y is τ_p −closed subset of X.
(4) $f(D_\alpha^p - int(U)) \subset cl(f(U))$
(5) $D_\alpha^p - cl(f^{-1}(V)) \subset f^{-1}(cl(V))$.
(6) $f^{-1}(int(V)) \subset D_\alpha^p - int(f^{-1}(V))$.

Proof: (1)→(2) $f^{-1}(V) \in D_\alpha^p O(X)\ \forall V \subseteq Y$. If $b \in f^{-1}(V)$, then $f(f^{-1}(V)) \subset V\ \forall b \in X$.
(3)→(4) Assume that K is a p −closed subset of Y and $K \subset f(U)$. Now, $U \subset f^{-1}(K)$ is $p - D_\alpha -$ closed subset of X. So, $D_\alpha^p - cl(U) \subset D_\alpha^p - cl\ (f^{-1}(K)) = f^{-1}(K)$. Hence, $f(D_\alpha^p - cl(U)) \subset K$. Thus, $f(D_\alpha^p - cl(f(K) \subset cl(f(U))$.

(4)→(5) Let F be a subset of Y. Then $f(D_\alpha^p - cl(f^{-1}(F)) \subset cl(f(f^{-1}(F)) \subset cl_i(F)$. Hence, $D_\alpha^p - cl((f^{-1}(F)) \subset clf^{-1}((F)$. Therefore, $D_\alpha^p - cl((f^{-1}(F)) \subset f^{-1}(cl(F))$.
(5)→(6) Assume that F be a subset of Y, then $D_\alpha^p - cl(f^{-1}(Y-F)) \subset f^{-1}(D_\alpha^p - cl\ (Y-F))$. Hence, $D_\alpha^p - cl\ (X - f^{-1}(F)) \subset f^{-1}(Y - int(F))$. Therefore, $X - D_\alpha^p - int\ (f^{-1}(F)) \subset X - f^{-1}(int(F))$. Hence, $f^{-1}(int(F)) \subset D_\alpha^p - int(f^{-1}(F))$.
(6)→(1) Assume that M be a p −open subset of Y. So $f^{-1}(int(M)) \subset D_\alpha^p - int(f^{-1}(M))$. So, $f^{-1}(int(M)) \subset D_\alpha^p - int(f^{-1}(M))$. Hence, $f^{-1}(M)$ is an $p - D_\alpha$ −open. Thus, f is a $p - D_\alpha$ −continuous function.

Theorem 2.28: The composition of D_α^p −continuous function and τ_p −continuous function is D_α^p −continuous.

Definition 2.29: The function $f:(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ has D_α^p −closed graph if $\forall(a,b) \in (X \times Y) - G(f)$, $\exists W_1 \in \tau_p - D_\alpha^p O(X,a)$ and $W_2 \in GO(Y,b)$: $(W_1 \times cl^*(W_2)) \cap G(f)$ is empty.

Lemma 2.30: Each closed graph is D_α^p −closed.

Theorem 2.31: The function $f:(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is $\tau_p - D_\alpha$ −closed graph if and only if $\forall(a,b) \in (X \times Y) - G(f)$, $\exists U_1 \in \tau_p - D_\alpha^p O(X,a)$ and $U_2 \in GO(Y,b)$: $(U_1 \times cl^*(U_2)) \cap G(f)$ is empty.

Proof: Assume that $f:(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is a D_α^p −closed graph. So, $\forall(a,b) \in (X \times Y) - G(f), \exists U_1 \in \tau_p - D_\alpha^p O(X,a)$ and $U_2 \in GO(Y,b)$ such that $(U \times cl^*(U_2)) \cap G(f)$ is empty. Hence, $f_p(x) \in f_p(U_1)$ and $b \in cl^*(U_2)$. Now, $b \neq f_i(a)$, hence $f_i(U_1) \cap cl^*(U_2)$ is empty. Conversely, assume that $(a,b) \in (X \times Y) - G(f)$, $\exists U_1 \in \tau_p - D_\alpha^p O(X,a)$ and $U_2 \in GO(Y,b)$: $(U \times cl^*(U_2)) \cap G(f)$ is empty. Thus, $f_p(a) \neq b$ and $f_p(U_1) \cap cl^*(U_2)$ is empty.

Theorem 2.32: The function $f:(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is $\tau_p - D_\alpha$ −closed graph if $\forall(a,b) \in (X \times Y) - G(f)$, $\exists U_1 \in \tau_p - D_\alpha^p O(X,a)$ and $U_2 \in \tau_p - D_\alpha(Y,b)$: $(\ U_1 \times cl^*(U_2) \cap G(f)$ is empty.

