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Intuitionistic fuzzy optimization method for solving multi-objective linear fractional programming problems

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ABSTRACT

An iterative technique based on the use of parametric functions is proposed in this paper to obtain the best preferred optimal solution of a multi-objective linear fractional programming problem (MOLFPP). Each fractional objective is transformed into a non-fractional parametric function using certain initial values of parameters. The parametric values are iteratively calculated and the intuitionistic fuzzy optimization method is used to solve a multi-objective linear programming problem. Also, some basic properties and operations of an intuitionistic fuzzy set are considered. The development of the proposed algorithm is based on the principle of optimal decision set achieved by the intersection of various intuitionistic fuzzy decision sets which are obtained corresponding to each objective function. Additionally, as the intuitionistic fuzzy optimization method utilizes the degree of belonging and degree of non-belonging, we used the linear membership function for belonging and non-belonging to see its impact on optimization and to get insight into such an optimization process. The proposed approaches have been illustrated with numerical examples.

1. Introduction

A fractional programming problem (FPP) is concerned with the optimization problem of one or many ratios of functions subject to some constraints. These ratios are quantities that measure the efficiency of the system, such as cost/time, cost/profit, output/worker, and cost/volume, while many ratios of functions are measured on different scales at the existence of some conflicts. The optimal solution for an objective function may not be an optimal solution for any other objective function. So, one needs to find the notion of the best compromise solution, also known as a non-dominant solution (Lai and Hwang, 1994; Stancu-Minasian and Pop, 2003).

By Hungarian mathematician Martos (1964), the linear fractional programming problem (LFPP) was developed in the 1960s and has a wide range of applications in several important fields such as science, engineering, economics, finance, management, business, information theory, marine transportation, water resources, health care, corporate planning and so forth. Multi-objective Fractional Programming Problem has attracted considerable research interest in recent few years and numerous methods have been suggested in this context for the determination of the optimal solutions.

In the literature, for various kinds of fractional programming, there are several different sorts of studies; some of them deal with theory (Jo and Lee, 1998; Liu and Yokoyama, 1999; Tigan and Stancu-Minasian, 2000; Patel, 2005), and some of them are concerned with solution methods (Stancu-Minasian and Pop, 2003; Dinkelbach, 1967; Arévalo et al., 1997; Calvete and Galé, 1999; Yadav and Mukherjee, 1990; Sakawa et al., 2000; Sakawa and Nishizaki, 2001; Gupta and Bhatia, 2001; Saad, 2005; Mohan and Nguyen, 2001; Güzel, 2013) and applications (Leber et al., 2005). Dinkelbach (1967) suggested the algorithm based on a theorem by Jagannathan (1966) concerning the relationship between fractions and parametric programming and restated and demonstrated this theorem in a somewhat simpler way. Leber et al. (2005) suggested using a fractional programming algorithm (Dinkelbach's (1967) algorithm) to calculate the melting temperature of pairings of two single DNA strands in biology.

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Regarding the parametric approach, Wolf (1986) used parametric linear programming to solve non-linear FPP. Pal. Costa (2007) suggested an approach to solve MOLFPP which goes on dividing the non-dominated region to search for the maximum value of the weighted sum of the objectives. Valipour et al. (2014) presented an algorithm to solve MOLFPP which is an extension of Dinkelbach's (1967) parametric approach to solving the Linear Fractional Programming Problem. Borza et al. (2013) used a parametric method to solve a single objective LFPP with interval coefficients in the objective function. Almogy and Levin (1971) used a parametric approach to solve a problem with the objective defined as the sum of fractional functions. Miettinen (2012) showed numerous approaches to solving multi-objective optimization problems. Zhong and You (2014) suggested a parametric approach to solving mixed integer linear and non-linear fractional programming problems by converting them into the equivalent parametric formulation.

Modeling of most real-world problems, including optimization process turns out to be a multi-objective programming problem in a natural way. Such multi-objective programming problems might in general comprise conflicting objectives. To illustrate that, if we consider a problem of agricultural production planning, the optimal model should have the objectives of maximizing profit and minimizing the inputs and costs of agriculture. Therefore, these objectives are conflicting in nature and hence the solution to such problems is in general compromise solutions which satisfy each objective function to a degree of satisfaction and a concept of belonging and non-belonging arises in such situations. It was Zimmermann (1978, 1983) who first used the fuzzy set presented by Zadeh (1965) for solving the fuzzy multi-objective linear programming problem. Optimization in the fuzzy environment was further studied and was applied in many areas by several researchers (Tanaka and Asai, 1984; Luhandjula, 1989; Sakawa and Yano, 1989). A brief review of studies of several research workers on optimization under uncertainty can be found in the work of Sahinidis (2004).

