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Robust fault-tolerant tracking control for a class of T-S fuzzy systems subject to actuator failure and external disturbance

S. Alkaik, M. Kchaou *

College of Engineering, University of Hail, Hail, Saudi Arabia

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ABSTRACT

Based on the Takagi-Sugeno (T-S) fuzzy model approach, this study discusses the robust fault-tolerant tracking controller design for non-linear systems affected by external disturbances, uncertainties, and actuator failures. In contrast to existing results, this study assumes the actuator fault model includes linear and nonlinear terms, and a state feedback controller is designed to improve the tracking and stability of the system when actuators fail. Using a non-quadratic Lyapunov function, new sufficient conditions for L_2-gain tracking performance analysis are derived to determine simultaneously the minimal level of the L_2-gain and controller gains. The robustness of the proposed approach is also investigated. An illustration of the theoretical developments is provided by a Duffing forced oscillation system.

1. Introduction

Since a number of practical applications of tracking control are very widespread, including missile trajectory control, aircraft attitude control, and robot tracking control, more and more attention has been paid to this problem for the past few years. As a potential research topic, tracking control is generally considered more challenging than stabilization (Chu and Li, 2019; Zhang et al., 2014; Chang and Yen, 2005). Due to the fact that tracking control requires the designed controller to guarantee system stability, as well as drive the nonlinear system's state to pursue those of the reference model.

Systems and control theory presents a number of challenges involved in the design and analysis of nonlinear systems. To cope with this class of systems, fuzzy logic might come up with an innovative solution for the design analysis and control synthesis of various industrial plants. Alternatively, Takagi–Sugeno (T-S) fuzzy models exhibit an excellent ability to express nonlinear systems through the combination of fuzzy logic and linear control theories (Takagi and Sugeno, 1985; Kchaou et al., 2011; Latrach et al., 2015). Therefore, the application of the T-S fuzzy model greatly expands the research field of nonlinear control theory. As a result, a rich literature related to controller design, filtering design, and stability analysis on T-S has been published (Latrech et al., 2018; Kang and Lee, 2018; Shi et al., 2020; Makni et al., 2019; Tao et al., 2018; Kchaou et al., 2018). In the quoted papers, the tracking control problem for T-S fuzzy models has been tackled (Fu et al., 2022; Li et al., 2022; Zhang et al., 2015; Gu et al., 2017). Another issue with feedback control is that, as industrial engineering has progressed, many practical plants have become highly complicated, and, in addition, various environmental factors have become more relevant, such as sudden changes in working conditions, corroded internal components, and aged sensors and actuators. Because actuator/sensor failures can have a negative impact on system performance, control communities are interested in this control problem. The goal of this issue is to introduce the concept of fault-tolerant control (FTC) and fault diagnosis as critical approaches for designing reliable controllers that are capable of maintaining the critical functionality of systems subject to problems and failures. Reviewing the literature, many elegant reported results related to this area have been proposed for different classes of systems (Wang et al., 2018; Kchaou et al., 2021; Yan et al., 2019). To mention a few, Kaviarasan et al. (2016) developed a method for designing fault-tolerant controllers for power systems subject to random changes and actuator failures. The FTC method for wind-diesel hybrid systems with time-

* Corresponding Author.
Email Address: m.kchaou@uoh.edu.sa (M. Kchaou)
https://doi.org/10.21833/ijaas.2023.02.013
Corresponding author's ORCID profile:
https://orcid.org/0000-0002-6849-1745

varying bounded sensor faults was proposed by Kamal et al. (2013). In Wang et al. (2015), the reliable observer-based control problem for discrete-time Takagi-Sugeno fuzzy systems with time-varying delay and stochastic actuator faults was formulated from the input-output approach.

