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A comparison between the nonhomogeneous Poisson and α-series processes for estimating the machines' fault time of thermal electricity generation

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ABSTRACT

This study aims to compare the stochastic process model designed as a nonhomogeneous Poisson process and α-series process, to obtain a better process for using monotonous trend data. The α-series process is a stochastic process with a monotone trend, while the NHPP is a general process of the ordinary Poisson process and it is used as a model for a series of events that occur randomly over a variable period of time. Data on the daily fault time of machines in Bahrri Thermal Station in Sudan was analyzed during the interval from first January 2021, to July 31, 2021, to acquire the best stochastic process model used to analyze monotone trend data. The results revealed that the NHPP model could be the most suitable process model for the description of the daily fault time of machines in Bahrri Thermal Station according to lowest MSE, RMSE, Bias, MPE, and highest. The current study concluded that through the NHPP, the fault time of machines and repair rate occur in an inconsistent way. The further value of this study is that it compared NHPP and α-series to obtain a better process for using monotone trend data and prediction. Meanwhile, the other studies in this field focused on comparing methods of estimation parameters of the NHPP and the α-series process. The distinctive scientific addition of this study stems from displaying the precision of the NHPP better than the α-series process in the case of monotone trend data.

1. Introduction

Electrical energy is the basis of contemporary development in its various economic and development aspects as it is the most important infrastructure and it affects the instability of the country due to the frequent faults of electricity generation machines, and this has an impact on economic development and human life (Butt et al., 2021). It is important to have a stochastic process model to study the fault time of machines for thermal electricity generation (Borges, 2012). Thermal electricity generation is considered one of the most important sectors of electricity generation in Sudan, which supplies the national electricity grid in Sudan and helps the competent authorities to develop their plans for the stability of the electric current. Generally, there are five stochastic process models to study the fault time of machines when it represents a monotonous trend (Louit et al., 2009); (i) renewal process model (ii) geometric process model (iii) α-series process model (iv) homogeneous Poisson process (HPP) (v) nonhomogeneous Poisson process (NHPP). The use of the α-series process with the NHPP is considered one of the important issues, especially in the field of service provision, which includes health, finance, telecommunications, and electricity sectors. The series can be used when successive fault times are a monotonous trend due to the influence of time and the cumulative form of the process. The NHPP can be described as a process of the fault rate that occurs in a variable manner and represents a monotonous trend due to the change of time.

The researchers have sought an understanding of the fault time of machines, and many of them have undertaken stochastic process models to build models. So the first research in this field was Non-homogeneous Poisson and linear regression models as approaches to studying time series with change

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points (de Oliveira et al., 2022). Another work studied Statistical Inference for Alpha-Series Process with the Generalized Rayleigh Distribution (Biçer, 2019). Some of the researchers studied Time-between-events monitoring using an NHPP with power law intensity (Ali, 2021). Another study had also a parameter estimation in the α-series process with lognormal distribution (Chumnaul, 2019). While the Previous studies have compared the α-series process and its related processes such as the geometric process, renewal process, and the Weibull process, this study makes the scientific addition of displaying the importance of the NHPP as well as α-series process to study fault time of machines. The objective of this study is to find the best stochastic process model that represents the fault time of electricity-generation machines for the period from first January 2021 to July 31, 2021.

Bahrri Thermal Plant is considered one of the largest thermal stations of the Sudanese Thermal Generation Company as it supplies the national electricity grid with a high percentage of electricity about 380 MW, as it compensates for the shortage in water generation when the water level drops during the summer.

2. Methodology

2.1. α-series process model

The α-series process is the first possible process of stochastic process models when successive fault times represent a monotonous trend due to the influence of time and the accumulative form of the process (Biçer, 2019). If we assume that the cumulative distribution function is $F_t(x)$, it has a positive mean μ and a specific variance σ^2, and α , μ and σ^2 then the main parameters of the α-series process A because it determines the mean and variance of σ^2, then if the dataset $[x_t, t = 1,2, \dots]$ represents the times of occurrence for the α-series process, so that:

$$x_t = \frac{y_t}{t^{\propto}} \tag{1}$$

and

$$y_t = t^{\propto} x_t \tag{2}$$

where, y_t represents a series of identical independent variables. Taking the natural logarithm of both sides of Eq. 2, we get:

$$y_t = \propto lnt + lnx_t \tag{3}$$

The expectation of x_t that it can be found in the following formula:

$$E(x_t) = \frac{E(y_t)}{t^{\alpha}} = \frac{\mu}{t^{\alpha}} \tag{4}$$

Also, the variance of x_t can be calculated by the following formula:

$$V(x_t) = \frac{V(y_t)}{(t^{\propto})^2} = \frac{\sigma^2}{t^{2\alpha}} \tag{5}$$

