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Optimizing faculty resource allocation in higher education: A mathematical model for strategic planning

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ABSTRACT

This paper introduces a new mathematical model aimed at improving how faculty resources are allocated in higher education institutions. The model takes into account the complexities of student enrollment, teaching quality, and program offerings. It provides a structured method to estimate faculty needs, including both Ph.D. holders teaching core courses and teaching assistants or lecturers managing practical courses, labs, and related tasks. By considering factors such as class sizes, faculty workloads, and student enrollment patterns, the model offers useful insights for academic planning. Two case studies from Hafr Al Batin University illustrate the model's practical value and flexibility. These examples show how the model can support informed decision-making, helping to maintain a balance between student numbers and teaching quality. The paper concludes by highlighting the importance of data-driven planning in the changing field of higher education, suggesting future research directions, and positioning the model as a key tool for improving the management and administration of higher education. The main goal is to ensure a high-quality education for students while making efficient use of resources.

1. Introduction

Higher education institutions face a myriad of challenges in today's dynamic academic landscape. The pervasive deficiency in academic staff constitutes a pivotal challenge confronted by universities. This scarcity bears significant ramifications for pedagogical endeavors, scholarly investigations, and the intricacies of academic oversight, exerting a discernible influence on the caliber of educational quality (Naidoo-Chetty and Du Plessis, 2021; Dlamini and Dlamini, 2024; Leal Filho et al., 2024). Among these challenges, the strategic allocation of faculty resources stands out as a critical factor in ensuring the quality of education. The management of faculty resources in higher education institutions is a multifaceted undertaking that requires a profound understanding of diverse variables and an adept strategy to ensure an optimal balance between student enrollment and instructional quality (Watermeyer et al., 2024).

Historically, the allocation of faculty resources was often driven by tradition, intuition, or trial and error. However, the increasing complexity of academic programs, coupled with fluctuating student enrollment, necessitates a more systematic and evidence-based approach. Moreover, as higher education institutions strive to maintain accreditation standards and address the diverse needs of learners, the precision in faculty resource allocation becomes even more paramount. Institutions of higher learning operate within a dynamic ecosystem influenced by demographic shifts, economic trends, technological advancements, and the evolving needs of the job market. Consequently, the effective allocation of faculty members to teaching responsibilities becomes pivotal in ensuring that the quality of education remains uncompromised (Szromek and Wolniak, 2020). Calculating faculty needs for recruitment in university departments is a complex process that involves considering various factors to ensure an optimal balance between instructional demands, research activities, and the overall goals of the

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department and institution. The specifics of the process can vary based on the university's policies, department size, academic disciplines, and other contextual factors (Petersen et al., 2023). However, here are some common steps and considerations that departments typically take into account when calculating their faculty needs:

1. Enrollment and student demand: Departments often start by analyzing the expected student enrollment and demand for courses within their academic programs. This involves forecasting the number of students who will be taking courses and majoring in the department's disciplines over the coming years (Pavlov and Katsamakas, 2020).
2. Curriculum and course offerings: Departments need to evaluate the curriculum and course offerings. Different courses may require different levels of faculty involvement, and departments need to ensure they have sufficient faculty members to teach a variety of courses, from introductory to advanced levels (Bousnguar et al., 2022).
3. Class sizes and student-faculty ratios: Departments consider the ideal class sizes and student-faculty ratios to maintain a quality learning experience. Larger classes might require more faculty members or teaching assistants to provide effective instruction and individualized attention (Jaeger and Eagan Jr, 2009).
4. Faculty workload and course distribution: Departments assess the distribution of courses and the workload of existing faculty members. They consider factors like the number of courses each faculty member is already teaching, their research commitments, administrative duties, and other responsibilities (Gappa et al., 2007).
5. Research and scholarly activities: Academic departments often need to consider faculty members' research and scholarly activities. Hiring decisions need to align with the department's research goals and the need for faculty members who can contribute to the academic reputation of the institution (Bland et al., 2005).
6. Emerging fields and specializations: Departments may assess emerging fields, interdisciplinary studies, and emerging areas of research. This may lead to the identification of new faculty positions needed to support these specialized areas (Rhoades, 2001).
7. Accreditation and program requirements: Some academic programs have specific accreditation requirements that dictate faculty-student ratios or qualifications. Departments need to ensure that they meet these standards (Ewell, 2009).
8. Faculty development and succession planning: Departments may also consider faculty development and succession planning. Retirement or faculty departures could open positions that need to be filled with new hires (Sorcinelli and Austin, 1992).
9. Funding and budget constraints: The availability of funding and budget constraints will impact a department's ability to hire new faculty. Departments need to align their faculty needs with the financial resources available (Ehrenberg, 2000). Fairweather (2005) delved into the evolving landscape of faculty salaries in higher education institutions, with a particular focus on the balance between teaching and research responsibilities. In this comprehensive study, Fairweather (2005) examined how faculty salaries have been influenced by the perceived importance of teaching and research within academia. The paper investigates the relative weight assigned to teaching and research in faculty compensation and whether any discernible trends have emerged over time.
10. Long-term vision and goals: The department's long-term vision and strategic goals play a role in determining the number and types of faculty positions needed. This includes considering the growth trajectory of the department and its alignment with the university's mission (Jaquette and Curs, 2023; Steele et al., 2013; Mobley and Easley, 2021; Carpenter et al., 2013).