When the information available is imprecise, imprecise, or uncertain, several extensions of fuzzy sets immerged there is the insight of growing use of a fuzzy set of modeling of problems under situations. In such extensions, Atanassov (2016, 1999) presented the intuitionistic fuzzy sets as a powerful extension of fuzzy sets. Atanassov (2016, 1999) in his studies, emphasized that in view of handling imprecision, vagueness, or uncertainty in information both the degree of belonging and degree of non-belonging should be considered as two independent properties as these are not complements of each other. Angelov (1997) considered the idea of membership and non-membership in optimization problems and gave an intuitionistic fuzzy approach to solve optimization problems. The multi-objective intuitionistic fuzzy linear programming problem applied to transportation problems was studied by Jana and Roy (2007). The inclusion degree of intuitionistic fuzzy set to multi-criteria decision-making, problem was applied by Luo and Yu (2008). Further, several researchers such as Mahapatra et al. (2010), Nachammai and Thangaraj (2012), and Nagoorgani and Ponnalagu (2012) have also considered the linear programming problem under an intuitionistic fuzzy environment. Linear programming problem in an intuitionistic fuzzy environment using intuitionistic fuzzy number and interval uncertainty in fuzzy numbers was studied by Dubey and Mehra (2011) and Dubey et al. (2012). Sharma et al. (2022) developed the concept of mediative fuzzy relation and meditative fuzzy projection in the context of fuzzy relation and fuzzy projection. They extended the basic operations of fuzzy projection into intuitionistic fuzzy projection and then into mediative fuzzy projection. They have shown the credibility and impact of the meditative index factor involving the mediative fuzzy projection in the context of prediction work in relation to their proposed model. After that, they applied the mediative fuzzy projection in the medical diagnosis of post-COVID-19 patients.

Proposed approaches to solve a MOLFPP using the concept of parametric functions and under intuitionistic fuzzy optimization together are considered in this paper. It converts the Linear Fractional Programming Problem to a suitable non-fractional problem using certain parameters to find a set of non-inferior solutions through iterative computations. Termination conditions are imposed on all the objectives by the Decision Maker to determine the best preferred optimal solution at which a certain level of satisfying optimality is attained by all the objective functions.

The organization of the paper is as follows: in Section. 2 regarding parametric approach transformation to MOLFPP. In section 3 some definitions of multi-objective linear programming problems. In Section. 4 intuitionistic Fuzzy optimization method to solve multi-objective linear programming problems. In Section. 5 computational algorithm and procedures of solution. In Section. 6 numerical examples for illustrating the solution of proposed approaches. Finally, concluding remarks are given in section 7.

2. Parametric approach

Consider the following single objective fractional programming and parametric, non-fractional programming problems respectively (Nayak and Ojha, 2019).

$$Problem - I: \underset{x \in S}{Min} \frac{N(x)}{D(x)} \quad (1)$$

$$Problem - II: \underset{x \in S}{Min} \{N(x) - \gamma D(x)\}, \quad (2)$$

where, γ is a parameter and S is the non-empty compact feasible region in which both N and D are continuous functions with $D(x) > 0, \ \forall \ x \in S$.

Theorem 2.1: x^* is an optimal solution to Problem-I if and only if $\underset{x \in S}{Min}\ \{N(x) - \gamma D(x)\} = 0$ where $\gamma^* = \frac{N(x^*)}{D(x^*)}$.

Consider the following multi-objective linear fractional programming and parametric linear programming problems respectively.

$$Problem - III: \underset{x \in S}{Min}\ \frac{N_i(x)}{D_i(x)} \quad i = 1, 2, \dots, k \tag{3}$$

$$Problem - IV: \underset{x \in S}{Min}\ \{N_i(x) - \gamma^* D_i(x)\} \quad i = 1, 2, \dots, k \tag{4}$$

Assume that $D(x) > 0,\ \forall\, x \in S$ and $\gamma_i{}^* = \frac{N_i(x^*)}{D_i(x^*)}$ $where\ x^* \in S$.

Remark 2.1: x^* is Pareto optimal solution of Problem-IV if for each $x \in S$, $N_i(x) - \gamma^* D_i(x) = 0\ \forall\, i\ or\ N_j(x) - \gamma^* D_j(x) > 0\ for\ at\ least\ one\ j \in \{1, 2, \dots, k\}$.

Using the above Remark 2.1 and Theorem 2.1 due to Dinkelbach (1967), the following results are achieved.