2.1.1. Least squares method

The least squares method is one of the important nonparametric methods for estimating, as it has been noted that the time rate of occurrence in the α-series is fit with this method (Suleiman, 2013), that the principle of this method depends on the error squares to get the best estimator for the parameters of the α-series. If the stochastic process $[x_t, t = 1,2, \dots]$ represents the inter-occurrence times of an alpha-series process, then the sum of the squares of the error in logarithmic scale is:

$$\sum_{t=1}^{n} e^2{}_t = \sum_{t=1}^{n} [lny_t - E(lny_t)]^2 \tag{6}$$

Substituting into Eq. 5, we get:

$$\sum_{t=1}^{n} e^2{}_t = \sum_{t=1}^{n} [lnx_t - \propto lnt - \beta]^2 \tag{7}$$

where, $E(lny_t) = \beta$

In order to reduce the sum of squares of the error to the least possible, the partial differential of Eq. 8 with respect to the two parameters α and β was taken, and setting the result to zero.

$$\frac{\partial \sum_{t=1}^{n} e^2{}_t}{\partial \beta} = -2 \sum_{t=1}^{n} [lnx_t - \propto lnt - \beta]^2 = 0 \tag{8}$$

$$\frac{\partial \sum_{t=1}^{n} e^2{}_t}{\partial \propto} = -2 \sum_{t=1}^{n} [lnx_t - \propto lnt - \beta]^2 \, lnt = 0 \tag{9}$$

From Eq. 10 it is obtained:

$$-2 \sum_{t=1}^{n} ln\, x_t - \propto \sum_{t=1}^{n} lnt - n\beta = 0 \tag{10}$$

Therefore,

$$\beta = \frac{1}{n} [\sum_{t=1}^{n} lnx_t - \alpha \sum_{t=1}^{n} lnt] = 0 \tag{11}$$

From Eq. 10 it is obtained:

$$\sum_{t=1}^{n} lntlnx_t + \propto \sum_{t=1}^{n} (lnt)^2 - \beta \sum_{t=1}^{n} lnt \tag{12}$$

Substituting Eq. 13 into Eq. 14, we get:

$$\mathrm{n} \sum_{t=1}^{n} \mathrm{lntlnx_t} + \sum_{t=1}^{n} \mathrm{lnt} \sum_{t=1}^{n} \mathrm{x_t} + \alpha[\mathrm{n} \sum_{t=1}^{n} (\mathrm{lnt})^2 - (\sum_{t=1}^{n} \mathrm{lnt})^2] = 0 \tag{13}$$

Therefore, the estimator of least squares for the parameter α is:

$$\hat{\alpha} = \frac{\sum_{t=1}^{n} lntlnx_t - n \sum_{t=1}^{n} lnt \sum_{t=1}^{n} x_t}{n \sum_{t=1}^{n} (lnt)^2 - (\sum_{t=1}^{n} lnt)^2} \tag{14}$$

To find the least squares estimator for parameter β, we substitute the estimator for parameter t into Eq. 13 as it comes:

$$\hat{\beta} = \frac{\sum_{t=1}^{n} lnt \sum_{t=1}^{n} lnx_t lnt - n \sum_{t=1}^{n} ln(lnt) \sum_{t=1}^{n} lnx_t}{\sum_{t=1}^{n} (lnt)^2 - n(\sum_{t=1}^{n} lnt)^2} \tag{15}$$

As for the estimation of the parameters β, σ^2, the variance of the error is first estimated by likening to Eq. 17, with a simple regression model.

$$y_t = a + bx_t + e_t \quad (16)$$
$$lnx_t = \beta - \alpha lnt + e_t \qquad t = 1,2, \dots, n \quad (17)$$

As:

$y_t = lnx_t$, $a = \beta$, $b = -\propto$, $x_t = lnt$

Since the variance of the regression line is:

$$\sigma_e^2 = \frac{\sum_{t=1}^{n} y^2{}_t - a\sum_{t=1}^{n} y_t - b\sum_{t=1}^{n} x_t y_t}{n-2} \quad (18)$$
$$\sigma^2 = \frac{\sum_{t=1}^{n}(lnx_t)^2 - \widehat{\beta}\sum_{t=1}^{n} lnx_t + \widehat{\alpha}\sum_{t=1}^{n} lnx_t lny_t}{n-2} \quad (19)$$

