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H_∞ reliable fuzzy control for vehicle dynamics stability

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ABSTRACT

This article introduces a reliable control scheme for a four-wheel vehicle. This scheme assumes that actuators fail and external disturbances occur to the system. In contrast to existing results, this study assumes the actuator fault model includes linear and nonlinear terms, and an output feedback controller is designed to improve vehicle stability and handling when actuators fail. Using Takagi-Sugeno (T-S) fuzzy models, a reliable fuzzy static output feedback (SOF) controller is designed to address the nonlinear aspect of the system. Based on the non-quadratic Lyapunov function with auxiliary matrices, less conservative sufficient conditions are established such that the closed-loop system is stable with a γ level of H_∞ performance against external disturbances. Furthermore, using an appropriate model transformation, a set of linear matrix inequalities (LMIs) is formulated to synthesize the controller gains. The proposed scheme is then tested using numerical experiments to demonstrate potential applications and validate its effectiveness.

1. Introduction

The automobile industry is continually striving to develop active control systems that enhance the stability and performance of vehicles in dangerous situations. Note that, the development of active control systems to achieve enhanced vehicle stability and comfort continues to be a topic of active investigation (Wang et al., 2020; 2015a; 2016b; Jin et al., 2018; Latrach et al., 2013). Ultimately, we want to produce vehicles that everyone can safely operate.

Due to their complex models, the development of vehicle control systems is a significant challenge. However, we know that to cope with nonlinear systems, fuzzy logic might come up with an innovative solution for the design analysis and control synthesis of various industrial plants. Alternatively, Takagi–Sugeno (T-S) fuzzy models exhibit an excellent ability to express nonlinear systems through the combination of fuzzy logic and linear control theories (Takagi and Sugeno, 1985; Latrach et al., 2015; Kchaou et al., 2011). Therefore, the application of the T-S fuzzy model greatly expands the research field of nonlinear control theory. As a result, a rich literature related to controller design, filtering design, and stability analysis on T-S has been published (Shi et al., 2020; Makni et al., 2019; Tao et al., 2018). Additionally, the T-S fuzzy model is recently investigated to deal with vehicle models. In Wang et al. (2016a), the yaw control issue for in-wheel-motor electric ground vehicles is investigated based on the differential steering and in the presence of the complete failure of the steering system. Dahmani et al. (2013) proposed a fuzzy-model-based roll state estimator for a three-degrees-of-freedom vehicle model in the presence of the road bank.

The aforementioned control results of vehicle systems are assumed to be under ideal working conditions. However, such systems may experience catastrophic results in the event of a component system failure. Because actuator/sensor failures can have a negative impact on system performance, control communities are interested in this control problem. The goal of this issue is to introduce the concept of fault-tolerant control (FTC) and fault diagnosis as critical approaches for designing reliable controllers that are capable of maintaining the critical functionality of systems subject to problems and failures. Reviewing the literature, many elegant reported results related to this area have been proposed for different classes of systems (Wang et al., 2018; Kchaou et al., 2021; Yan et al., 2019). Kaviarasan et al. (2016) developed a method for designing fault-tolerant controllers for power systems subject to random changes and actuator

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failures in Kaviarasan et al. (2016). The FTC method for wind-diesel hybrid systems with time-varying bounded sensor faults has been proposed by Kamal et al. (2013). In Wang et al. (2015b), the reliable observer-based control problem for discrete-time Takagi-Sugeno fuzzy systems with time-varying delay and stochastic actuator faults is formulated from the input-output approach. The state variables of feedback control are not all measurable, which is another vulnerability in feedback control. The concept of static output feedback (SOF) as an alternative to state feedback has long been admitted as a compelling yet challenging implementation method in T-S fuzzy model-based design (Latrech et al., 2018; Regaieg et al., 2019).

Having been inspired by the statements above, in this article, a reliable output feedback control scheme will be investigated to control the vehicle lateral motion in the presence of exogenous disturbance and actuator faults. The main contributions of this paper are highlighted as follows:

1. Instead of existing fuzzy static output control schemes (Latrech et al., 2018; Kang and Lee, 2018), this study introduces a new model of the fault including a non-linear part to deal with reliable control problems for T-S fuzzy systems subject to exogenous disturbance and nonlinear actuator faults.
2. Based on non-quadratic Lyapunov functional, sufficient conditions for stability with H_∞ performance of the resulting closed-loop system is derived.
3. Different from the results suggested in Latrech et al. (2018) and Kang and Lee (2018), where the descriptor-redundancy scheme is adopted to design the SOF controller, in this study, we examine the properties of some specific slack matrices to convert bi-linear matrix conditions to LMI ones using an appropriate model transformation. Moreover, the design conditions are derived as a one-step LMI problem without ensuing equality constraints.