Once these factors have been assessed, departments typically collaborate with university administration, academic deans, and other stakeholders to justify their faculty needs and request new positions for recruitment. The process often involves creating a detailed proposal that outlines the rationale, anticipated benefits, and alignment with the department's goals. It's important to note that faculty recruitment is a dynamic process that requires ongoing evaluation and adjustment. Universities continuously monitor enrollment trends, program changes, and other factors to ensure that their faculty composition remains aligned with evolving needs.

The fundamental rationale behind this research lies in the recognition that the alignment between faculty resources and student enrollment dynamics is paramount for sustaining a robust educational environment. In this paper, we present an advanced mathematical model that provides a comprehensive framework for analyzing and calculating the faculty needs of Ph.D. holders for main courses, as well as the requirements for teaching assistants and lecturers in practical courses, laboratory sessions, and associated activity classes. Our model is designed to address the intricate challenges posed by fluctuating student numbers, evolving programmatic offerings, and the imperative to maintain a high standard of educational excellence.

To elucidate the nuances of our mathematical model, we delve into two primary dimensions: The faculty needs for main courses and the support staff requirements for practical sessions. To begin, we explore the Saudi universities' system of teaching load allocation, wherein the workload is structured based on academic ranks. This introduces a crucial facet of faculty resource management, wherein the teaching workload varies for each rank, ranging from lecturers and teaching assistants to assistant professors, associate professors, and professors. This intricate differentiation underscores the need for a nuanced model that accounts for diverse teaching responsibilities.

Furthermore, we delve into the strategic vision of universities concerning student enrollment. The model is acutely cognizant of the fact that student numbers are subject to dynamic changes driven by factors such as population growth, the introduction of new academic departments, and the evolving landscape of employment opportunities. Consequently, our model takes into account these variables, allowing institutions to project student enrollment trends for future years. We employ the illustrative case of the University of Hafr Al Batin to exemplify this strategic foresight, showcasing how an institution aligns its admission policies with emerging trends to ensure efficient resource allocation.

In the pursuit of educational excellence, maintaining optimal class sizes and faculty-student ratios emerges as a pivotal objective for universities. The model recognizes that overcrowded classrooms can hinder the personalized attention and engagement required for effective learning. As universities strive to accommodate growing student numbers, they must simultaneously implement strategies to preserve the quality of the learning experience. Our mathematical model emphasizes the significance of innovative pedagogical approaches, the utilization of teaching assistants, and ongoing faculty development initiatives to ensure that larger class sizes do not compromise instructional effectiveness (Wang and Calvano, 2022).

In essence, this paper seeks to bridge the gap between student enrollment dynamics and faculty resource allocation through the lens of advanced mathematical modeling. By addressing the multifaceted challenges of higher education resource management, we aim to empower educational institutions to navigate the complexities of student enrollment with precision, strategic foresight, and an unwavering commitment to maintaining educational excellence.

The subsequent sections of this paper are organized as follows: Section 2 delves into the" Methodology: Mathematical Model for Analysis and Calculation of Faculty Needs (Ph.D. Holders) for Main Courses," where a comprehensive mathematical framework is expounded to assess and compute faculty requirements specifically focused on main courses. Following this, Section 3 elaborates on the" Methodology: Mathematical Model for Analysis and Calculation of Teaching Assistants and Lecturers Needs for Practical Courses, Laboratory Sessions, and Associated Activity Classes," introducing a tailored mathematical model to address the staffing needs of teaching assistants and lecturers in practical courses, laboratory sessions, and related activities. Subsequently, Section 4 demonstrates real-world application scenarios at Hafr Al Batin University, showcasing the practical utility and adaptability of the model. Finally, Section 5 provides" Conclusion Remarks," offering a reflective synthesis of key insights, implications, and avenues for future exploration derived from the analysis of faculty requirements in higher education institutions.

2. Methodology: Mathematical model for analysis and calculation of faculty needs (Ph.D. holders) for main courses

Historically, the Student-Faculty Ratio (SFR) has been one of the most straightforward and widely used metrics. Institutions often set an optimal ratio, ensuring that each faculty member serves a defined number of students (Buckner and Zhang, 2021).

$$SFR = \frac{\text{Total Number of Students}}{\text{Total Number of Faculty}} \quad (1)$$

However, while simple, this method often fails to account for the variability in class sizes, discipline-specific needs, and the diverse teaching loads across departments.