By substituting t into Eq. 17, and by making simplifications on the above expression, the amount of error variance of the regression line equation was obtained.

$$\sigma_e^2 = \frac{\left[\sum_{t=1}^{n}(lnx_t)^2 - \frac{1}{n}\left(\sum_{t=1}^{n} lnx_t\right)^2\right] - \widehat{\alpha}\left[\sum_{t=1}^{n}(lnt)^2 - \frac{1}{n}\left(\sum_{t=1}^{n} lnt\right)^2\right]}{n-2} \quad (20)$$

From Eq. 20 it is possible to find the value of parameter m for an α-series process as follows:

$$y_t = e^{(\beta + e_t)} \quad t = 1,2, \dots, n \quad (21)$$

Taking the expectation from both sides:

$$E(y_t) = \left[e^{\beta + e}\right] = e^{\beta} E\left(e^{e_t}\right) \quad (22)$$

The estimator can be found using the Taylor series on the following:

$$e_t = 1 + e_i + \frac{e^2{}_i}{2} + \cdots \quad (23)$$

As: $E(e_i) = 0, E(e_i) = \sigma^2{}_e$

Therefore, the estimator of the parameter μ for an α-series process is:

$$\hat{\mu} = E[y_t] = e^{\widehat{\beta}}\, E\left[1 + e_i + \frac{e^2{}_i}{2} + \cdots\right] \approx e^{\widehat{\beta}}\left(1 + \frac{\widehat{\sigma}^2}{2}\right) \quad (24)$$

The estimator of the process σ^2 for an α-series process is.

$$V(y_t) = V\left(e^{(\beta + e_t)}\right) = e^{2\beta} V\left(e^{e_t}\right) = e^{2\beta} V(e_t) \quad (25)$$
$$\hat{\sigma}^2 = \hat{\sigma}_e e^{2\beta}$$

2.2. Homogeneous Poisson process (HPP)

It is the counting process $[N(t); t \geq 0]$, it is called the Poisson process with rate λ when it contains the following conditions (Qurashi and Hamdi, 2016):

1. The number of events at time zero is zero, $N(t) = 0$.
2. The process $[N(t); t \geq 0]$ has independent increments.
3. The number of events in any interval of length t is distributed as a Poisson distribution with parameter λt.

$$P[N(t) = n] = \frac{(\lambda t)^n e^{-\lambda t}}{n!} \qquad \text{n=0,1,2,...} \quad (26)$$

The number of events in the period $[t + s, s]$ is also a random variable that follows a Poisson distribution with a mean $\lambda(t + s, s)$.

$$P[N(t + s) - N(s) = n] = \frac{(\lambda(t+s-s)^n e^{-\lambda(t+s-s)}}{n!} \quad \text{n=0,1,2,...} \quad (27)$$

2.3. Nonhomogeneous Poisson process (NHPP)

NHPP is a general process of the ordinary Poisson process which is one of the advanced stochastic processes used in reliability engineering, and it has been used successfully in the study of reliability and machine failure problems. The number of events that occur randomly in time t with rate λ for events for every unit of time t. It is suitable for modeling a series of events that occur randomly for a variable length of time (Ali, 2021).

Definition: The counting processes $[N(t); t \geq 0]$ are said to be an NHPP with of Intensity function λt, $t \geq 0$, if the following conditions are available:

1. The counting processes $[N(t); t \geq 0]$, i.e. the number of events in time t, has independent increments but is unstable.
2. The probability of more than one event occurring in a period of time h is close to zero.

$$P[N(t + h) - N(t) \geq 2] = 0h \quad (28)$$

3. The probability of one event occurring during time.

$$P[N(t + w) - N(t) \geq 1] = \lambda(t)w + U(w) \quad (29)$$

Since the term $U(w)$ denotes any quantity that leads to zero, when approaching zero. Thus, the Poisson process $[N(t); t \geq 0]$ follows the Poisson distribution with a probability mass function:

$$P[N(t - s) - N(t) = n] = \frac{(\lambda(t))^n e^{-\lambda t}}{n!} \quad \text{n=0,1,2,...} \quad (30)$$

where, m represents the process parameter is the cumulative rate of occurrence of failure (ROCOF). If λt is a constant quantity for all values of t i.e. λt linear in time t, then the process $[N(t); t \geq 0]$ is a HPP with event rate λt, if λt is variable and it changes with time the process $[N(t); t \geq 0]$ is an NHPP, and the scientist Feller is the first scientist who gave a definition of the Poisson process and put its most important characteristics (de Oliveira et al., 2022).