Proof: Assume that f is a D_α^p −closed graph, then $\forall(a,b) \in (X \times Y) - G(f)$, $\exists U_1 \in \tau_p - D_\alpha^p O(X,a)$ and $U_2 \in GO(Y,b)$. But $\tau_p - g$ −subset of X is $\tau_p - D_\alpha$ −open, then $D_\alpha^p - cl(U_2) \subset cl(U_2)$. Thus, $(U_1 \times D_\alpha^p - cl(U_2)) \cap G(f)$ is empty.

Corollary 2.33: The function $f(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is D_α^p −closed graph $\forall(a,b) \in (X \times Y) - G(f), \exists W_1 \in D_\alpha^p O(X,a)$ and $W_2 \in D_\alpha\ \ (Y,b) \cap W_1 \times D_\alpha^p - cl(W_2)) \cap G(f)$ is empty if $\forall(a,b) \in (X \times Y) - G(f), \exists U \in \tau_p - D_\alpha^p O(X,a)$ and $V \in \tau_p - D_\alpha - (Y,b)$: $f_p(U) \cap D_\alpha^p - cl(V)$ is empty.

Definition 2.34: The function $f:(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ has a strongly D_α^p −closed graph if $\forall(a,b) \in (X \times Y) - G(f)$, $\exists W_1 \in D_\alpha^p O(X,a)$ and $W_2 \in O(Y,b)$: $(W_1 \times cl(W_2)) \cap G(f)$ is empty.

Lemma 2.35: (i) The strongly D_α^p −closed graph is D_α^p −closed in (X,τ_1,τ_2).
(ii) Each strongly $\tau_p - \alpha$ −closed graph is strongly D_α^p −closed graph in (X,τ_1,τ_2).

Theorem 2.36: For the function $f:(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$, the following are equivalent:

(i) f has a strongly D_α^p −closed graph.
(ii) $\forall(a,b) \in (X \times Y) - G(f)$, $\exists W_1 \in D_\alpha^p O(X,a)$ and $W_2 \in O(Y,b)$: $f(W_1) \cap cl(W_2)$ is empty.
(iii) $\forall(a,b) \in (X \times Y) - G(f), \exists W_1 \in D_\alpha^p O(X,a)$ and $W_2 \in O(Y,b)$: $(W_1 \times cl(W_2)) \cap G(f)$ is empty.

Corollary 2.37: If the function $f:(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ has a strongly D_α^p −closed graph, then $\forall a \in X, f(x) = \cap \{cl(V): V \in D_\alpha^p O(X, xa\}$.

Proof: Assume that $f:(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is a strongly D_α^p −closed graph, then $\exists b \neq f(a): b \in \cap \{cl(V): V \in D_\alpha^p O(X,x)\}$.Hence, $b \in cl(f(V))$ for some $V \in D_\alpha^p O(X,x)$. So, $\forall W \in \alpha - O(Y,b)$, $W \cap f(V)$ is empty. Therefore, $f(V)$ is non-empty and $f(V) \subset W \subset cl_\alpha^p(W)$ which is a contradiction since f has a strongly D_α^p −closed graph. Consequently, $a \in X$, $f(a) = \cap \{cl(V): V \in D_\alpha^p O(X,a)\}$.

Corollary 2.38: If $f:(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is a D_α^p −continuous function, and Y is p −Housdorff, hence $G(f)$ is strongly D_α^p −closed.

Proof: Assume that $(a,b) \in (X \times Y) - G(f)$. Now, because Y is p −Housdorff, $\exists U \in O(Y,b)$: $f(a) \notin cl(U)$. Since $cl(U)$ is τ_p −closed, we have $Y - cl(U) \in O(Y,b)$. Therefore, $\exists W \in D_\alpha^p O(X, xa: f(W) \subset Y - cl(U)$. Thus, $f(W) \cap cl(U)$ is empty. Consequently, $G(f)$ has a strongly D_α^p −closed graph.

3. Conclusion

Every closed graph is D_α^p −closed and the $\tau_p - \alpha$ −closed subset is D_α^p −closed. The strongly D_α^p −closed graph is D_α^p −closed. The composition of D_α^p −continuous function and τ_p −continuous function is D_α^p −continuous. Moreover, each $\tau_p - g$ −continuous function is D_α^p −continuous.

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Compliance with ethical standards

Conflict of interest

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