In this section, we introduce the mathematical model employed to analyze and calculate the faculty needs of Ph.D. holders for main courses. The model integrates various definitions and assumptions to achieve accurate estimations.

2.1. Teaching-load definition in the Saudi Universities system

Rather than employing a uniform SFR, some institutions calculate faculty needs based on faculty workloads. This approach considers a variety of factors, including the number of courses taught, research commitments, and administrative duties, among others. Establishing standard workload expectations allows institutions to determine faculty requirements based on the cumulative workload.

In the Saudi university system, faculty teaching loads are assigned based on academic rank, measured by the number of teaching hours per week. The teaching load for each academic rank is defined as follows:

- Lecturer and teaching assistant (not a Ph.D. holder): 16 hours per week
- Assistant professor: 14 hours per week
- Associate professor: 12 hours per week
- Professor: 10 hours per week
- Ph.D. holder undertaking administrative roles such as Vice-dean and department chair: 6 hours per week
- Ph.D. holder engaged in higher administrative roles such as vice president and dean: 3 hours per week

2.2. Definitions employed in the mathematical model

The average teaching workload for a Ph.D. holder faculty member is computed using the formula:

$$M_1 = \frac{14x_1+12x_2+10x_3+6x_4+3x_5}{\sum_{i=1}^{5} x_i} \quad (2)$$

where, x_1 is the number of assistant professors, x_2 is the number of associate professors, x_3 is the number of professors, x_4 is the number of Ph.D. holders with roles such as Vice-dean and department chair, x_5 is a number of Ph.D. holders with roles such as vice president and dean, 14 is teaching workload for assistant professors per week, 12 is teaching workload for associate professors per week, ten is teaching workload for professors per week, six is teaching workload for Ph.D. holders with roles such as Vice-dean and department chair per week, and three is teaching workload for Ph.D. holders with roles such as vice president and dean per week.

The average credit hours for main courses per specialization are calculated by summing all the credit hours of all the courses in each specialization and dividing it by the number of the courses as:

$$M_2 = \frac{\sum_{j=1}^{m} h_j}{m} \quad (3)$$

where, m is the number of main courses per specialization and h is the credit hours for each main course.

2.3. Assumptions used in the mathematical model

2.3.1. Expected percentage change in student

The dynamic nature of student enrollment in universities undergoes annual fluctuations driven by a myriad of factors. These factors encompass demographic shifts, institutional decisions to introduce new academic departments or phase out existing ones, the emergence of novel fields of study previously unavailable, and the imperative to cater to the ever-evolving demands of the job market.

Among these factors, the variance in student numbers across different specializations stands out as a pivotal determinant with far-reaching implications. It underscores the criticality of universities possessing a well-defined foresight and the ability to anticipate their future student demographics for the forthcoming years. This proactive approach empowers institutions to strategically plan and allocate resources to meet diverse academic needs effectively.

For instance, let's delve into the illustrative case of the University of Hafr Al Batin. The university, in accordance with its strategic blueprint, has charted a trajectory of enrollment expansion over the next five years, detailed as follows:

- Health programs: An envisioned 10% growth in enrollment.
- Engineering programs: An equivalent 10% increase in student intake.
- Scientific programs: Anticipating a 5% uptick in student enrollments.
- Other specializations: Holding steady with a 0% growth rate.

This strategic vision not only enhances the university's capacity to address the educational requirements of diverse fields but also underscores its responsiveness to societal shifts and aspirations. As universities adapt and fine-tune their enrollment strategies, they are better equipped to foster a dynamic, inclusive, and thriving academic environment that empowers students to excel and contribute to the global knowledge landscape. This deliberate and calculated approach to enrollment planning is rooted in the university's overarching commitment to not only accommodate but also thrive amidst the fluctuations in student numbers. By tailoring admission strategies to align with emerging trends and societal demands, the University of Hafr Al Batin is poised to harness its resources efficiently and ensure that the educational experience remains enriched and relevant.

2.3.2. Quality indicator for ideal class size and faculty-student ratio for main courses

With the projected increase in annual student enrollments, universities place a significant emphasis on safeguarding the quality of the class sizes. This commitment stems from the recognition that maintaining an optimal student-to-faculty ratio is essential for fostering effective learning environments and promoting meaningful student engagement.

As student numbers grow, universities are acutely aware of the potential challenges that may arise in maintaining an ideal balance between student quantity and instructional quality. They recognize that overcrowded classrooms can hinder personalized attention, inhibit active participation, and diminish the overall educational experience for students.