2.3.1. Properties of NHPP

1. Independent Of The Number: If we have an NHPP within the period $(0, t)$ The number of events during the same period is $N(t) = n$, So the moment we get n events are independently distributed within the period$(0, t)$ with intensity function $\lambda(t)/\Lambda(t)$.

2. Superposition: Compound of two or more NHPP with intensity functions $\lambda_1(t), \lambda_2(t)$ It is also an NHPP that means: $\lambda(t) = \lambda_1(t) + \lambda_2(t) + \cdots$
3. Random Selection: If we have an NHPP with an intensity function $\lambda(t)$, our selection of any event is random and independent of the other events and with a probability $P(t)$, which depends on time and therefore has an intensity function $P(t) = \lambda(t)$.
4. Random Split: If an NHPP with an intensity function $\lambda(t)$ is randomly split into two partial processes with probabilities $P_1(t), P_2(t)$, if $P_1(t) + P_2(t) = 1$, Therefore, the results of partial processes are NHPP with intensity functions $\lambda_1(t)P_1(t), \lambda_2(t)P_2(t)$.

The probability distribution of the intervals between the occurrences of events in the NHPP follows the Exponential distribution with probability distribution: $(t) = \lambda(t)e^{-\int_0^\infty \lambda(u)du}, t > 0$

$$f(t) = \lambda(t)e^{-\int_0^\infty \lambda(u)du}, \quad t > 0, \tag{31}$$

2.3.2. Power law model

Duane (1964) proposed a model called the power law as a function of the time rate of an event with two parameters λ, β

$$u(t) = \lambda\beta t^{\beta-1} \tag{32}$$

The cumulative intensity function of the power law is:

$$\varphi(t) = \lambda t^{\beta} \tag{33}$$

where, $N(t)$ is Number of observed failures in (0, t); $u(t)$ is Failure intensity (sometimes called the "instantaneous failure rate"); λ, β is Model parameters ($\lambda > 0, \beta > 0$).

If the parameter β in the power law function of the time rate of occurrence in Eq. 33, if:

1. $\beta > 1$: Indicates that NHPP is an increase with time.
2. $\beta < 1$: Indicates that NHPP is a decrease with time.
3. $\beta = 1$: Indicates that the time rate of occurrence in the process under study is a constant quantity with time. Therefore, the power law function turns into an exponential distribution function (Chumnaul, 2019).

2.3.3. Maximum likelihood method

Estimating the time rate is an estimate of the parameters in the function model that was chosen to represent the time rate function of the occurrence of events in the NHPP and one of the most used methods is the maximum likelihood (Chumnaul, 2019).

$$f_n(t_1, t_2, \dots, t_n) = \prod_{i=1}^{n} \lambda(t_i) Exp^{\left[-\int_0^\infty \lambda(u)du\right]} \tag{34}$$

When substituting the two Eqs. 32 and 33, they yield:

$$y = ln[u(t)]\,,\ a = ln\lambda\,, b = \beta\,, x = lnt \tag{35}$$
$$L = \lambda^n\beta^n \prod_{i=1}^{n} t_i{}^{\beta-1} Exp^{\left[-\lambda(t_n)^{\beta}\right]} \tag{36}$$
$$lnL = nln\lambda + nln\beta + (\beta - 1)\sum_{i=1}^{n} lnt_i - \lambda t_n^{\beta} \tag{37}$$
$$\hat{a} = \frac{\sum y}{n} - \hat{b}\frac{\sum x}{n} \tag{38}$$
$$\hat{\lambda} = \frac{n}{t_n^{\beta}} \tag{39}$$

Substituting the values of *x, y, a, and b* with their corresponding values from Eq. 37, we get the following:

$$\frac{\partial lnL}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n} t_{i-} \lambda t_n^{\beta} lnt_n = \frac{n + \beta\sum_{i=1}^{n} lnt_{i-}\lambda\beta t_n^{\beta} lnt_n}{\beta} \tag{40}$$

Equating Eq. 40 to zero:

$$\frac{\partial lnL}{\partial \beta} = \frac{n + \beta\sum_{i=1}^{n} lnt_{i-}\lambda\beta t_n^{\beta} lnt_n}{\beta} = 0 \tag{41}$$
$$\hat{\beta} = \frac{n}{\lambda\beta t_n^{\beta} lnt_n - \sum_{i=1}^{n} lnt_i} \tag{42}$$