The purpose of this paper is to discuss fault-tolerant tracking control of T-S fuzzy systems. The tracking control objective is to drive the states of the T-S fuzzy system by means of a state feedback controller so that it tracks a reference model as closely as possible despite uncertainties, exogenous disturbances, and actuator faults. This study is motivated by some real-world usage cases. For example, tracking control of Duffing oscillator systems can be used in engineering for master-slave synchronization, as well as in communication systems using chaotic signals. The main contributions of this paper are highlighted as follows:

1. As an alternative to existing fuzzy trucking control schemes (Gu et al., 2017; Zhang et al., 2015), this study proposes a new model of the fault including a nonlinear part for reliable tracking control of fuzzy systems subject to exogenous disturbances and nonlinear actuator failures.
2. In accordance with the non-quadratic Lyapunov function, sufficient conditions are derived for the L_2-gain tracking performance analysis of the resulting closed-loop system error.
3. The introduction of slack variables allows for the simultaneous computation of the minimum L_2-gain tracking performance and the synthesis of the controller gains.

Notations: The notations used in this paper are standard, where $X \in \mathbb{R}^n$ is the set of n −dimensional Euclidean space; $X \in \mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices; $X > 0$ is the real symmetric positive definite matrix; $sym(X)$ stands for $X + X^T$. It is noted also $X_\rho = \sum_{i=1}^{r} \rho_i$, $X_{\rho\rho} = \sum_{i=1}^{r}\sum_{j=1}^{r} \rho_i\rho_j X_{ij}$, and $\dot{X}_\rho = \sum_{i=1}^{r} \dot{\rho}_l X_l$, $*$ denotes the term that is induced by symmetry.

2. System description and problem formulation

In this paper, the class of non-linear systems described by the following T-S fuzzy model is considered:

Plant rule i: IF $\theta_1(t)$ is M_{i1} and $\cdots$ $\theta_s(t)$ is M_{is} then,

$$\dot{x}(t) = A_i(t)x(t) + B_{2i}u^f(t) + B_{1i}w(t) \tag{1}$$

where, $\theta(t) = [\theta_1(t),\ \theta_2(t),\ \cdots \theta_s(t)]$ is the premise variable, and $Mij(i = 1, \cdots, r, j = 1, \cdots, s)$ are the fuzzy sets. r denotes the index number of the fuzzy rules. $x(t) \in \mathbb{R}^n$ is the state vector, $u^f(t) \in \mathbb{R}^m$ is the control input vector, and $w(t) \in \mathbb{R}^w$ is exogenous disturbance input taking value in L_2. Matrices $A_i(t)$, B_{2i}, B_{1i}, define the ith local model, where $A_i(t) = A_i + \Delta A_i(t)$ is an uncertain matrix such that:

$$\Delta A_i(t) = M_i\Delta(t)N_i,\ \Delta^T(t)\Delta(t) \le R \tag{2}$$

where, M_i and N_i are known matrices and $\Delta(t)$ is an unknown time-varying matrix.

By using the inference method, the (T-S) fuzzy system can be inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} \rho_i(\theta(t))\{A_i(t)x(t) + B_{2i}u^f(t) + B_{1i}w(t)\} \tag{3}$$

where, $\rho_i(\theta(t)) = \frac{\varrho_i(\theta(t))}{\sum_{i=1}^{r}\varrho_i(\theta(t))}$, and $\varrho_i(\theta(t)) = \prod_{j=1}^{s} M_{ij}(\theta_j)$.

Thus, the normalized membership functions satisfy that:

$$\rho_i(\theta(t)) \in [0\ \ 1], \qquad \sum_{i=1}^{r} \rho_i(\theta(t)) = 1.$$

The objective of this study is to design a fuzzy tracking controller to make the system states in Eq. 1 track those of the following reference model:

$$\dot{x}_r(t) = A_r x_r(t) + B_r r(t) \tag{4}$$

where, $x_r(t)$ represents the reference state, A_r is an asymptotically stable matrix, $r(t)$ is a bounded reference input.