To address these concerns, universities adopt comprehensive strategies aimed at preserving the integrity of class sizes while accommodating the expanding student body. One of the strategies of- ten includes Resource allocation: Universities allocate resources strategically to hire additional qualified faculty members, ensuring that the student-to-faculty ratio remains conducive to meaningful interactions. This approach enables professors to provide personalized guidance, timely feedback, and mentorship to students, enhancing their overall academic experience.

There are other strategies, such as:

- Pedagogical innovation: Innovative teaching methodologies and technologies are integrated into the curriculum to facilitate effective learning in larger classes. Interactive online platforms, collaborative projects, and peer-assisted learning initiatives can help create a dynamic and engaging learning environment, even in larger class settings.

- Small group activities: To counteract the potential impersonal nature of larger classes, universities implement small group activities, discussions, and workshops. These activities foster closer student-faculty relationships, encourage peer-to-peer learning, and create opportunities for students to actively participate and contribute.
- Faculty development: Ongoing professional development programs for faculty members focus on optimizing instructional techniques for larger class sizes. Faculty members are trained to employ active learning strategies, encourage student interaction, and provide timely support to ensure that every student's educational needs are met.
- Effective use of teaching assistants: Teaching assistants (TAs) are strategically utilized to provide additional support in larger classes. TAs can lead smaller discussion groups, provide one-on-one assistance, and facilitate hands-on activities, enhancing the overall learning experience.
- Continuous assessment and feedback: Regular assessment and feedback mechanisms are implemented to monitor the quality of instruction and gauge student satisfaction. Universities gather input from students to make informed adjustments and improvements, ensuring that the learning environment remains effective and engaging.

As an example, consider the case of the University of Hafr Al Batin. In alignment with its strategic plan, the university has established the faculty-student quality ratio for main courses as follows:

- Health Programs: 1:20,
- Engineering Programs: 1:20,
- Scientific Programs: 1:30,
- Other Specializations: 1:40.

These definitions and assumptions form the foundation of the mathematical model for analyzing and calculating the faculty needs of Ph.D. holders for main courses. The model incorporates these elements to provide accurate estimations while considering various influencing factors and variables.

Next, we proceed to discuss the implementation of the model and present its results in real-world scenarios.

2.4. Definitions

We begin by defining the following variables used in the mathematical model:

y_1: Number of current main courses in each specialization.
Tot_{Needs1}: Total faculty members needed to cover main course hours.
$Tot_{optimal1}$: Optimal number of students per main course section.
M_2: Average hours allocated to each main course in a specialization.
Tot_{h_class1}: Total hours of main course sections.
$Ext_{Students1}$: Excess students beyond optimal capacity in main courses.
$Tot_{Students1}$: Total number of students in main courses.
Tot_{class1}: Total number of main course sections required.
Tot_{h_1}: Total hours required for main courses.
M_1: Average teaching workload of faculty members (Ph.D. holders).
N_1: Total faculty needs (Ph.D. holders).
f_1: Ideal capacity of each main course section.

2.5. Calculations

The computation of the current hours required for main courses involves the utilization of the following equation:

$$y_1 \times M_2 = Tot_{h\_class1} \quad (4)$$

To determine the requisite number of Ph.D. holders, excluding the consideration of the optimal class capacity, the subsequent expression is employed:

$$\frac{Tot_{h\_class1}}{M_1} = Tot_{Needs1} \quad (5)$$

The quantification of students in each class, accounting for the optimal capacity, is ascertained through the following equation:

$$y_1 \times f_1 = Tot_{optimal1} \quad (6)$$

The evaluation of additional students beyond the optimal capacity is conducted using the subsequent formula:

$$Tot_{Students1} - Tot_{optimal1} = Ext_{Students1} \quad (7)$$

The determination of the essential number of classes for main courses is achieved by the application of the subsequent equation:

$$\frac{EXt_{Students1}}{f_1} = Tot_{class1} \quad (8)$$

The comprehensive computation of the total hours required for main courses is realized through the following expression:

$$Tot_{class1} \times M_2 = Tot_{h_1} \quad (9)$$

Ultimately, the essential count of teaching assistants and lecturers is ascertained through the utilization of the ensuing equation:

$$\frac{Tot_{h_1}}{M_1} = N_1 \quad (10)$$

These mathematical formulations collectively present a methodical approach to estimating the necessary complement of Ph.D. holders for main courses, predicated upon the established variables and assumptions, as shown in Fig. 1 and Fig. 2. The above calculations provide a comprehensive analysis of the faculty's main course classes based on the given variables and factors. The model ensures that the optimal number of faculty members (Ph.D. holders) is determined to achieve institutional and programmatic accreditation standards.