Thus, the model of the estimated power law function is:

$$\lambda(t) = \hat{\lambda}\hat{\beta}(t)^{\hat{\beta}-1} \tag{43}$$

2.4. Trend analysis of process

The trend analysis of the stochastic processes of the α-series, Geometric process, renewal process, and NHPP is important in determining the general form of the process, when applied to actual data, several basic problems were faced, the most important of which is matching the study data with the process. To test this, Yeh (1992) suggested a number of tests on the process, which helps us know whether the data follows the stochastic process or not. The trend analysis test in the process is a trend analysis monotone test. There are some simple methods to test the general trend of the stochastic process: (1) Technical graphic (2) The Mann test (3) the Laplace test (Kara et al., 2019). In this study, the general trend analysis test was used:

$H_0: \gamma = 0$ Faults rate is constant HPP
$H_1: \gamma \neq 0$ Faults rate is not Constant NHPP

The statistical test for the above hypothesis is as follows:

$$Z = \frac{\sum_{i=1}^{n} t_i - \frac{1}{n} n t_n}{\sqrt{\frac{nt^2{}_n}{2}}} \quad Z \sim N(0,1) \tag{44}$$

where, t_i is Times of occurrence of events (hours, minutes,...) for the time period $(t_n, 0)$; n is The number of events that occur in the time period $(t_n, 0)$; Z is the Test value. When a value of Z in the range $\left(-Z_{\frac{\alpha}{2}} < Z < Z_{\frac{\alpha}{2}}\right)$ accept H_0, $\left(Z < Z_{\frac{\alpha}{2}}\right)$or $\left(Z < -Z_{\frac{\alpha}{2}}\right)$ reject H_0. statistically significant at a 5% level.

2.5. Model selection criteria

When using several stochastic processes, how do we choose the best process that fits the data? There are several criteria that can be used to find the best process. The most important of these criteria is the coefficient of determination (R^2) and criteria for the quality of the fit: Bias, mean squared error (MSE), root mean squared error (RMSE), and mean absolute error (MAE). The following numerical indices are commonly used in model evaluation (Song et al., 2019):

$$R^2 = 1 - \frac{\sum_{t=1}^{n}(y_t - \hat{y}_t)^2}{\sum_{t=1}^{n}(y_t - \bar{y})^2} \quad (45)$$

$$RMSE = \sqrt{MSE} = \sqrt{\frac{SSe}{n-P-1}} \quad (46)$$

$$Bias = \frac{1}{n}\sum_{t=1}^{n} e_t \quad (47)$$

$$\text{MAE} = \frac{1}{n}\sum_{t=1}^{n}|e_t| \quad (48)$$

2.6. Reliability function

The reliability function is defined as the probability of success or the probability that the machine will perform its intended function under specified design limits. Mathematical reliability R(t) is the probability that a system will be successful in the interval from time 0 to time t (Choudhury et al., 2021):

$$R(t) = P(T > t), t > 0 \quad (49)$$

where, t is a random variable denoting the time-to-failure or failure time. Unreliability F(t), a measure of failure, is defined as the probability that the system will fail by time t:

$$F(t) = P(T \le t) \text{ for } t > 0 \quad (50)$$

2.7. Hazard function

Defined as limits of the rate of faults for a period of the near-zero equation can be written in the form:

$$h(t) = \frac{f(t)}{R(t)} \quad (51)$$

3. Results and discussion

In this part, the α-series process is compared with the NHPP by conducting trend analysis, then estimating the α-series process and estimating the NHPP parameters, and finally, the comparison between the two processes using some statistical criteria to obtain the best process that fits the data of this study. Fig. 1 shows that the fault time of machines in Bahrri Thermal Station increases in certain periods, stabilizes, and decreases in other periods, which indicates the random occurrence of faults in machines during the study period.

3.1. Trend analysis of process

The first procedure in the statistical analysis of the stochastic process on the fault time of the electricity machines at the Bahrri Thermal Station during the period from January 1, 2021, to July 31, 2021, is whether the process has a monotonous general trend or not. To do this, the Laplace test was used.