Theorem 2.2: x^* is Pareto's optimal solution to Problem-III if and only if x^* is Pareto's optimal solution to Problem-IV. The Proof of this theorem is in Nayak and Ojha (2019).

Theorem 2.3: The Pareto optimal solutions of Problem-IV are also Pareto optimal of Problem-III if x^* is Pareto optimal of Problem-IV. The Proof of this theorem is in Nayak and Ojha (2019).

3. Multi-objective linear programming problem

Many objective functions are optimized simultaneously with respect to a common set of constraints in the multi-objective optimization problem. Frequently there doesn´t exist a single optimal solution that optimizes all the objectives together with their respective best satisfactory level. In such cases, a set of Pareto optimal solutions are generated using an appropriate method available in the literature, and the best preferred (compromise) optimal solution that satisfies all the objectives with the best possibility, is determined by the decision maker comparing the objective values in accordance to own desire on a priority basis or the requirement of the system. A multi-objective optimization problem (Ehrgott, 2005; Miettinen, 2012) can be mathematically stated as:

$$\text{Min}\ z(x) = \left(z_1(x), z_2(x), \dots, z_k(x)\right)$$
$$\text{s.t} \quad x \in S \tag{5}$$

where, $x \in \mathbb{R}^n$ and $z_i : \mathbb{R}^n \to \mathbb{R}$ $\quad i = 1, 2, \dots, k$; S is the set of constraints, considered as a non-empty compact feasible region. A multi-objective linear programming problem is otherwise called a multi-criterion optimization or vector optimization problem, where as a Pareto optimal solution is otherwise called a non-inferior or non-dominated, or efficient solution.

Definition 3.1: $x^* \in S$ is a Pareto optimal solution of the multi-objective linear programming (Eq. 5) if there does not exist another feasible solution $\bar{x} \in S$ such that $z_i(\bar{x}) \leq z_i(x^*)\ \forall\, i\ and\ z_j(\bar{x}) < z_j(x^*)$ for at least one j.

Definition 3.2: $x^* \in S$ is a weak Pareto optimal solution of the multi-objective linear programming (Eq. 5) if there does not exist another feasible solution $\bar{x} \in S$ such that $z_i(\bar{x}) < z_i(x^*)\ \forall\, i$.

Definition 3.3: Trade-off or Pareto front is a part of the objective feasible region which consists of the objective values evaluated at the Pareto optimal solutions of the multi-objective linear programming.

Definition 3.4: Ideal objective vector has the coordinates which are obtained by evaluating the values of the objectives at their respective individual minimal points.

Definition 3.5: Nadir's objective vector has the coordinates which are the respective worst objective values when the set of solutions is restricted to the trade-off.

4. Intuitionistic fuzzy optimization

4.1. Intuitionistic fuzzy set (IFS)

Let X be a non-empty set and I=[0, 1], then an IFS $\tilde{A}$ is defined as a set $\tilde{A} = \{< x,\ \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) >: x \in X\}$ where: $\mu_{\tilde{A}}: X \to I$ and $\nu_{\tilde{A}}: X \to I$ denotes the degree of belonging and the degree of non-belonging with $0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1\ for\ each\ x \in X$.

Additional, every fuzzy set A on a non-empty set X with a membership function $\mu_{\tilde{A}}$ is obviously AX if with $\nu_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x)$ and so IFS is a generalization of a fuzzy set.

Here union and intersection of two intuitionistic fuzzy sets are defined as:

$$\tilde{A} \cap \tilde{B} = \{[x, \text{Min}(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)), \text{Max}(\nu_{\tilde{A}}(x), \nu_{\tilde{B}}(x))] \mid x \in X\}$$
$$\tilde{A} \cup \tilde{B} = \{[x, \text{Max}(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)), \text{Min}(\nu_{\tilde{A}}(x), \nu_{\tilde{B}}(x))] \mid x \in X\}.$$

Fuzzy Optimization Technique Max-min approach Zimmermann (1978, 1983) first used the max-min operator given by Bellman and Zadeh (1970) to solve multi-objective linear programming (MOLP) problems and considered the problem in Eq. 5 as:

Find X

$$\text{s.t}\ Z_k(x) \gtrsim g_k, \quad k = 1, 2, \dots, p \qquad \text{v} \tag{6}$$
$$g_j(x) \leq 0,\ i = 1, 2, \dots, q$$
$$x \geq 0$$

where, $g_k\ \forall\, x$ denote goals and all objective functions are assumed to be maximized. Here objective functions are considered fuzzy constraints. To find membership functions of objective functions, we could first obtain the table of positive ideal solutions (PIS). Under the concept of a min-operator, the feasible solution set is defined by the interaction of the fuzzy objective set. This feasible solution set is then characterized by its membership $\mu_D(x)$ which is:

$$\mu_D(x) = \text{Min}(\mu_1(x), \dots, \mu_k(x))$$

Additionally, a decision maker makes a decision with a maximum μ_D Value in the feasible decision sets. The decision solution can be obtained by solving the problem of maximizing $\mu_D(x)$ subject to the given constraints i.e.,

$$\begin{aligned}&\text{Max}\,[\text{Min}\,\mu_k(x)]\\&\text{s.t}\\&g_j(x) \leq 0\ , i = 1, 2, \dots, q\end{aligned} \tag{7}$$

Now, if suppose $\alpha = min_k\, \mu_k(x)$ be the overall satisfaction level of compromise, then we obtain the following equivalent model:

$$\begin{aligned}&\text{Max}\,\alpha\\&\text{s.t}\quad \mu_k(x) \geq \alpha\,, \forall\, k\\&g_j(x) \leq 0\ , i = 1, 2, \dots, q\\&x \geq 0\end{aligned} \tag{8}$$

4.2. Intuitionistic fuzzy method (IFM)

Consider the intuitionistic fuzzy optimization problem as a generalization of the above problem undertaken by Angelov (1997).

$$\begin{aligned}&\min f_i(x),\ i = 1, 2, \dots, p\\&g_j(x) \leq 0\ , i = 1, 2, \dots, q\end{aligned} \tag{9}$$

where, x is the decision variable, $f_i(x)$ denotes objective functions, $g_j(x)$ denotes the constraint functions, p and q denote the number of objective functions and constraints respectively.

The optimal solution to this problem must satisfy all constraints exacting. Thus an analogous fuzzy optimization model of the problem the degree of acceptance of objectives and constraints is maximized as:

$$\begin{aligned}&\widetilde{\min}\, f_i(x),\ i = 1, 2, \dots, p\\&g_j(x) \lesssim 0\ , i = 1, 2, \dots, q\end{aligned} \tag{10}$$

where, $\widetilde{min}$ Denotes fuzzy minimization and$\lesssim$denotes fuzzy inequality.

For the solution of this system (Eq. 10), Bellman and Zadeh (1970) used fuzzy set maximize for the degree of membership of the objectives and constraints as:

$$\text{Max}\ \mu_k(x)\,,\ x \in X\ ,\ k = 1, 2, \dots, p+q \quad 0 \leq \mu_k(x) \leq 1 \tag{11}$$

where, $\mu_k(x)$ Denotes the degree of satisfaction to respective fuzzy sets.

It is important to understand that in a fuzzy set the degree of non-membership complements membership, hence maximization of membership function will automatically minimize the non-membership. But in the intuitionistic fuzzy set degree of rejection is defined simultaneously by the degree of acceptance and both these degrees are not complementing each other, hence IFS may give a more general tool for describing this uncertainty-based optimization model.

Thus, the intuitionistic fuzzy optimization (IFO) model for the problem in Eq. 8 is given as:

$$\begin{aligned}&\underset{x}{\text{Max}}\{\mu_k(x)\}\,, x \in X\ ,\ k = 1, 2, \dots, p+q\\&\underset{x}{\text{Min}}\{v_k(x)\}\,,\quad k = 1, 2, \dots, p+q\\&\text{s.t}\\&v_k(x) \geq 0\,,\ \ k = 1, 2, \dots, p+q\\&\mu_k(x) \geq v_k(x),\ \ k = 1, 2, \dots, p+q\\&\qquad \mu_k(x) + v_k(x) \leq 1,\ \ k = 1, 2, \dots, p+q\end{aligned} \tag{12}$$

where, $\mu_k(x)$ denotes the degree of acceptance of x to the k^{th} IFS and $v_k(x)$ Denotes the degree of rejection of x from the k^{th} IFS. These IFS include intuitionistic fuzzy objectives and constraints.