Notations: The notations used in this paper are standard, where $X \in \mathbb{R}^n$ is the set of $n-$dimensional Euclidean space; $X \in \mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices; $X > 0$ is the real symmetric positive definite matrix; $sym(X)$ stands for $X + X^T$. We note also $X_\eta = \sum_{i=1}^{r} \eta_i$, $X_{\eta\eta} = \sum_{i=1}^{r}\sum_{j=1}^{r} \eta_i\eta_j X_{ij}$, and $\dot{X}_\eta = \sum_{l=1}^{r} \dot{\eta}_l X_l$. $*$ denotes the term that is induced by symmetry.

2. Vehicle model

In this section, the lateral dynamics of a vehicle are modeled based on a bicycle model as shown in Fig. 1. Based on the assumption that the front-wheel steering angle is small, the vehicle's dynamics in the yaw plane are defined by the following differential equations (Latrech et al., 2018; Dahmani et al., 2013).

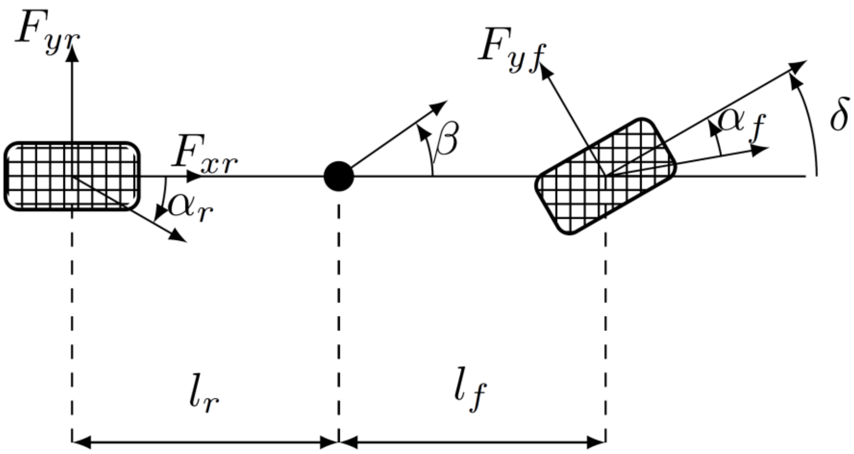

Fig. 1: Vehicle lateral yaw dynamics model

$$\begin{cases} \dot{\beta} &= \frac{2F_f+2F_r}{mV} - \Omega_z \\ \dot{\Omega}_z &= \frac{2l_fF_f-2l_rF_r+M_z}{J_z} \end{cases} \quad (1)$$

where, β denotes the slide slip angle, Ω_z is the yaw velocity, F_f is the nonlinear cornering force of the two front tires, F_r is the nonlinear cornering force of the two rear tires and M_z is yaw moment. V is the vehicle velocity, J_z is the yaw moment of inertia, m is the vehicle mass.

Using the T-S fuzzy approach, the forces F_f and F_r can be described as follows (Latrech et al., 2018; Dahmani et al., 2013):

$$\begin{cases} F_f &= \eta_1(|\alpha_f|)C_{f1}\alpha_f + \eta_2(|\alpha_f|)C_{f2}\alpha_f \\ F_r &= \eta_1(|\alpha_f|)C_{r1}\alpha_r + \eta_2(|\alpha_f|)C_{r2}\alpha_r \end{cases} \quad (2)$$

where, $\eta_i (i = 1, 2)$ identify the bell membership functions as below:

$$\eta_1(\alpha_f) = \frac{\omega_1(\alpha_f)}{\omega_1(\alpha_f)+\omega_2(\alpha_f)}, \eta_2(\alpha_f) = \frac{\omega_2(\alpha_f)}{\omega_1(\alpha_f)+\omega_2(\alpha_f)}, \quad (3)$$

$$\omega_1(\alpha_f) = \frac{1}{(1+|\frac{|\alpha_f|-c_1}{a_1}|)^{2b_1}}, \omega_2(\alpha_f) = \frac{1}{(1+|\frac{|\alpha_f|-c_2}{a_2}|)^{2b_2}}. \quad (4)$$

C_{fi}, C_{ri} represent the consequence parameters of rules, α_f, α_r are the front and rear tire side slip which can be determined using the following expression where δ_f stands for the steering input angle:

$$\begin{cases} \alpha_f & \cong -\beta - \frac{l_f \Omega_z}{V} + \delta_f \\ \alpha_r & \cong -\beta - \frac{l_r \Omega_z}{V} \end{cases} \tag{5}$$