Let $e(t) = x(t) - x_r(t)$. Then the input signal is generated by,

$$u(t) = \sum_{i=1}^{r} \rho_i(\theta(t))K_i e(t) \tag{5}$$

where, K_i is the controller gain to be designed. Assume that the actuator failure happens. The following new actuator fault input model, which involves linear and nonlinear terms, is used:

$$u^f(t) = \Omega u(t) + \Phi\phi(u(t)) \tag{6}$$

where, $\Omega = diag(\Omega_1,\ \Omega_i, \cdots, \Omega_m)$, $0 < \Omega_i \le 1$ stand for the actuator fault matrix, and $\phi(u(t))$ is a nonlinear vector function satisfying:

$$\phi^T(u(t))\phi(u(t)) \le \sigma^2 u^T(t)H^T H u(t) \tag{7}$$

where, H is a matrix with an appropriate dimension.

Remark: Two cases can be obtained according to model (6):

1. If $\Omega = 1$, and $\Phi = 1$, the actuator is working in normal mode.
2. If $\Omega \ne 1$, and $\Phi \ne 1$ the actuator is working in failure mode.

From Eq. 8 and Eq. 5, the closed-loop system can be written as:

$$\begin{aligned}\dot{e}(t) \quad &= \sum_{i=1}^{r}\sum_{j=1}^{r} \rho_i(\theta(t))\rho_j(\theta(t))(A_{ij}(t)e(t) \\ &+B_{2i}\Phi\phi(u(t)) + v(t)),\end{aligned} \tag{8}$$

where, $A_{ij}(t) = A_i(t) + B_{2i}\Omega K_j$, and $v(t) = (A_i(t) - A_r)x_r(t) + B_i w(t) - B_r r(t)$.

Equivalently, one gets:

$$\dot{e}(t) \quad = A_{\rho\rho}(t)e(t) + B_{2\rho}\Phi\phi(u(t)) + v(t) \tag{9}$$

The objective of this paper is to design a reliable fuzzy controller (5) such that the closed-loop system error in Eq. 8 is robustly stable and satisfying following tracking performance,

$$\int_0^\infty e^T(t)Qe(t)dt \le V(0) + \gamma^2 \int_0^\infty v^T(t)v(t))dt \tag{10}$$

where, $\gamma > 0$ denotes the desired tracking performance level, $Q > 0$ is the weighting matrix, and $V(0)$ is the initial energy function.

3. Main results

This subsection focuses on the development of a reliable control law such that the resulting closed-loop system is stable with L_2-gain performance. We need the following lemma.

Lemma 1: The following inequality holds (Tuan et al., 2001):

$$\sum_{i=1}^r \sum_{j=1}^r \eta_i\eta_j\Upsilon_{ij} < 0 \tag{11}$$

if

$$\Upsilon_{ii} < 0,\ i = 1,2,\cdots,r \tag{12}$$

$$\frac{2}{r-1}\Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} < 0,\quad j > i \tag{13}$$

3.1. L_2-gain tracking analysis and reliable controller design

In this section, it assumed that $\Delta A_i = 0$.

Theorem 1: For given scalars $\phi_i < 0$, μ_1 and μ_2, closed-loop system (8) is stable with L_2-gain performance γ, if there exist matrices $P_i > 0$, Z, G, Y_i and scalars $\varepsilon_{1i} > 0$ and $\gamma > 0$ such that the following LMIs hold for $i, j, l = 1,2,\cdots,r$:

$$P_i + Z \ge 0 \tag{14}$$

$$\Phi_{ii}^l < 0,\quad i = 1,\cdots,r \tag{15}$$

$$\frac{2}{r-1}\Phi_{ii}^l + \Phi_{ij}^l + \Phi_{ji}^l < 0,\quad i \ne j \tag{16}$$

where,

$$\Phi_{ij}^l = \begin{bmatrix} \Phi_{ij}^{11l} & \Phi_{ij}^{12} & \varepsilon_{1i}B_{2i}\Phi & I & \mu_1(\sigma HY_j)^T \\ & \Phi_{ij}^{22} & 0 & 0 & \mu_2(\sigma HY_j)^T \\ & * & -\varepsilon_{1i}I & 0 & 0 \\ & * & * & -\gamma^2 I & 0 \\ & * & * & * & -\varepsilon_{1i}I \end{bmatrix}$$