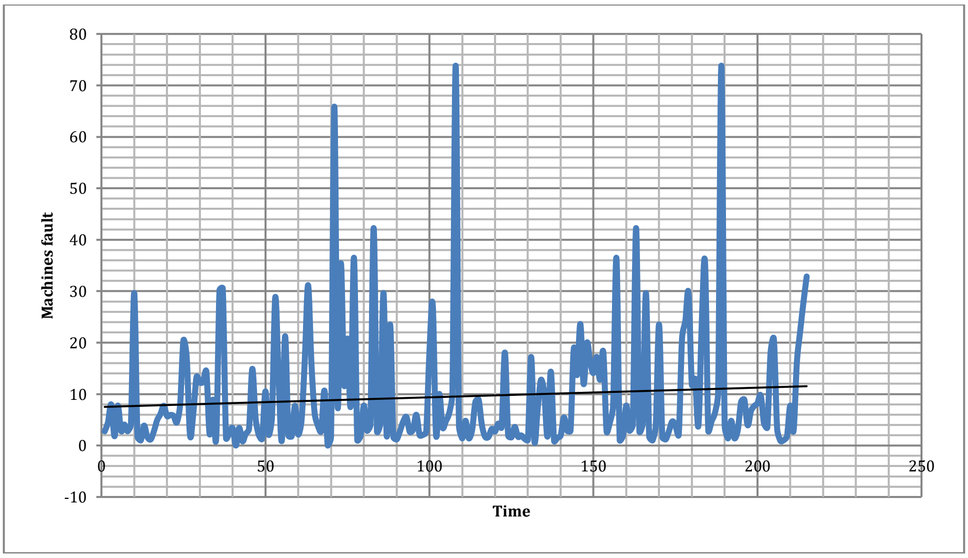

Fig. 1: The daily faults time of machines in Bahrri station for thermal generation

3.1.1. Laplace test for trend analysis

Here we test the following hypothesis:

H_0: Fault time of machines has no monotonous trend (HPP)

H_1: Fault time of machines has monotonous trend (NHPP)

Table 1 shows that the p-value of the Laplace test (0.000) is less than the significance level of 5%, we reject H_0 that indicates the fault time of machines has a monotonous trend. That is, the data fit the α-series process and the NHPP.

Table 1: Laplace test for trend analysis

	Statistic	P-value
Laplace value	50.670	0.000

Fig. 2 shows that the general form of the process is in the trend of increasing, which shows that the process is a stochastically increasing and monotonous process; this was confirmed by Laplace's test in Table 1.

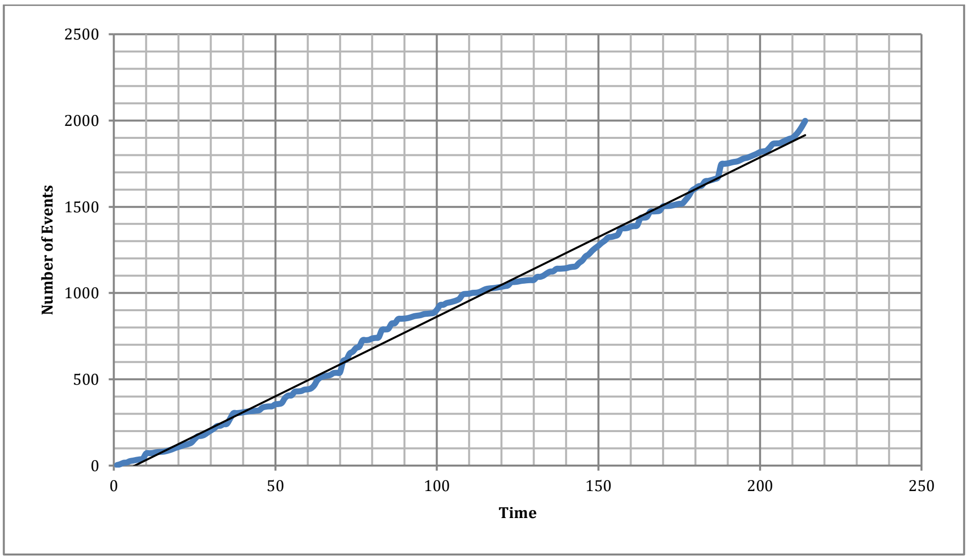

Fig. 2: The cumulative number of faults vs time

3.2. Estimation of the parameters of the α-series process

To estimate the parameters of the α-series of machines' failure time by the least squares method, Mathcad 2000 software was used, Table 2 shows that:

Table 2: Estimators of the α-series process of machines failure time

Parameters	Estimate
$\hat{\alpha}$	-0.1289
$\hat{\gamma}$	1.1523
$\hat{\sigma}_e^2$	1.0131
$\hat{\mu}$	4.1494
$\hat{\sigma}^2$	10.1514
α-series process model:	$\hat{y}_t = 1.5323 + 0.1289x_t$

3.3. Estimation of the parameters of the NHPP

To estimate the parameters of the NHPP of machines' fault time by the Maximum likelihood method, Statgraphics-18 software was used to estimate the parameters in Table 3.