Thus, the following T-S fuzzy model for the vehicle lateral dynamic is obtained:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \eta_i(\alpha_f)\{A_i x + B_{2i} u(t) + B_{1i} w(t)\} \\ z(t) = \sum_{i=1}^{2} \eta_i(\alpha_f)\{C_{1i} x(t) + D_{1i} w(t)\} \\ y(t) = \sum_{i=1}^{2} \eta_i(\alpha_f)\{C_{2i} x(t)\} \end{cases} \tag{6}$$

where, $x(t) = [\beta^T \quad \Omega_z^T]^T, u(t) = M_z, w(t) = \delta_f$,

$$\begin{aligned} A_i &= \begin{bmatrix} -2\frac{C_{fi}+C_{ri}}{mV} & -2\frac{C_{fi}l_f - C_{ri}l_r}{mV^2} - 1 \\ -2\frac{C_{fi}l_f - C_{ri}l_r}{J_z} & -2\frac{C_{fi}l_f^2 + C_{ri}l_r^2}{J_z V} \end{bmatrix}, \\ B_{1i} &= \begin{bmatrix} 2\frac{C_{fi}}{mV} \\ 2\frac{a_f C_{fi}}{J_z} \end{bmatrix} \quad i = 1,2, \\ B_2 &= \begin{bmatrix} 0 \\ \frac{1}{J_z} \end{bmatrix}, C_{2i} = [0 \quad 1] \\ C_{1i} &= [0.1 \quad 0], D_{1i} = 0.1 \end{aligned} \tag{7}$$

3. Main results

Assume that the actuator failure happens. The following new actuator fault input model, which involves linear and nonlinear terms, is used:

$$u^f(t) = \Omega u(t) + \Phi\varphi(u(t)) \tag{8}$$

where, $0 < \Omega \leq 1$ stand for the actuator fault matrix, and $\phi(u(t))$ is a nonlinear vector function satisfying:

$$\varphi^T(u(t))\varphi(u(t)) \leq \sigma^2 u^T(t) H^T H u(t) \tag{9}$$

where, H is a matrix with an appropriate dimension.

Based on the PDC approach, the following output feedback control law is suggested:

$$u(t) = \sum_{i=1}^{2} \eta_i(|\alpha_f(t)|) K_i y(t) \tag{10}$$

where, K_i is the controller gains. Then, the closed-loop system is defined as

$$\begin{cases} \dot{x}(t) & = \sum_{i=1}^{2}\sum_{j=1}^{2} \eta_i\eta_j(A_{ij}x(t) + B_{2i}\Phi\varphi(u(t)) + B_{1i}w(t)) \\ z(t) & = \sum_{i=1}^{2} \eta_i(C_{1i}x(t) + D_{1i}w(t)) \end{cases} \tag{11}$$

where,

$$A_{ij} = A_i + B_{2i}\Omega K_j C_{2j}.$$

Equivalently, the system in Eq. 11 can be written as:

$$\begin{cases} \dot{x}(t) & = (A_{\eta\eta}x(t) + B_{2\eta}\Phi\varphi(u(t)) + B_{1\eta}w(t)) \\ z(t) & = (C_{1\eta}x(t) + D_{1\eta}w(t)) \end{cases} \tag{12}$$

Two cases can be obtained according to model in Eq. 8:

1. If $\Omega = 1$ and $\Phi = 0$, the actuator is working in normal mode.
2. If $\Omega \neq 1$ and $\Phi \neq 0$, the actuator is working in failure mode.

The control design purpose of this study is to design an output feedback control law (Eq. 10) in order to improve vehicle stability and maneuverability when this latter is subject to lane-changing maneuvers. The control system can tolerate the presence of failures.

Before proceeding, we recall the following lemma.

Lemma 1 (Tuan et al., 2001): The following inequality holds:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} \eta_i\eta_j\Upsilon_{ij} < 0 \tag{13}$$

if,

$$\Upsilon_{ii} < 0, \; i = 1,2,\cdots,r \tag{14}$$

$$\frac{2}{r-1}\Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} < 0, \quad j > i \tag{15}$$

3.1. Stability analysis

This subsection of the paper focuses on the development of a reliable control law such that the resulting closed-loop system is H_∞ stable.

Theorem 1: For given scalars $\phi_i < 0$, μ_1 and μ_2, closed-loop system (Eq. 12) is stable with H_∞ performance γ, if there exist matrices $P_i > 0$, Z, G, F and scalars $\varepsilon_{1i} > 0$ and $\gamma > 0$ such that the following inequalities hold for $i, j, l = 1,2,\cdots,r$:

$$P_i + Z \geq 0 \tag{16}$$

$$\mathbf{\Phi}_{ij}^{l} = \begin{bmatrix} \Phi_{ij}^{11l} & \Phi_{ij}^{12} & B_{2i}\Phi & B_{1i} & \mu_1(C_{1i}G)^T & \Phi_{ij}^{16} \\ & \Phi_{ij}^{22} & 0 & 0 & \mu_2(C_{1i}F)^T & \Phi_{ij}^{26} \\ & * & -\varepsilon_{1i}^{-1}I & 0 & 0 & 0 \\ & * & * & -\gamma I & D_{1i}^T & 0 \\ & * & * & * & -\gamma I & 0 \\ & * & * & * & * & -\varepsilon_{1i}I \end{bmatrix} < 0, \tag{17}$$

where,

$$\begin{aligned} \Phi_{ij}^{11l} &= -\sum_{l=1}^{2}\phi_l(P_l + Z) + sym(A_{ij}G) \\ \Phi_{ij}^{12} &= P_i - \mu_1 G^T + \mu_2 A_{ij} F \\ \Phi_{ij}^{22} &= -\mu_2 sym(F) \\ \Phi_{ij}^{16} &= \mu_1(\sigma H K_j C_{2j} G)^T \\ \Phi_{ij}^{26} &= \mu_2(\sigma H K_j C_{2j} F)^T \end{aligned} \tag{18}$$