Now the decision set $\tilde{D}$ the conjunction of intuitionistic fuzzy objectives and constraints is defined as:

$$\tilde{F} \cap \tilde{C} = \{[x, \text{Min}(\mu_{\tilde{F}}(x), \mu_{\tilde{C}}(x)), \text{Max}(v_{\tilde{F}}(x), v_{\tilde{C}}(x))], \mid x \in X\} \tag{13}$$

where, $\tilde{F}$ is integrated intuitionistic fuzzy objective and $\tilde{C}$ denotes integrated intuitionistic fuzzy constraints and is defined as:

$$\tilde{F} = \{x, \mu_{\tilde{F}}(x), v_{\tilde{F}}(x)] \mid x \in X\} = \cap_{i=1}^{p} \tilde{F}^{(i)} = \left\{x, min_{i=1}^{p}\, \mu_i^f(x), max_{i=1}^{p}\, v_i^f(x)\, \middle|\, x \in X\right\} \tag{14}$$

$$\tilde{C} = \{x, \mu_{\tilde{C}}(x), v_{\tilde{C}}(x)] \mid x \in X\} = \cap_{j=1}^{q} \tilde{C}^{(j)} = \left\{x, min_{j=1}^{q}\, \mu_j^g(x), max_{j=1}^{q}\, v_j^g(x)\, \middle|\, x \in X\right\} \tag{15}$$

Further, the intuitionistic fuzzy decision set (IFDS) denoted as $\widetilde{D}$:

$$\widetilde{D} = \tilde{F} \cap \tilde{C} = \{x, \mu_{\widetilde{D}}(x), v_{\widetilde{D}}(x)] \mid x \in X\} \tag{16}$$

$$\mu_{\widetilde{D}}(x) = Min\,[\mu_{\tilde{F}}(x), \mu_{\tilde{C}}(x)] = min_{k=1}^{p+q} \mu_k(x) \tag{17}$$

$$v_{\widetilde{D}}(x) = Max\,[v_{\tilde{F}}(x), v_{\tilde{C}}(x)] = max_{k=1}^{p+q} v_k(x) \tag{18}$$

where, $\mu_{\widetilde{D}}(x)$ Denotes the degree of acceptance of IFDS and $v_{\widetilde{D}}(x)$ Denotes the degree of rejection of IFDS.

Now for the feasible solution, the degree of acceptance of IFDS is always less than or equal to the degree of acceptance of any objective and constraint and the degree of rejection of IFDS is always more than or equal to the degree of rejection of any objective and constraint, i.e.,

$$\mu_{\widetilde{D}}(x) \leq \mu_k(x)\,, v_{\widetilde{D}}(x) \geq v_k(x)\,, \forall\ \ k = 1, \dots\dots, p+q$$

Thus the above system can be transformed into the following system of inequalities:

$$\begin{aligned}&\alpha \leq \mu_k(x)\,,\ k = 1, \dots\dots, p+q\\&\beta \geq v_k(x)\,,\quad k = 1, \dots\dots, p+q\\&\alpha + \beta \leq 1\\&\alpha - \beta \geq 0\\&\beta \geq 0\,,\quad x \in X\end{aligned} \tag{19}$$

where, α denotes the minimum acceptable degree of objective(s) and constraints, and β denotes the maximum degree of rejection of objective(s) and constraints.

Now using the Intuitionistic fuzzy optimization problem, Eq. 8 is transformed into the linear programming problem given as:

$$\begin{aligned}
&\text{Max } (\alpha - \beta)\\
&\text{s.t}\\
&\alpha \le \mu_k(x)\,,\ k = 1, \dots\dots, p+q\\
&\beta \ge v_k(x)\,,\quad k = 1, \dots\dots, p+q \qquad (20)\\
&\alpha + \beta \le 1\\
&\alpha - \beta \ge 0\\
&\beta \ge 0\\
&x \in X
\end{aligned}$$

Now this linear programming problem can be easily solved by a simplex method to give a solution to the multi-objective linear programming problem (Eq. 8) by an intuitionistic fuzzy optimization approach. Fig. 1 illustrates the linear membership and linear non-membership functions.

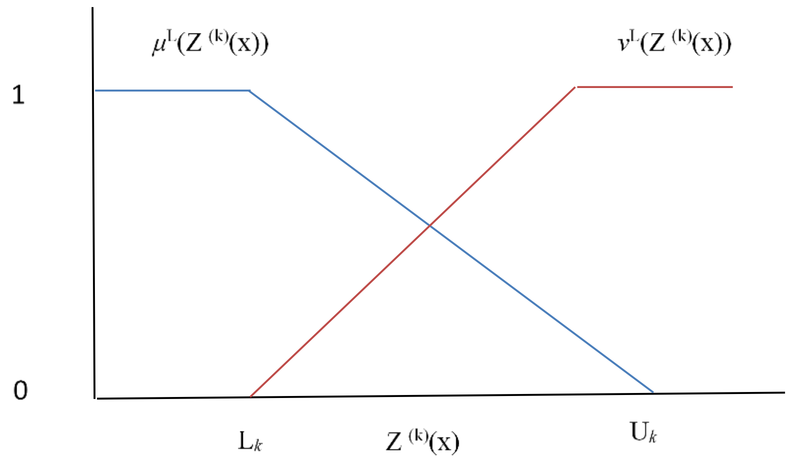

Fig. 1: The linear membership and the linear non-membership functions

5. Computational algorithm

The algorithm of intuitionistic fuzzy optimization with linear membership function is as follows:

After transforming each fractional objective to a non-fractional parametric function using certain initial values of parameters. The steps for solving the multi-objective linear programming problem will be:

- Step 1: Pick the first objective function and solve it as a single objective subject to the constraints. Continue the process k-times for k different objective functions. Find the value of objective functions and decision variables.
- Step 2: To build membership functions, goals, and tolerances should be determined first.