Table 3: Estimators of NHPP of machines faults time

Parameters	Estimate
$\hat{\lambda}$	0.2492
$\hat{\beta}$	0.1294
NHPP model	$(0.2492)(0.1294) * t^{0.8706}$
Fault rate	$(0.2492) * t^{0.1294}$

3.4. Goodness of fit

From Table 4, it was observed that the NHPP model provided the best fit since the model gives the lowest value of MSE 12.1876 which is about 3% less than the α-series process model. On the other hand, this result reflects that the predicted faults time of machines by the NHPP model is very close (in the mean) to actual cumulative faults data.

Table 4: Analysis results comparison

Criteria	Process Model	
	NHPP	α-series
MSE	12.1876	15.4277
RMSE	3.49108	3.9083
Bias	0.0129	3.9278
MPE	1.9767	7.5500
R^2	0.9661	0.9346

We see from the results of Table 5 and Table 2, that the NHPP model is more suitable than the α-series process models to describe the fault time of machines in Bahrri Station for thermal generation. According to the above results we estimated fault rate, mean cumulative number of faults, and mean interevent time for selected values of time or distance, rate of occurrence of machines occurs at an inconstant rate, and the repair rates of machines increase with the increase in time and this indicates the frequent fault time of machines in Bahrri station for thermal generation show in Table 5.

Table 5: Faults rate and mean cumulative number of faults

Time	Rate	Mean cum. faults	Repair rate
0	0.249229	0.000	4.0125
400	0.11482	52.7515	8.7093
800	0.104973	96.4553	9.5263
1200	0.0996097	137.29	10.0392
1600	0.0959711	176.367	11.4198
2000	0.0932406	214.186	12.7249

From Fig. 3, it is clear that the repair rate of machines increases with time and this also confirms the frequent faults time of machines during the study period.

Fig. 4 shows that the reliability of machines decreases with the time the machine increase in until equal to zero. Fig. 5 shows the hazard function increases whenever the time increases too.

Therefore, the unique finding of this study is that it compared between nonhomogeneous Poisson's process and α-series to obtain a better process for using in monotone trend data and prediction, meanwhile, other researchers compared methods of estimation parameters of the α-series process with a related process such as Geometric process, Renewal process.