Proof: Based on the property of the membership functions, we know from Eq. 17 that:

$$\mathbf{\Phi}_{\eta\eta}^{\eta} = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} \eta_i\eta_j\eta_l\mathbf{\Phi}_{ij}^{l} < 0 \tag{19}$$

Define,

$$\mathbb{T}_{\eta\eta} = \begin{bmatrix} I & A_{\eta\eta} & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & C_{1\eta} & 0 & 0 & I & 0 \\ 0 & \sigma H K_\eta C_{2\eta} & 0 & 0 & 0 & I \end{bmatrix}.$$

Pre- and post-multiplying Eq. 19 by $\mathbb{T}_{\eta\eta}$ and its transpose, respectively, and assuming that $\dot{\eta}_l \geq \phi_l$, the following condition holds:

$$\begin{bmatrix} \Phi_{\eta\eta}^{11\eta} & B_{2\eta}\Phi & B_{1\eta} & P_\eta C_{1\eta}^T & (\sigma H K_\eta C_{2\eta} P_\eta)^T \\ & -\varepsilon_{1\eta}^{-1} I & 0 & 0 & 0 \\ & * & -\gamma I & D_{1\eta}^T & 0 \\ & * & * & -\gamma I & 0 \\ & * & * & * & -\varepsilon_{1\eta} I \end{bmatrix} < 0. \quad (20)$$

where,

$$\Phi_{\eta\eta}^{11\eta} = -(\dot{P}_\eta + \dot{Z}) + sym(A_{\eta\eta} P_\eta).$$

To address the stability analysis, the following non-quadratic fuzzy Lyapunov function is introduced:

$$V(x(t)) = x^T(t) P_\eta^{-1} x(t) \quad (21)$$

The evaluation of $\dot{V}(x(t))$ onward the solutions of system in Eq. 12, with $\varphi(u(t)) = 0$ and $w(t) = 0$, provides

$$\begin{aligned} \dot{V}(x(t)) &= 2x^T(t) P_\eta^{-1} \dot{x}(t) + x^T(t) \dot{P}_\eta^{-1} x(t) \\ &= 2x^T(t) P_\eta^{-1} A_{\eta\eta} x(t) - x^T(t) P_\eta^{-1} \dot{P}_\eta {P_\eta}^{-1} x(t) \end{aligned} \quad (22)$$

On the other hand, we can easily deduce from $\sum_{i=1}^{2} \eta_i = 1$ that $\sum_{i=1}^{2} \dot{\eta}_i = 0$. Thus, for any matrix Z, we get:

$$P_\eta^{-T} \sum_{i=1}^{2} \dot{\eta}_i Z {P_\eta}^{-1} = P_\eta^{-1} \dot{Z} {P_\eta}^{-1} = 0 \quad (23)$$

and,

$$\dot{V}(x(t)) = x^T(t)\left(sym\left(P_\eta^{-1} A_{\eta\eta}\right) - P_\eta^{-1}\left(\dot{P}_\eta + \dot{Z}\right){P_\eta}^{-1}\right)x(t). \quad (24)$$

From Eq. 17, the following condition is verified:

$$\Phi_{\eta\eta}^{11\eta} = -(\dot{P}_\eta + \dot{Z}) + sym(A_{\eta\eta} P_\eta) < 0. \quad (25)$$

By performing the congruence transformation by ${P_\eta}^{-1}$ to Eq. 25, we know that:

$$sym(P_\eta^{-1} A_{\eta\eta}) - P_\eta^{-1}(\dot{P}_\eta + \dot{Z}){P_\eta}^{-1} < 0. \quad (26)$$

Thus, it can be verified, for $x(t) \neq 0$, that $\dot{V}(x(t)) < 0$ and the system in Eq. 12 is stable.

Now, the following index is introduced to examine the H_∞ performance for the system in Eq. 12,

$$J = \int_0^\infty (\gamma^{-1} z^T(t) z(t) - \gamma w^T(t) w(t)) dt. \quad (27)$$

Based on the condition in Eq. 9, inequality in Eq. 28 holds for any scalar $\varepsilon_{1h} > 0$:

$$\begin{aligned} &-\varepsilon_{1h} \varphi^T(u(t)) \varphi(u(t)) \\ &+\varepsilon_{1h} \sigma^2 x^T(t) (K_h C_{2h})^T H^T H (K_h C_{2h}) x(t) \geq 0 \end{aligned} \quad (28)$$