$$\begin{aligned} &\Phi_{ij}^{11l} = -\textstyle\sum_{l=1}^2 \phi_l(P_l + Z) + sym(\mathbf{A}_{ij}) + \bar{Q}_i \\ &\Phi_{ij}^{12} = P_i - \mu_1 G^T + \mu_2\mathbf{A}_{ij} \\ &\Phi_{ij}^{22} = -\mu_2 sym(G) \\ &\mathbf{A}_{ij} = A_{ij}G = A_iG + B_{2i}Y_j. \end{aligned} \tag{17}$$

Moreover, the gain K_i is calculated from $K_i = Y_iG^{-1}$.

Proof. Using the conditions stated in the theorem, it is easy to check that matrix G is non-singular, using the fact that $-sym(G) < 0$.

Set $Y_i = K_iG$. Based on Lemma 1, we know from Eqs. 15-16 that:

$$\Phi_{\eta\eta}^\eta = \sum_{i=1}^r \sum_{j=1}^r \sum_{l=1}^r \eta_i\eta_j\eta_l\Phi_{ij}^l < 0. \tag{18}$$

Define $\mathbb{T}_{\eta\eta} = \begin{bmatrix} I & A_{\eta\eta} & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & \sigma HK_\eta & 0 & 0 & I \end{bmatrix}$: Pre- and post-multiplying Eq. 18 by $\mathbb{T}_{\eta\eta}$ and its transpose, respectively, and assuming that $\dot{\eta}_l \ge \phi_l$, the following condition holds:

$$\begin{bmatrix} \Phi_{\eta\eta}^{11\eta} & B_{2\eta}\Phi & I & (\sigma HK_\eta P_\eta)^T \\ & -\varepsilon_{1\eta}^{-1}I & 0 & 0 \\ & * & -\gamma^2 I & 0 \\ & * & * & -\varepsilon_{1\eta}I \end{bmatrix} < 0 \tag{19}$$

where,

$$\Phi_{\eta\eta}^{11\eta} = -(\dot{P}_\eta + \dot{Z}) + sym(A_{\eta\eta}P_\eta) + P_\eta QP_\eta.$$

To address the stability analysis, the following non-quadratic fuzzy Lyapunov function is introduced:

$$V(x(t)) = x^T(t)P_\eta^{-1}x(t) \tag{20}$$

The evaluation of $\dot{V}(x(t))$ onward the solutions of system (8), with $\varphi(u(t)) = 0$ and $w(t) = 0$, provides:

$$\begin{aligned} \dot{V}(x(t)) \quad &= 2x^T(t)P_\eta^{-1}\dot{x}(t) + x^T(t)\dot{P}_\eta^{-1}x(t) \\ &= 2x^T(t)P_\eta^{-1}A_{\eta\eta}x(t) \\ &-x^T(t)P_\eta^{-1}\dot{P}_\eta {P_\eta}^{-1}x(t). \end{aligned} \tag{21}$$

On the other hand, we can easily deduce from $\sum_{i=1}^r \eta_i = 1$ that $\sum_{i=1}^r \dot{\eta}_i = 0$. Thus, for any matrix Z, we get:

$$P_\eta^{-T}\sum_{i=1}^r \dot{\eta}_i Z{P_\eta}^{-1} = P_\eta^{-1}\dot{Z}{P_\eta}^{-1} = 0 \tag{22}$$

and

$$\begin{aligned} \dot{V}(x(t)) \quad &= x^T(t)(sym(P_\eta^{-1}A_{\eta\eta}) \\ &-P_\eta^{-1}(\dot{P}_\eta + \dot{Z}){P_\eta}^{-1})x(t) \end{aligned} \tag{23}$$

From Eq. 15, the following condition is verified,

$$\Phi_{\eta\eta}^{11\eta} = -(\dot{P}_\eta + \dot{Z}) + sym(A_{\eta\eta}P_\eta) + P_\eta QP_\eta < 0. \tag{24}$$