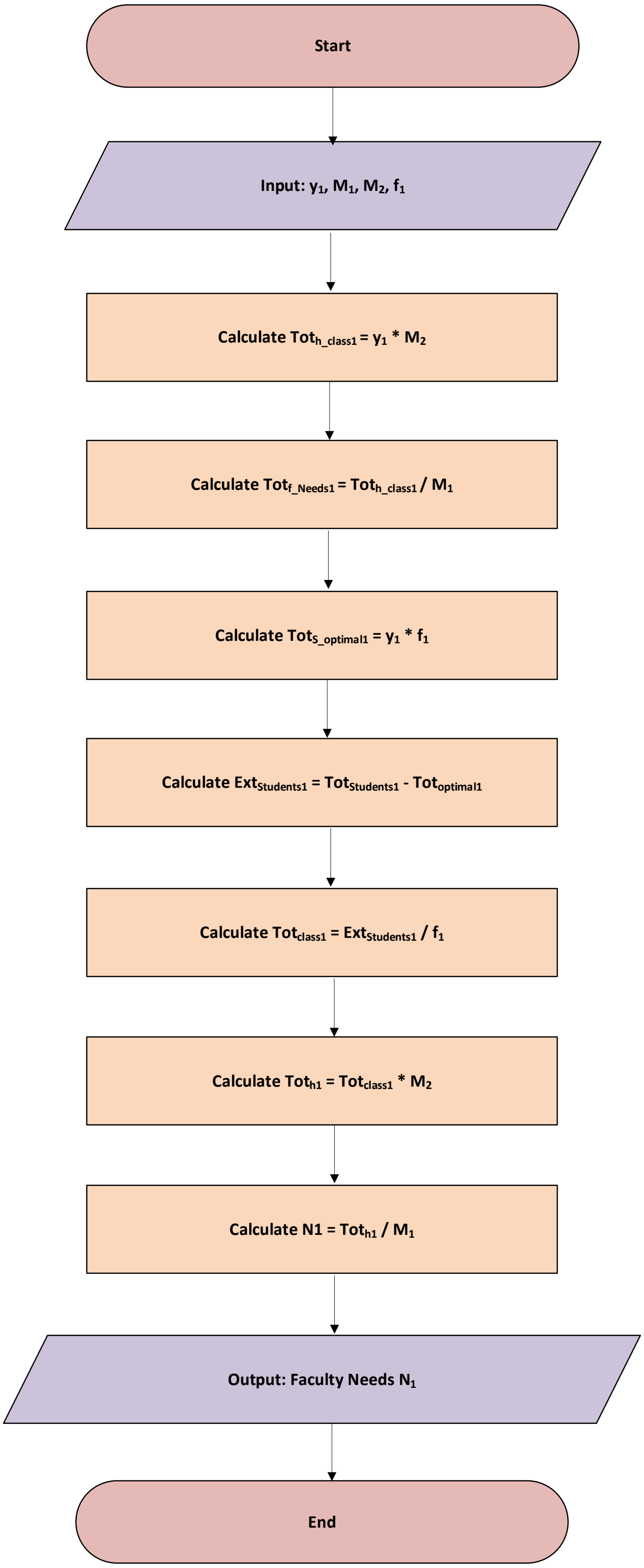

Fig. 1: Flowchart depicting the process for calculating faculty needs for main courses for Ph.D. holders

Fig. 2: Flowchart depicting the methodology for calculating faculty needs based on class sizes, workloads, and enrollment trends, assessing optimality of current allocations and suggesting adjustments

3. Methodology: Mathematical model for assessing teaching assistant and lecturer needs in practical and laboratory courses

In this section, we present the mathematical model that has been developed to analyze and calculate the requirements for teaching assistants and lecturers in practical courses, laboratory sessions, and associated activity classes. The model incorporates various definitions, equations, and assumptions to provide a comprehensive framework for estimating the needed faculty members.

3.1. Definitions used in the mathematical model

This section initiates by establishing precise definitions for the ensuing variables integral to the mathematical model:

y_2: Number of current sections in practical courses, laboratory sessions, and associated activities.
Tot_{Needs2}: Aggregate of teaching assistants and lecturers required to encompass the instructional hours in practical courses, laboratory sessions, and associated activity classes.
$Tot_{optimal2}$: Optimal quantity of students per section in practical courses, laboratory sessions, and associated activity classes.
M_4: Average hours allocated to each section in practical courses, laboratory sessions, and associated activity classes within a specific specialization.
Tot_{h_class2}: Cumulative hours of sections in practical courses, laboratory sessions, and associated activity classes.
$Ext_{Students2}$: Surplus students beyond the optimal capacity in sections of practical courses, laboratory sessions, and associated activity classes.
$Tot_{Students2}$: Overall count of students enrolled in sections of practical courses, laboratory sessions, and associated activity classes.
Tot_{class2}: Total number of sections in practical courses, laboratory sessions, and associated activity classes required.
Tot_{h_2}: Total hours requisite for practical courses, laboratory sessions, and associated activities.
M_3: Average teaching workload of teaching assistants and lecturers (Non-Ph.D. holders).
N_2: Aggregate necessities of teaching assistants and lecturers (Non-Ph.D. holders).
f_2: Desired capacity of each section in practical courses, laboratory sessions, and associated activity classes.