Defining

$$J_{zw}(t) = \gamma^{-1} z^T(t) z(t) - \gamma w^T(t) w(t).$$

According to the similar procedure outlined above, we can conclude that:

$$\dot{V}(x(t)) + J_{zw}(t) + (28) = \xi(t) \widehat{\Phi}_{\eta\eta} \xi(t) \quad (29)$$

where,

$$\xi(t) = [x^T(t) \;\; \varphi^T(u(t)) \;\; w^T(t)]^T,$$

and:

$$\widehat{\Phi}_{\eta\eta}^{\eta} = \begin{bmatrix} \widehat{\Phi}_{\eta\eta}^{11\eta} & {P_\eta}^{-1} B_{2\eta}\Phi & {P_\eta}^{-1} B_{1\eta} \\ & \varepsilon_{1\eta}^{-1} I & 0 \\ & * & -\gamma I \end{bmatrix} + \gamma^{-1} \begin{bmatrix} C_{1\eta}^T \\ 0 \\ D_{1\eta}^T \end{bmatrix} \begin{bmatrix} C_{1\eta}^T \\ 0 \\ D_{1\eta}^T \end{bmatrix}^T \quad (30)$$

and

$$\Phi_{\eta\eta}^{11\eta} = sym(P_\eta^{-1} A_{\eta\eta}) - P_\eta^{-1}(\dot{P}_\eta + \dot{Z}){P_\eta}^{-1} + \varepsilon_{1h} \sigma^2 (K_h C_{2h})^T H^T H (K_h C_{2h}).$$

By performing firstly, the congruence transformation to Eq. 19 by $diag\{P_\eta^{-1}, I, I, I, I\}$, and secondly the Schur Complement Lemma, we deduce that $\widehat{\Phi}_{\eta\eta}^{\eta} < 0$, and,

$$J \leq \int_0^\infty (\dot{V}(x(t)) + J_{zw}(t)) dt < 0 \quad (31)$$

Hence, we can conclude that the closed-loop system is stable and achieve a γ level of the H_∞ performance.

- As can be seen from the proof of Theorem 1, a non-quadratic Lyapunov function is investigated, in which the property of fuzzy membership functions is exploited to derive the H_∞ stability for the system in Eq. 12. The method is further reduced in conservatism by introducing some slack matrixes.
- In cases where the membership functions are not differentiable, a quadratic Lyapunov should be used, while the matrices themselves are independent of the membership functions. It can be shown that the conditions in Theorem 1 hold for this case by setting ϕ_i to be sufficiently small, and restraining P_i variables to be P.

4. Controller synthesis

In the sequel, intend to develop a method to synthesize the gains K_i such that a closed-loop system in Eq. 12 is stable with a H_∞ performance γ.

Theorem 2: For given scalars μ_1, μ_2, and $\phi_i < 0$, closed-loop system in Eq. 12 is stable with γ level of H_∞ performance, if there exist scalars $\gamma > 0$, $\varepsilon_{1i} > 0$, matrices $P_i > 0$, Z, G, F and Y_i such that the following LMIs hold:

$$P_i + Z > 0 \quad (32)$$

$$\Upsilon_{ii}^{l} < 0, \quad i = 1, \cdots, r \quad (33)$$

$$\frac{2}{r-1}\Upsilon_{ii}^{l}+\Upsilon_{ij}^{l}+\Upsilon_{ji}^{l}<0,\quad i\neq j=1,\cdots,r,\ \ l=1,\cdots,r \qquad (34)$$

where,

$$\Upsilon_{ij}^{l}=\begin{bmatrix}\Upsilon_{ij}^{11l} & \Upsilon_{ij}^{12l} & \varepsilon_{1i}\bar{B}_{2i}\Phi & \bar{B}_{1i} & \mu_1(\bar{C}_{1i}G)^T & \Upsilon_{ij}^{16}\\ & \Upsilon_{ij}^{22l} & 0 & 0 & \mu_2(\bar{C}_{1i}F)^T & \Upsilon_{ij}^{26}\\ & * & -\varepsilon_{1i}I & 0 & 0 & 0\\ & * & * & -\gamma I & D_{1i}^T & 0\\ & * & * & * & -\gamma I & 0\\ & * & * & * & * & -\varepsilon_{1i}I\end{bmatrix}$$

$$\Upsilon_{ij}^{11l}=-\sum_{l=1}^{2}\phi_l(P_l+Z)+\mu_1 sym(\bar{A}_iG+\bar{B}_{2i}\Omega Y_j\bar{C}_{2i})$$
$$\Upsilon_{ij}^{12l}=P_i+\mu_2(\bar{A}_iF+\bar{B}_{2i}\Omega Y_j\bar{C}_{2i})-\mu_1G^T$$
$$\Upsilon_{ij}^{16}=\mu_1(\sigma H\bar{B}_{2i}Y_j\bar{C}_{2i})^T$$
$$\Upsilon_{ij}^{26}=\mu_2(\sigma H\bar{B}_{2i}Y_j\bar{C}_{2i})^T \qquad (35)$$
$$G=\begin{bmatrix}G_{11} & 0\\ G_{21} & G_{22}\end{bmatrix}, F=\begin{bmatrix}G_{11} & 0\\ F_{21} & F_{22}\end{bmatrix},$$

Moreover, the gain K_i is calculated from $K_i = Y_iG_{11}^{-1}$, where $\bar{A}_i = T_i^{-1}A_iT_i$ $\bar{B}_{2i} = T_i^{-1}B_{2i}$, $\bar{B}_{1i} = T_i^{-1}B_{1i}$, $\bar{C}_{1i} = C_{1i}T_i$, and T_i is any matrix satisfying $\bar{C}_{2i} = C_{2i}T_i = [I \quad 0]$.