By performing the congruence transformation by ${P_\eta}^{-1}$ to Eq. 24, we know that:

$$sym(P_\eta^{-1}A_{\eta\eta}) - P_\eta^{-1}(\dot{P}_\eta + \dot{Z}){P_\eta}^{-1} < -Q < 0 \tag{25}$$

Thus, it can be verified, for $x(t) \ne 0$, that $\dot{V}(x(t)) < 0$ and the system (8) is stable. Now, the following index is introduced to examine the L_2-gain performance for the system in Eq. 8,

$$J \quad = \int_0^\infty \left(e^T(t)Qe(t) - \gamma^2 v^T(t)v(t)\right)dt. \tag{26}$$

Based on the condition in Eq. 7, inequality (27) holds for any scalar $\varepsilon_{1\eta} > 0$:

$$-\varepsilon_{1\eta}\varphi^T(u(t))\varphi(u(t)) \\ +\varepsilon_{1\eta}\sigma^2 x^T(t)(K_\eta)^T H^T H(K_\eta)x(t) \geq 0. \tag{27}$$

Defining $J_e(t) = e^T(t)Qe(t) - \gamma^2 v^T(t)v(t)$: According to the similar procedure outlined above, we can conclude that:

$$\dot{V}(x(t)) + J_e(t) = \xi(t)\widehat{\Phi}_{\eta\eta}\xi(t) \tag{28}$$

where, $\xi(t) = [x^T(t) \;\; \varphi^T(u(t)) \;\; v^T(t)]^T$, and,

$$\widehat{\Phi}^{\eta}_{\eta\eta} = \begin{bmatrix} \widehat{\Phi}^{11\eta}_{\eta\eta} & P_\eta^{-1}B_{2\eta}\Phi & P_\eta^{-1} \\ & \varepsilon_{1\eta}^{-1}I & 0 \\ & * & -\gamma^2 I \end{bmatrix} \tag{29}$$

and $\Phi^{11\eta}_{\eta\eta} = sym(P_\eta^{-1}A_{\eta\eta}) - P_\eta^{-1}(\dot{P}_\eta + \dot{Z})P_\eta^{-1} + Q + \varepsilon_{1\eta}\sigma^2(K_\eta)^T H^T H(K_\eta)$.

By performing firstly, the congruence transformation to Eq. 19 by $diag\{P_\eta^{-1},\ I,\ I,\ I\}$, and secondly the Schur complement lemma, we deduce that $\widehat{\Phi}^{\eta}_{\eta\eta} < 0$, and,

$$\dot{V}(x(t)) + J_e(t) < 0 \tag{30}$$

Integrating Eq. 30 from 0 to ∞, we get:

$$\int_0^\infty (\dot{V}(x(t)) + J_e(t))dt = \\ \int_0^\infty (e^T(t)Qe(t) - \gamma^2 v^T(t)v(t))dt - V(0) < 0. \tag{31}$$

Hence, we can conclude that the closed-loop system is stable and achieve a γ level of the L_2-gain performance.

Remark:

- As can be seen from the proof of Theorem 1, a non-quadratic Lyapunov function is investigated, in which the property of fuzzy membership functions is exploited to derive the L_2-gain stability for the system (8). The method is further reduced in conservatism by introducing some slack matrices.
- In cases where the membership functions are not differentiable, a quadratic Lyapunov should be used, while the matrices themselves are independent of the membership functions. It can be shown that the conditions in Theorem 1 hold for this case by setting ϕ_i to be sufficiently small, and restraining P_i variables to be P.

4. Robust reliable controller design

Lemma 2: For given constant matrices M and N and a symmetric constant matrix P of appropriate dimensions, then (Xie, 1996),

$$P + sym(P + M\Delta N) < 0$$

holds for any Δ satisfying $\Delta^T(t)\Delta(t) \leq R$ if and only if there exists a scalar $\epsilon > 0$ such that:

$$P + sym(M\Delta N) \leq \epsilon MM^T + \epsilon^{-1}N^T RN. \tag{32}$$

In the sequel, intend to develop a method to synthesize the gains K_i such that the closed-loop system (8) is stable with a L_2-gain performance γ.