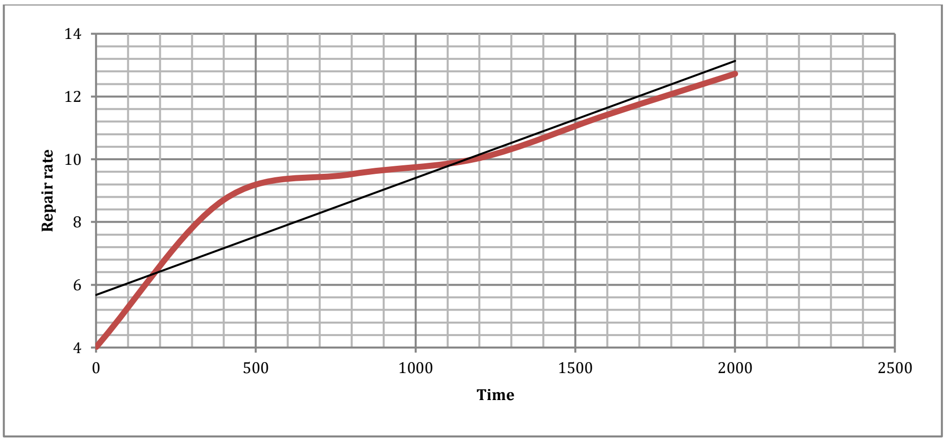

Fig. 3: The daily fault time of machines in Bahrri station of thermal generation

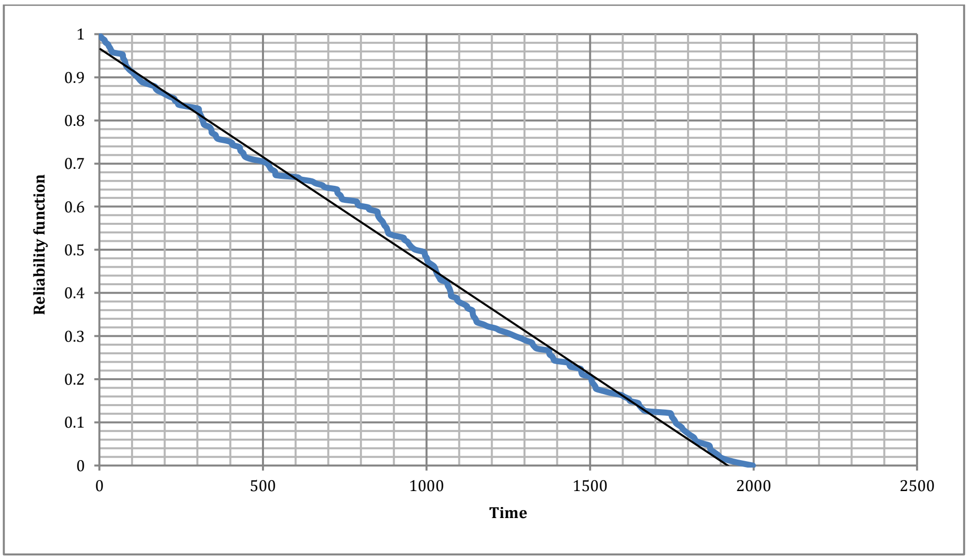

Fig. 4: Reliability function vs time for machine

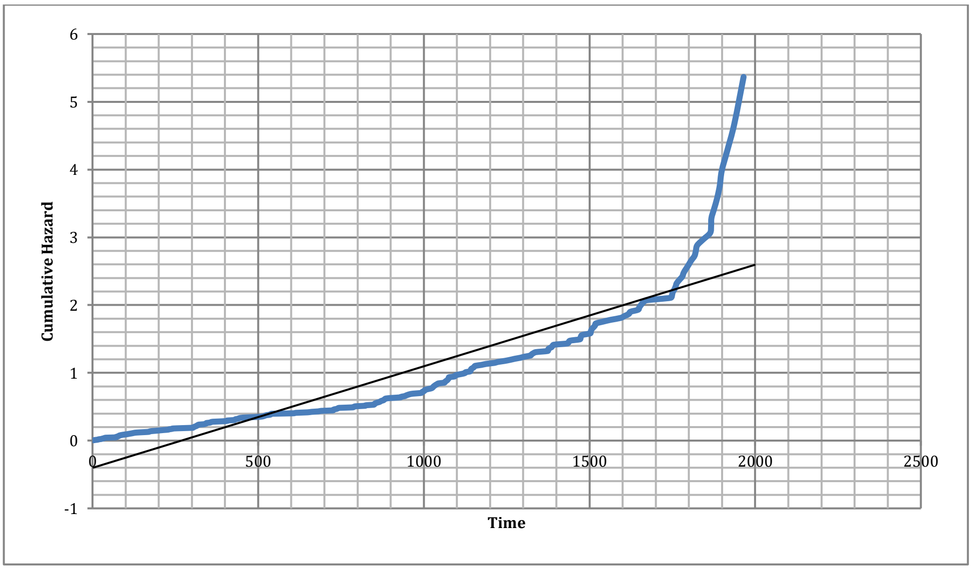

Fig. 5: Cumulative hazard function vs time for machine

4. Conclusion

This study concludes that the NHPP model proved to be very effective in describing the fault time of machines, thus allowing a predicted fault time of machines, and that represents the unique and scientific addition of this study by showing the preference for NHPP model in predicting successive faults of machines. The NHPP model gives the best results with the lowest MSE 12.1876, RMSE 3.49108, Bias 0.0129, and MPE 1.9767, in addition to the highest 0.9661 compared with the α-series process model. On this basis, the rate fault and repair rate increase with time, which confirms that the NHPP is the best model for describing the daily fault time of machines in Bahrri Thermal Station. The estimated reliability function and hazard function proved that as operating time increases the performance of the machines decreases. This confirms the frequent faults of machines in Bahrri Thermal Station. The study recommends depending on the applied study results in the Bahrri Thermal Station. Therefore, the accuracy of the results is conducted. As for future studies, the researchers recommend that to conduct further research, the results of this study should be compared with other processes such as the Point process, geometric process, and renewal process, in order to find out which processes are more suitable for the data.

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Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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