Proof: Using the conditions stated in the theorem, it is easy to check that matrix F is non-singular, using the fact that $-sym(F) < 0$. Thus, G_{11} is non-singular. Set $Y_i = K_iG_{11} = K_i\bar{C}_{2i}F = K_i\bar{C}_{2i}G$. According to Lemma 1, we have:

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\eta_i\eta_j\eta_l\Upsilon_{ij}^{l}<0 \qquad (36)$$

By substituting matrices A_i, B_{2i}, B_{1i}, C_{2i}, C_{1i} by $\bar{A}_i$, $\bar{B}_{2i}$, $\bar{B}_{1i}$, $\bar{C}_{2i}$, $\bar{C}_{1i}$, respectively, condition in Eq. 17 holds, and in view of Theorem 1, we know that closed-loop system in Eq. 12 is stable with a γ level of H_∞ performance for all $0 \neq w(k) \in L_2[0,\infty)$.

Remark: Note that the optimal performance index γ^* for H_∞ performance can be obtained by solving the following optimization problem:

$$\min \gamma^2$$
$$\text{s.t, LMIs Eqs. } 32-34 \qquad (37)$$

5. Simulation results

To validate the effectiveness of the designed controller, we conduct this section of numerical simulations to show that the control system can tolerate the effect of the actuator failure. Assume that the parameters of the model are selected from Latrech et al. (2018) as $m = 1500kg$, $I_z = 3000kg.m^2$, $a_r = 1.3$ $a_f = 1.2$ and $U = 20m/s$, $C_{f1} = 60712$, $C_{f2} = 4812$, $C_{r1} = 60088$, $C_{r2} = 3455$. For $\eta = 0.7$ the parameters of membership functions are $a_1 = 0.0908$, $a_2 = 23.3421$, $b_1 = 0.7237$, $b_2 = 204.0533$, $c_1 = 0.0415$ and $c_2 = 23.4094$.

By selecting $\Omega = 0.25$, $\sigma = 0.1$, $H = 1$, $\mu_1 = 5$, $\mu_2 = 7$, $\alpha = 1270$, $\phi_1 = \phi_2 = -1$, the optimization problem in Eq. 37 produces a feasible solution with $\gamma^* = 0.18915$ and the following parameters:

$$K_1 = -242224.92,\quad K_2 = -228116.31 \qquad (38)$$

For the simulation, we choose the nonlinear fault function as $\varphi(u(t)) = 0.1sin(5u(t))$, and the initial condition as $x_0(t) = [-0.05 \quad 0.1]^T$.

In real life, the driver may give a steering correction to control the vehicle's yaw motion when the vehicle deviates from the desired trajectory. Interestingly, the reaction lag of the driver can drastically affect the lateral movement of the vehicle. Moreover, since the rate of the vehicle's yaw will increase rapidly when a fault occurs, the driver will become panicked and may give inappropriate steering input, which may cause an accident. So, using the proposed reliable controller the stability of the vehicle is maintained by reducing the effect of the steering input angle δ_f (Fig. 2).

To further show the merit of the proposed control scheme, we apply the controller designed by Latrech et al. (2018) with the following gains:

$$K_1 = 41871,\quad K_2 = -57066 \qquad (39)$$

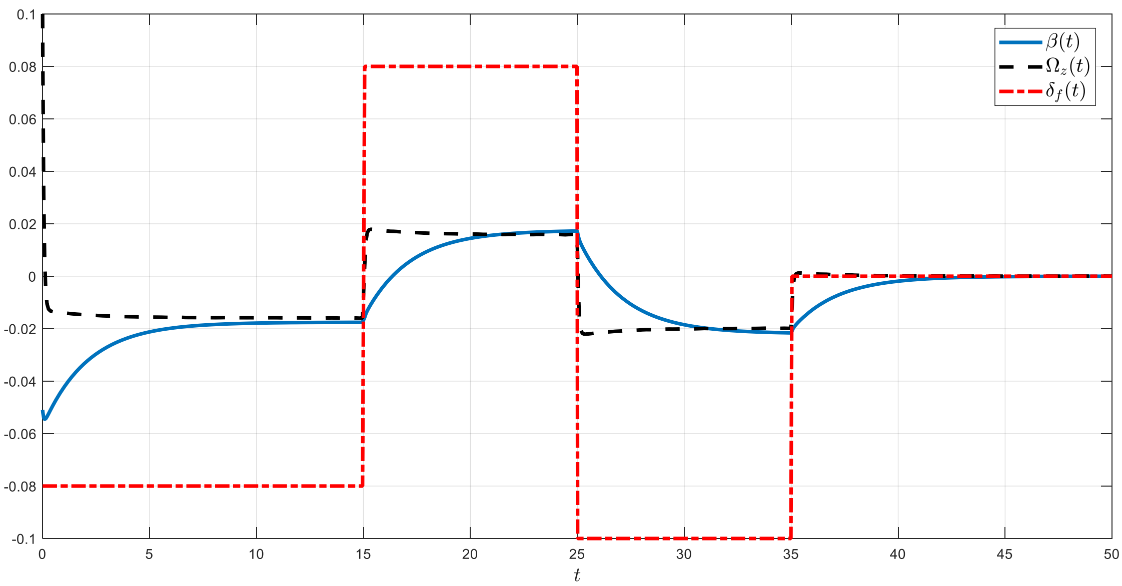

Fig. 2: State trajectories using controller in Eq. 38

Figs. 2-5 depicts the states and the input of the vehicle when the controller with the gains of Eqs. 38-39 is applied. These Figs. 2-5 confirm the performance of the proposed controller and its robustness to failures in actuators and to external disturbances as is evident from the comparative simulation results.

Simulation results suggest that the designed controller stabilizes the vehicle system, while the fuzzy static output controller has the benefit of tolerating and accommodating the actuator-fault constraints as well as exogenous disturbances.