For given scalars μ_1, μ_2, and $\phi_i < 0$, closed-loop system (8) is robustly stable with γ level of L_2-gain performance, if there exist scalars $\gamma > 0$, $\varepsilon_{1i} > 0$, matrices $P_i > 0, Z, G, F$ and Y_i such that the following LMIs hold:

$$P_i + Z > 0 \tag{33}$$
$$\Upsilon^l_{ii} < 0, \quad i = 1, \cdots, r \tag{34}$$
$$\frac{2}{r-1}\Upsilon^l_{ii} + \Upsilon^l_{ij} + \Upsilon^l_{ji} < 0, \quad i \neq j \tag{35}$$

where,

$$\Upsilon^l_{ij} = \begin{bmatrix} \Phi^l_{ij} & -\epsilon_i\Upsilon^{12}_i & \Upsilon^{13}_i \\ & -\epsilon_i I & 0 \\ & * & -\epsilon_i R^{-1} \end{bmatrix} \\ \Upsilon^{12}_i = [M_i^T \quad 0 \quad 0 \quad 0 \quad 0]^T \\ \Upsilon^{13}_i = [\mu_1(N_i G) \quad \mu_2(N_i G) \quad 0 \quad 0 \quad 0]^T. \tag{36}$$

Based on Lemma 2, we know from Eq. 34 that:

$$\Upsilon^{\eta}_{\eta\eta} = \sum_{i=1}^r \sum_{j=1}^r \sum_{l=1}^r \eta_i\eta_j\eta_l\Upsilon^l_{ij} < 0. \tag{37}$$

According to Lemma 2, it is easy to verify that:

$$\Phi_{\eta\eta} + sym(\Upsilon^{12}_\eta\Delta(t)\Upsilon^{13T}_\eta) < 0 \tag{38}$$

By subsuming $A_{\eta\eta}$ with $A_{\eta\eta}(t)$ and following the same lines to prove Theorem 1 we can demonstrate that a closed-loop system (8) is robustly stable with a γ level of L_2-gain performance for all $0 \neq w(k) \in L_2[0,\infty)$. Note that the optimal performance index γ^* for L_2-gain performance can be obtained by solving the following optimization problem:

$$\min \gamma^2 \\ \text{s.t. LMIs Eq. 33} - \text{Eq. 35} \tag{39}$$

5. Simulation results

Our discussion in this section provides a concise description of the computational framework and illustrates the efficacy and advantages of the proposed control scheme through a Duffing forced oscillation system.

5.1. Computational framework

The computation experiments are performed on a computer with the following characteristics:(i) [OS] Windows 10 Enterprise for 64 bits; (ii) [RAM] 8 Gigabytes; and (iii) [Processor] Intel(R) Core(TM) i7-4790T CPU @ 2.70 GigaHertz.

5.2. Example

This example illustrates a simulation of the Duffing forced-oscillation system tracking those of a linear reference model using the developed approach. This nonlinear system has the following dynamics:

$$\begin{cases}\dot{x}_1(t) & = x_2(t) \\ \dot{x}_2(t) & = -x_1^3(t) - 0.1x_2(t) + 12cos(t) + u(t)\end{cases} \quad (40)$$

The system can be expressed by T-S fuzzy model with the format of Eq. 3 by assuming that the state satisfies $x_1(t) \in [-5,5]$, whose parameters are as follows:

$$\begin{aligned} A_1 &= \begin{bmatrix}0 & 1 \\ 0 & -0.1\end{bmatrix}, A_2 = \begin{bmatrix}0 & 1 \\ -25 & -0.1\end{bmatrix}, B_{21} = \begin{bmatrix}0 \\ 1\end{bmatrix} \\ B_{22} &= B_{11} = B_{12} = B_{21}\end{aligned} \quad (41)$$

and the membership functions $\rho_1(x_1) = 1 - \frac{x_1^2}{25}$ and $\rho_2(x_1) = 1 - \rho_1(x_1)$.