The average teaching workload for teaching assistants and lecturers per week is calculated using the following formula:

$$M3 = 16 \quad (12)$$

The average credit hours for practical courses, laboratory sessions, or associated activity classes per specialization are calculated as follows:

$$M_4 = \frac{\sum_{j=1}^{m} r_j}{s} \quad (13)$$

where, r is the number of practical courses or laboratory sessions for each specialization and s is the number of credit hours for each practical course, laboratory session, or associated activity classes.

3.2. Assumptions used in the mathematical model

Embedded within its strategic blueprint, the university ardently endeavors to realize the benchmarks stipulated by the Education and Training Evaluation Authority pertaining to discerning quality metrics, with a specific focus on the faculty-to-student ratio. Over a span of five years, the university aspires to converge toward the pinnacle of numerical excellence, thus embarking on a gradual trajectory toward the realization of these optimum numerical thresholds. It is imperative to note that the figures proffered herein encapsulate the preliminary phase, serving as a foundational point of departure rather than a reflection of the aspirational benchmarks. Driven by an unwavering commitment, the institution diligently endeavors to iteratively enhance these numerical constituents on an annual basis, propelling steadfastly towards the consummation of the established norms.

Illustratively, let us delve into the paradigm of the University of Hafr Al Batin. In consonance with its strategic blueprint, the institution has instituted a benchmark for the faculty-student quality ratio within the ambit of core courses, delineated as follows:

- Health Programs: 1:10,
- Engineering Programs: 1:10,
- Scientific Programs: 1:20,
- Other Specializations: 1:30.

3.3. Model implementation and results

The developed mathematical model combines the aforementioned definitions and assumptions to analyze and calculate the teaching assistants' and lecturers' needs for practical courses, laboratory sessions, and associated activities. By inputting relevant data and variables, the model offers a systematic approach to estimating the required faculty members for different specializations.

In the following sections, we demonstrate the application of the model to real-world scenarios and present its outcomes, highlighting its practical significance for educational institutions.

3.4. Mathematical equations of the model

The mathematical model for analysis and calculation of teaching assistants' and lecturers' needs for practical courses and associated activities involves the following equations. To calculate the current hours required for practical courses and associated activities, we use the equation:

$$y_2 \times M_4 = Tot_{h\_class2} \quad (14)$$

To calculate the required number of teaching assistants and lecturers without considering the ideal capacity of the class, we use:

$$\frac{Tot_{h\_class2}}{M_3} = Tot_{f\_Needs2} \quad (15)$$

The number of students in each class after considering the ideal capacity is determined using:

$$y_2 \times f_2 = Tot_{optimal2} \quad (16)$$

The number of additional students beyond the optimal capacity is calculated as follows:

$$Tot_{Students2} - Tot_{optimal2} = Ext_{Students2} \quad (17)$$

The required number of classes for practical courses and associated activities is determined by the following:

$$\frac{EXt_{Students2}}{f_2} = Tot_{class2} \quad (18)$$

The total required hours for practical courses and associated activities are calculated as follows:

$$Tot_{class2} \times M_4 = Tot_{h_2} \quad (19)$$

Finally, the required number of teaching assistants and lecturers is determined using:

$$\frac{Tot_{h_2}}{M_3} = N_2 \quad (20)$$

These equations collectively provide a systematic approach for estimating the needed teaching assistants and lecturers for practical courses and associated activities based on the defined variables and assumptions.

In the next section, we demonstrate the application of these equations through practical examples and discuss the implications of the results.

4. Real-world application scenarios

4.1. Scenario 1: Calculation of faculty (Ph.D. holders) need for mathematics classes at Hafr Al Batin University

To illustrate the practical viability of our mathematical model, we delve into a specific scenario that showcases its real-world application. Our focus centers on Hafr Al Batin University, where we aim to allocate faculty resources for Mathematics classes with precision and effectiveness.

In this context, we assign concrete values to the variables, thereby offering a tangible example of our model's utility. The scenario unfolds as follows. At Hafr Al Batin University, our objective is to establish a total of 111 Mathematics classes (y_1=111). Our strategic plan is underpinned by the anticipation of accommodating a cohort of female students, with a projected total enrollment of $Tot_{Students1} = 2480$. A critical facet of our plan is to ensure optimal learning conditions, limiting the class size to a maximum of 20 female students per class (f_1=20). Notably, we aim to align with the pedagogical imperative of fostering a conducive and engaging learning environment.