Using the ideal solutions, obtained in Step 1 we find the values of all the objective functions at each ideal solution and construct a payoff table as Table 1.

Table 1: Payoff table

Z	Z_1	Z_2	...	Z_K
Z^1	Z_1^*	Z_{12}	...	Z_{1K}
Z^2	Z_{21}	Z_2^*	...	Z_{2K}
.	.	.		.
.	.	.	...	.
.	.	.		.
Z^K	Z_{K1}	Z_{K2}	...	Z_K^*

where, z^* *and* z_K are the maximum and the minimum values (in max problems and opposite in min problems) respectively.

- Step 3: From Step 2 the upper and lower bounds of each objective function are as follows:

$$U_k^\mu = max\,\{Z_k(x_r^*)\} \text{ and } L_k^\mu = min\,\{Z_k(x_r^*)\} \qquad where\ 1 \le r \le k$$

For membership of objectives.

- Step 4: the upper and lower bounds for non-membership of objectives as follows:

$$U_k^v = U_k^\mu$$
$$L_k^v = L_k^\mu + \tau\left(U_k^\mu - L_k^\mu\right) \qquad 0 \le \tau \le 1$$

- Step 5: Use the following linear membership function $\mu_k(z_k(x))$ and non-membership function $v_k(z_k(x))$ for each objective function:

$$\mu_k(z_k(x)) = \begin{cases} 0, & z_k(x) \le L_k^\mu \\ \frac{z_k(x) - L_k^\mu}{U_k^\mu - L_k^\mu} & L_k^\mu \le z_k(x) \le U_k^\mu \\ 1, & z_k(x) \ge U_k^\mu \end{cases} \qquad (21)$$

and

$$v_k(z_k(x)) = \begin{cases} 0, & z_k(x) \ge U_k^\mu \\ \frac{U_k^\mu - z_k(x)}{U_k^\mu - L_k^\mu} & L_k^\mu \le z_k(x) \le U_k^\mu \\ 1, & z_k(x) \le L_k^\mu \end{cases} \qquad (22)$$

- Step 6: Now the intuitionistic fuzzy optimization model for multi-objective linear programming problems gives an equivalent linear programming problem as:

$$\begin{aligned}&\max\ (\alpha-\beta)\\&\text{s.t}\\&z_k(x)-\alpha\left(U_k^{\mu}-L_k^{\mu}\right)\le L_k^{\mu}\\&z_k(x)+\beta\left(U_k^{\mu}-L_k^{\mu}\right)\le U_k^{\mu}\\&\alpha+\beta\le 1\\&\alpha-\beta\ge 0\\&\beta\ge 0\\&g_j\le b_i\\&x\ge 0\end{aligned}\tag{23}$$

- Step 7: The above linear programming in problem of Eq. 23 can be easily solved by the simplex method in any program like LINDO, WinQSP, and TORA.

6. Numerical examples

Example 1 (Nayak and Ojha, 2019):

$$\begin{aligned}&\text{Min } z_1(x)=\frac{-x_1+3x_2+2}{x_1+2x_2+1}\\&\text{Min } z_2(x)=\frac{5x_1+2x_2+2}{2x_1+3x_2+1}\\&\text{s.t}\\&2x_1+x_2\le 4\\&3x_1-2x_2\le 5\\&x_1+2x_2\le 3\\&x_1+3x_2\ge 2\\&x_1,x_2\ge 0\end{aligned}\tag{24}$$

- Step 1: Solution due to proposed method Using (Charnes and Cooper, 1962) variable transformation technique, it is obtained that $X_1 = (1.7272, 0.0910)$ *and* $X_2 = (0, 1.5)$ are the individual optimal solutions of the objectives $z_1(x)$ *and* $z_2(x)$ respectively.
- Step 2: The range of best and worst values of the objectives is determined using payoff Table 1 as:

$$0.1875\le z_1(x)\le 1.6250$$
$$0.9091\le z_2(x)\le 2.2884$$

Assigning equal weights, i.e., $w_1 = w_2 = 0.5$, the initial solution of the proposed iterative method is obtained as $X^{(0)} = w_1X_1 + w_2X_2 = (0.8636, 0.7955)$. So the initial vector of parameters is obtained as:

$$\gamma^{(1)}=\left(\gamma_1^{(1)},\gamma_2^{(1)}\right)=\left(z_1\left(X^{(0)}\right),z_2\left(X^{(0)}\right)\right)=(1.0198, 1.5466)$$