For the sake of simplicity, we assume that the uncertain subsystem matrices are a 5% deviation from the nominal subsystem matrices. Indeed, we let

$\Delta(t) = 1.2sin(0.1t)$ and,

$\Delta A_i = 0.05\Delta(t)A_i,\ i = 1,2.$

By choosing $\Omega = 0.3$, $\eta_1 = \eta_2 = -1$, $\mu_1 = 11$, $\mu_2 = 0.1$ and $Q = diag(100,100)$, using Yalmip's toolbox in conjunction with Mosek's solver the optimization problem in Eq. 39 gives feasible solutions for the following cases:

1. $\Phi = 0.4$, and $\sigma = 0.3$, (Failure case)
2. $\Phi = 1$, and $\sigma = 0$, (Normal case)

Table 1 provides the controller gains using the proposed method for both cases and the methods proposed by Gu et al. (2017). For all methods, the simulations are undergone with a nonlinear fault function selected as $g(u(t)) = 0.3sin(4u(t))$, and the initial conditions $x(t) = [0.2\ \ 0.1]^T$, and $x_r(t) = [-0.5\ \ 0.1]^T$.

Table 1: Controller gains for different methods

Methods	controller gains
Proposed method for case 1	$K_1 = [-319.49\quad -40.56]$ $K_2 = [-295.38\quad -41.677]$
Proposed method for case 2	$K_1 = [-256.37\quad -39.947]$ $K_2 = [-243.09\quad -52.358]$
Method proposed by Gu et al. (2017)	$K_1 = [-15.0101\quad -8.3216]$ $K_2 = [-8.6828\quad -9.8111]$

In this example, we choose the following reference model:

$$\dot{x}_r(t) = \begin{bmatrix}0 & 1 \\ -1 & -2\end{bmatrix} x_r(t) + \begin{bmatrix}0 \\ 1\end{bmatrix} sin(t). \quad (42)$$

The results of the numerical simulation analysis are depicted in Figs. 1-3, from where it is represented for each method the system and reference model states variables.