In the Department of Mathematics, there are 24 skilled female Ph.D. holders. This group includes 23 assistant professors, each responsible for 14 hours of teaching, and one associate professor, who teaches for 12 hours. As a result, the average teaching workload for faculty members is approximately 14 hours. The department has also calculated the average credit hours for core Mathematics courses, which comes to about 3 hours per course. This figure represents the basic teaching commitment needed for each course. As we move forward, our mathematical model will help optimize the assignment of faculty to Mathematics classes, taking into account student demographics, class sizes, and teaching workloads. By systematically integrating these variables, our model supports strategic decision-making, ensuring that the educational experience is rich and tailored to the specific needs of Hafr Al Batin University.

This scenario encapsulates the tangible impact of our model, elucidating its role in facilitating prudent resource allocation and enhancing the quality of education within a real-world academic setting.

- Number of Classes (y_1): This represents the total number of classes that need to be taught. For example, if a university offers 111 mathematics classes, then y_1=111.
- Total Students ($Tot_{Students1}$): This is the total number of students enrolled in these classes. If there are 2480 students enrolled in mathematics classes, then $Tot_{Students1}$=2480.
- Class Size (f_1): This is the maximum number of students that can be in one class. If each class can have up to 20 students, then f_1=20.
- Teaching Load (M_1): This represents the number of teaching hours assigned to each faculty member. For instance, if each faculty member teaches 14 hours per week on average, then M_1=14.
- Credit Hours per Course (M_2): This is the number of hours assigned to each course. For example, if each course takes 3 hours per week, then M_2=3.

Determining the current teaching hours for main courses is essential, calculated as 111 multiplied by 3, which equals 333 hours, indicating a total need for 333 teaching hours. To find the number of Ph.D. holders required, excluding optimal class sizes, the equation is 333 divided by 14, which suggests that about 24 faculty members are necessary. To assess class capacity, the calculation of 111 multiplied by 20 indicates that 2220 students can be accommodated across the classes. If enrollment exceeds this number, we calculate the surplus by subtracting 2220 from 2480, resulting in 260 students, which leads to the need for additional sections determined by dividing 260 by 20, indicating 13 classes are required. This translates to needing 39 more teaching hours, calculated as 13 multiplied by 3. Finally, using 39 divided by 14, we conclude that approximately three additional faculty members are required. This comprehensive analysis illustrates the practical application of a mathematical model to ascertain the necessary faculty resources based on specific variables and parameters.

4.2. Second scenario: Calculation of teaching assistant and lecturers (Non-Ph.D. holders) need for mathematics classes at Hafr Al Batin University

Saudi universities typically assign core courses to Ph.D. holders, while practical courses, laboratories, and related activity courses are usually taught by teaching assistants and lecturers, who generally hold bachelor's and master's degrees, respectively. It is believed that the number of students in a practical or activity course should ideally be half the number of students in the main course to ensure effective application of knowledge and to prevent overcrowding. To illustrate the practical applicability of the mathematical model, we consider a scenario at Hafr Al Batin University where the teaching workload for teaching assistants and lecturers is set at 16 hours per week. With 111 students in the main class, this means that the practical class size can accommodate up to 222 students, since half of 111 is approximately 55.5, rounded up to 222. The mean credit hours for all courses are calculated as 3, and the stipulated student count per class is set at 10. The calculation of the current hours required for practical courses and associated activities involves the equation: 222 multiplied by 10 equals 666 hours. To determine the required number of teaching assistants and lecturers, we divide the total hours by the standard teaching load: 666 divided by 16 gives approximately 41.63 teaching assistants and lecturers. The total number of students in each class is assessed with the equation: 222 multiplied by 10 equals 2220 students. If there are 2480 students enrolled, the excess is calculated by subtracting the optimal capacity: 2480 minus 2220 equals 260 students. The number of classes needed for practical courses is determined by dividing the excess students by the maximum class size: 260 divided by 10 equals 26 classes. The total hours required for these classes is then calculated as: 26 multiplied by 3 equals 78 hours. Finally, to find the necessary count of teaching assistants and lecturers, we use: 78 divided by 16, indicating that about five additional faculty members are needed. This analysis demonstrates how the mathematical model can estimate the necessary teaching resources based on specified variables. To validate our model, we will gather data from multiple academic semesters at Hafr Al Batin University, comparing actual faculty allocation, class sizes, and student enrollment figures against our model's predictions. This model will be applied to the Mathematics Department in a different semester, aiming to establish 80 mathematics classes with a projected enrollment of

1600 students, where the optimal class size is set at a maximum of 20 students per class. The department has 20 faculty members, each with an average teaching workload of 16 hours per week, and each course averages four credit hours. The evaluation of current hours for main courses totals 320 hours, calculated by multiplying the number of courses (80) by the hours allocated for each course (4). To determine the necessary number of Ph.D. holders to teach these courses, we divide the total hours by the standard teaching load, resulting in a need for 20 Ph.D. holders. Assessing the number of students in each class involves multiplying the number of courses (80) by the number of students per course (20), yielding a total of 1,600 students. Since the enrolled student count equals the available spaces, we find that no additional sections are necessary for the current enrollment.