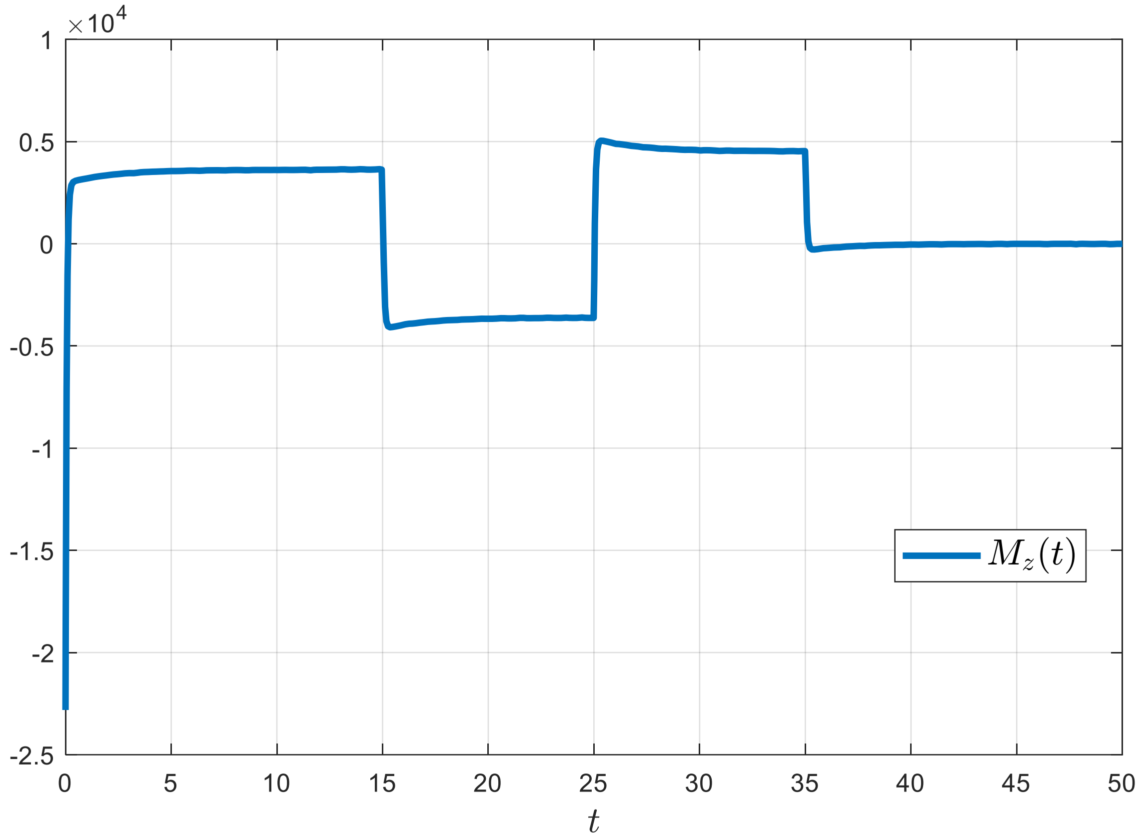

Fig. 3: Input trajectory using controller in Eq. 38

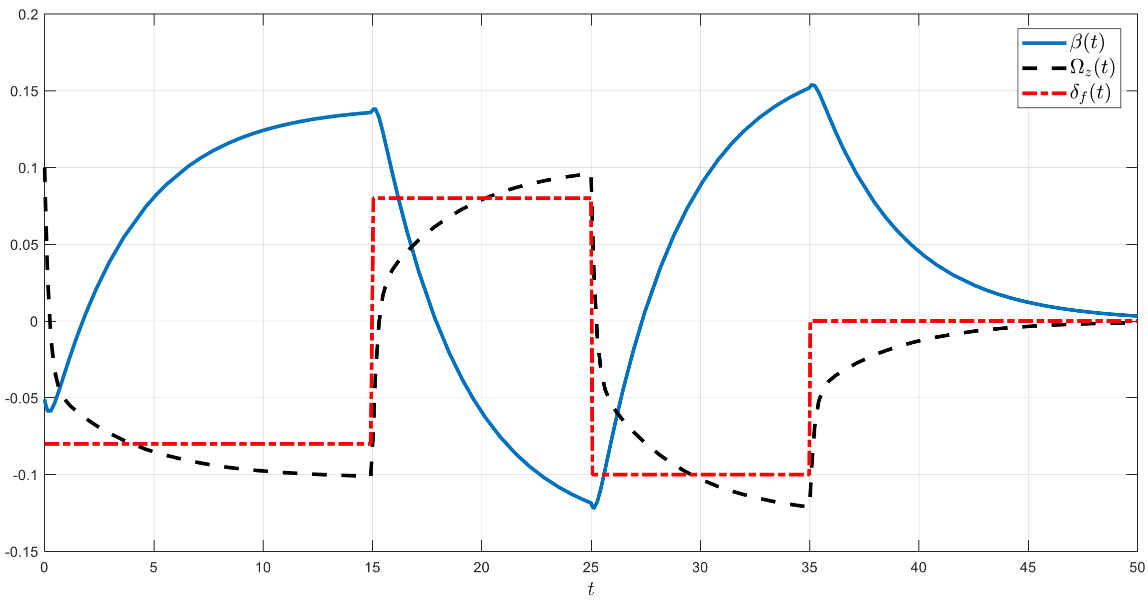

Fig. 4: State trajectories using controller in Eq. 39

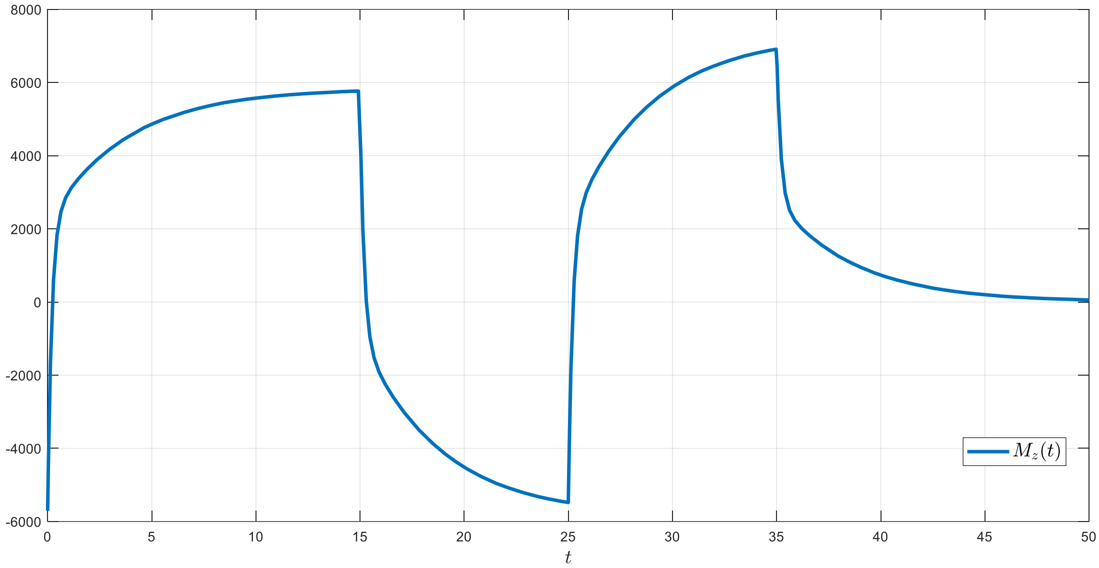

Fig. 5: Input trajectory using controller in Eq. 39

6. Conclusion

A fuzzy reliable control scheme for a lateral dynamic is proposed. The developed controller is based on the T-S fuzzy representation. First, the nonlinear vehicle model is introduced, then its representation by a T-S fuzzy model is provided. Next, based on a non-quadratic Lyapunov function, sufficient conditions are formulated in LMI terms to design a static output feedback controller able to tolerate the effect of the actuator failures which can affect the vehicle. The numerical simulations have shown the effectiveness of this proposed control scheme.

List of symbols

m	Vehicle mass
J_z	yaw moment of inertia
l_f	Distance of gravity from front axle
l_r	Distance of gravity from rear axle
Ω_z	Yaw velocity,
M_z	Yaw moment.
V	Vehicle velocity,
F_f	Front tire lateral forces
F_r	Rear tire lateral forces
β	Sideslip angle
α_f	Front tire slip angle
α_r	Rear tire slip angle
δ	Steering angle

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Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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