The fractional objectives can be parametrically linearized as:

$$Z_1\left(\gamma^{(1)}\right)=(-x_1+3x_2+2)-\gamma_1^{(1)}(x_1+2x_2+1)$$
$$Z_2\left(\gamma^{(1)}\right)=(5x_1+2x_2+2)-\gamma_1^{(1)}(2x_1+3x_2+1)$$

Then the new objectives (non-fractional) will be like this:

$$Z_1\left(\gamma^{(1)}\right)=-2.0198x_1+0.9604x_2+0.9802$$
$$Z_2\left(\gamma^{(1)}\right)=1.9068x_1-2.6398x_2+0.4534$$

- Step 3: $U_1^{\mu} = 1.6250$ *and* $L_1^{\mu} = 0.1875$ $U_2^{\mu} = 2.2884$ *and* $L_2^{\mu} = 0.9091$
- Step 4: $U_1^{\nu} = 1.6250$ *and* $L_1^{\nu} = 1.4375$ $U_2^{\nu} = 2.2884$ *and* $L_2^{\nu} = 1.3793$
- Steps 5, 6:

$$\begin{aligned}&\max\ (\alpha-\beta)\\&\text{s.t}\\&2.0198x_1-0.9604x_2-1.4375\alpha\ge 0.7927\\&-2.0198x_1+0.9604x_2+1.4375\beta\le 0.6448\\&1.9068x_1-2.6398x_2-1.3793\alpha\le 0.4548\\&1.9068x_1-2.6398x_2+1.3793\beta\le 1.8350\\&\alpha+\beta\le 1\\&\alpha-\beta\ge 0\\&2x_1+x_2\le 4\\&3x_1-2x_2\le 5\\&x_1+2x_2\le 3\\&x_1+3x_2\ge 2\\&x_1,x_2,\alpha,\beta\ge 0\end{aligned}\tag{25}$$

After solving the problem in Eq. 25, the solution is:

$$X^{(1)}=(1.28884, 0.237054)$$
$$\left(z_1\left(X^{(1)}\right),z_2\left(X^{(1)}\right)\right)=(0.5147, 2.07942)$$

The new vector of parameters is computed as:

$$\gamma^{(2)}=\left(\gamma_1^{(2)},\gamma_2^{(2)}\right)=\left(z_1\left(X^{(1)}\right),z_2\left(X^{(1)}\right)\right)=(0.5147, 2.07942)$$

After repeating all steps with a new model, the solution will be:

$$X^{(2)}=(0.54055, 0.486483)$$
$$\left(z_1\left(X^{(2)}\right),z_2\left(X^{(2)}\right)\right)=(1.16128, 1.6030)$$

Then the optimal solution will be $X^{(1)}$ According to remark 1 in section 2.

In Table 2, It is observed that the optimal objective values $z_1(x), z_2(x)$ obtained due to the proposed method are considerably closer and comparable to that of ε-constraint method in Nayak and Ojha (2019) where as $\left(f_1(x), f_2(x)\right) = (0.9561, 1.5378)$.

Example 2 (Güzel, 2013):

$$\begin{aligned}&\text{Max } z_1(x)=\frac{-3x_1+2x_2}{x_1+x_2+3}\\&\text{Max } z_2(x)=\frac{7x_1+x_2}{5x_1+2x_2+1}\\&\text{s.t}\\&x_1-x_2\ge 1\\&2x_1+3x_2\le 15\\&x_1\ge 3\\&x_1,x_2\ge 0\end{aligned}\tag{26}$$

- Step 1: Solution due to proposed method Using (Charnes and Cooper, 1962) variable transformation technique, it is obtained that $X_1 = (3.597, 2.603)$ *and* $X_2 = (0.7500, 0)$ are the individual optimal solutions of the objectives $z_1(x)$ *and* $z_2(x)$ respectively.