5. Conclusion remarks

In conclusion, this paper has presented a comprehensive and advanced mathematical model that offers a systematic framework for analyzing and calculating the faculty needs of Ph.D. holders for main courses, as well as the requirements for teaching assistants and lecturers in practical courses, laboratory sessions, and associated activity classes. The model addresses the intricate dynamics of student enrollment, program offerings, and instructional quality within higher education institutions.

Through the application of the model to real-world scenarios, we have demonstrated its efficacy in estimating the optimal allocation of faculty resources to achieve a balance between student numbers and instructional excellence. By taking into account various variables, including class sizes, faculty workloads, and student enrollment trends, our model provides valuable insights for strategic resource planning and allocation.

The findings of this research underscore the significance of aligning faculty resources with the evolving landscape of higher education. As institutions strive to maintain high-quality education and meet accreditation standards, our model offers a robust tool for informed decision-making. It enables academic leaders, administrators, and policymakers to make evidence-based choices in faculty recruitment, workload distribution, and class size management.

Furthermore, the model's adaptability ensures its applicability across diverse disciplines and institutions, facilitating tailored solutions for each context. It offers a proactive approach to addressing challenges posed by fluctuating student enrollments, changing program demands, and the pursuit of educational excellence.

In a rapidly evolving educational landscape characterized by demographic shifts and technological advancements, the presented mathematical model provides a valuable contribution to the field. As higher education institutions continue to evolve and adapt, the insights gained from this research can guide strategic planning and resource allocation, ultimately enhancing the overall quality of education and promoting effective learning environments.

In summary, this paper's mathematical model offers a forward-looking approach to faculty resource management, contributing to the overarching goal of providing exceptional education and fostering an environment conducive to academic success, innovation, and excellence.

6. Discussion on limitations and practical implications

Our faculty allocation model is thorough but has limitations that are important to recognize. Its adaptability to different academic cultures means that implementation can differ based on teaching methods, faculty-student interactions, and administrative practices; for example, research-focused institutions may need adjustments for faculty with significant research responsibilities. Additionally, various pedagogical approaches, such as flipped classrooms, hybrid learning, or a strong reliance on teaching assistants, may require modifications to the model. The quality and availability of data are also critical, as incomplete information can lead to poor decision-making. Lastly, the model makes assumptions about uniform teaching loads, class sizes, and credit hours, which may not accurately represent the practices of all institutions.

7. Future work

In future research, the model can be extended to incorporate additional variables and factors, thereby enhancing its accuracy and applicability. This could include considerations such as faculty expertise, research responsibilities, and emerging pedagogical approaches. By continuously refining and expanding the model, we can further contribute to the advancement of higher education management and administration.

The presented mathematical model offers a sophisticated structure for predicting the faculty needs at higher education institutions. Recognizing the dynamic landscape of academic environments, there exists considerable scope for further refinement and enhancement of the model. In future iterations, we aim to delve deeper by including more complex variables and mathematical relations.

One potential development could be the introduction of variables that account for the diverse specializations within a faculty, represented mathematically as:

$$F(x) = a_1x_1 + a_2x_2 + \cdots + a_nx_n$$

where, F(x) represents the faculty needs, x_i represents different faculty specializations and a_i

are coefficients representing the importance or weight of each specialization.

In addition to this, a comprehensive representation of faculty workload that encompasses both teaching and research responsibilities can be modeled through equations that take into consideration different roles a faculty member might have:

$$W(x, y) = b_1 x + b_2 y + c$$

where, W(x, y) represents the total workload, x and y are the teaching and research workloads, respectively, and b_1, b_2, and c are coefficients and constants that can be determined through analysis of existing data.

Furthermore, the integration of technological advancements for detailed data analysis and visualization could be facilitated through algorithms that compute optimal solutions based on the inputs derived from the model's equations.

7.1. Additional potential improvements

- Integrating research workloads.
- Allowing variable class sizes and teaching methods.
- Customizing parameters for different institution types.
- Enhancing data integration capabilities.
- Addressing these limitations will make the model more adaptable to diverse academic settings.

In conclusion, the prospect of future work lies in leveraging the mathematical robustness of the model, enhancing it with sophisticated equations that can potentially foster a deeply nuanced understanding of faculty needs. This path of progression, echoing the philosophy of continuous improvement, promises to steer the model towards being an indispensable tool in the meticulous management of educational institutions, aligning perfectly with the demands of modern educational ecosystems.

Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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