- Step 2: The range of best and worst values of the objectives is determined using payoff Table 1 as:

$$-2.1428 \leq z_1(x) \leq -0.6086$$
$$1.1487 \leq z_2(x) \leq 1.3636$$

Assigning equal weights, i.e., $w_1 = w_2 = 0.5$, the initial solution of the proposed iterative method is obtained as $X^{(0)} = w_1 X_1 + w_2 X_2 = (2.1735, 1.3015)$. So the initial vector of parameters is obtained as:

$$\gamma^{(1)} = \left(\gamma_1^{(1)}, \gamma_2^{(1)}\right) = \left(z_1(X^{(0)}), z_2(X^{(0)})\right) = (-0.6050, 1.1413)$$

The fractional objectives can be parametrically linearized as:

$$Z_1(\gamma^{(1)}) = (-3x_1 + 2x_2) - \gamma_1^{(1)}(x_1 + x_2 + 3)$$
$$Z_2(\gamma^{(1)}) = (7x_1 + x_2) - \gamma_1^{(1)}(5x_1 + 2x_2 + 1)$$

Then the new objectives (non-fractional) will be like this:

$$Z_1(\gamma^{(1)}) = -2.395x_1 + 2.605x_2 + 1.815$$
$$Z_2(\gamma^{(1)}) = 1.2935x_1 - 1.2826x_2 - 1.1413$$

- Step 3: $U_1^{\mu} = -0.6086$ *and* $L_1^{\mu} = -2.1428$ $U_2^{\mu} = 1.3636$ *and* $L_2^{\mu} = 1.1487$
- Step 4: $U_1^{\nu} = -0.6086$ *and* $L_1^{\nu} = -1.9893$ $U_2^{\nu} = 1.3636$ *and* $L_2^{\nu} = 1.17019$
- Steps 5, 6:

Max $(\alpha - \beta)$
s.t

$$\begin{aligned}
&2.395x_1 + 2.605x_2 + 1.5342\alpha \geq 0.3278\\
&2.395x_1 - 2.605x_2 - 1.3807\beta \geq 2.4236\\
&1.2935x_1 - 1.2826x_2 - 0.2149\alpha \leq 2.29\\
&1.2935x_1 - 1.2826x_2 + 0.1934\beta \leq 25049\\
&\alpha + \beta \leq 1\\
&\alpha - \beta \geq 0\\
&x_1 - x_2 \geq 1\\
&2x_1 + 3x_2 \leq 15\\
&x_1 \geq 3\\
&x_1, x_2, \alpha, \beta \geq 0.
\end{aligned} \tag{27}$$

After solving the problem in Eq. 27, the solution is:

$$X^{(1)} = (3, 1.82779)$$
$$\left(z_1(X^{(1)}), z_2(X^{(1)})\right) = (0.6827, 1.1613)$$

The new vector of parameters is computed as:

$$\gamma^{(2)} = \left(\gamma_1^{(2)}, \gamma_2^{(2)}\right) = \left(z_1(X^{(1)}), z_2(X^{(1)})\right) = (-0.6827, 1.1613)$$

After repeating all steps with a new model, the solution will be:

$$X^{(2)} = (3, 1.60107)$$
$$\left(z_1(X^{(2)}), z_2(X^{(2)})\right) = (-0.7627, 1.1770)$$

Then the optimal solution will be $X^{(1)}$ According to remark 1 in section 2.

In Table 2, It is observed that the optimal objective values $z_1(x), z_2(x)$ Obtained due to the proposed method are considerably closer and comparable to that of the method by Güzel (2013) where as $(z_1, z_2) = (-0.625, 1.15)$.

Table 2: Comparative results

Obj. Fun.	Min $z_1(x)$	Min $z_2(x)$	Obj. Fun.	Max $z_1(x)$	Max $z_2(x)$
Proposed	0.5147	2.07942	Proposed	-0.6827	1.1613
ε-constraint	0.9561	1.5378	Güzel (2013)	-0.625	1.15

7. Conclusion

In this paper, a parametric approach is used to transform the multi-objective linear fractional programming into non-fractional multi-objective linear programming by using a vector of parameters. The values of the parameters are changed from one to another step in order to generate a new set of Pareto optimal solutions using an intuitionistic fuzzy optimization algorithm for solving MOLPP. The purpose of this paper is to give the most effective algorithm for an intuitionistic fuzzy optimization method for getting optimal solutions after transforming the MOLFPP into a multi-objective linear programming problem. The value of the method lies in the fact that it gives a set of solutions with various levels of satisfaction to the decision makers. The decision makers may choose a suitable optimal solution according to the demand of the actual situation. The solution achieved due to the proposed approaches is compared with the ε constraint method, fuzzy programming, and Güzel (2013) proposed method which verifies the effectiveness of its performance and considerably closer and comparable to them. The computational works in the numerical examples are carried out using the softwares LINGO and WinQSP.

Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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