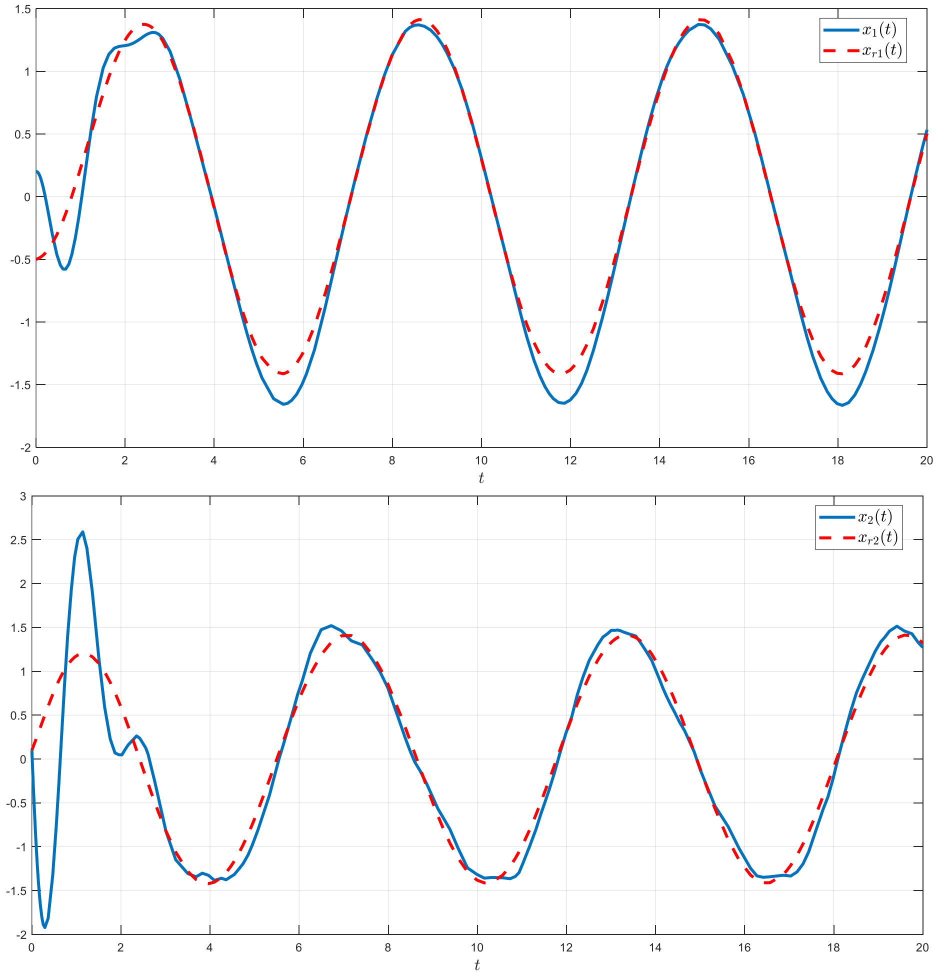

Fig. 1: State and reference responses using the reliable controller (case 2)

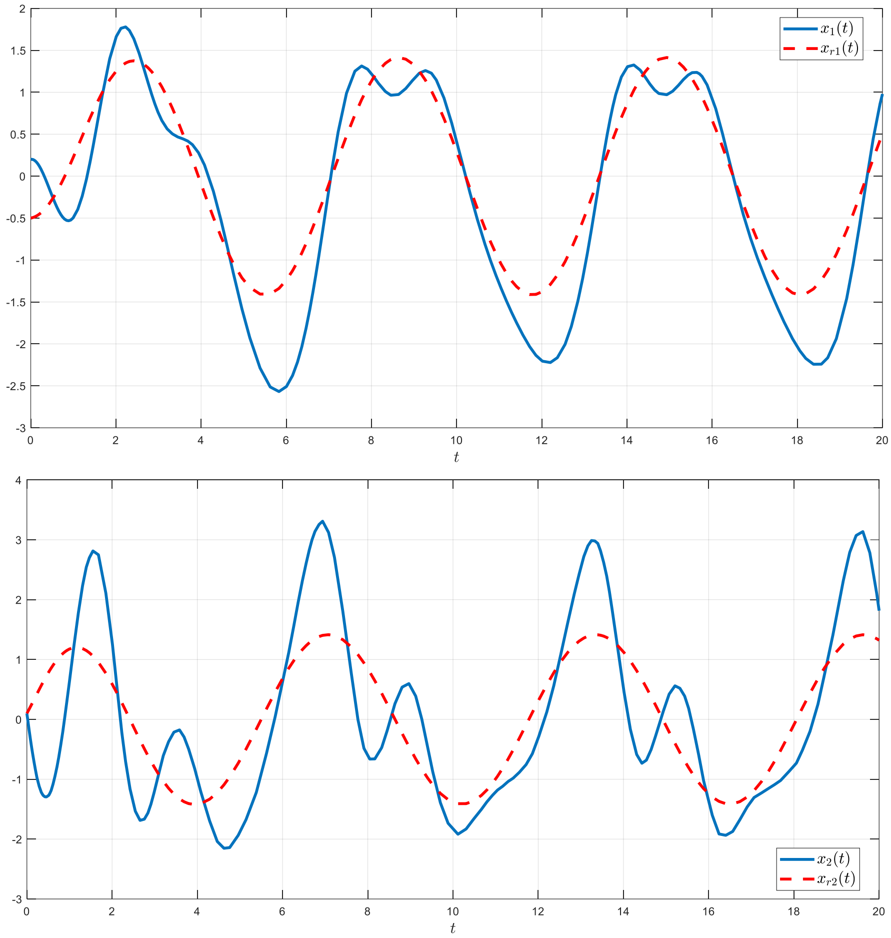

Fig. 2: State and reference responses using the non-reliable controller (case 1)

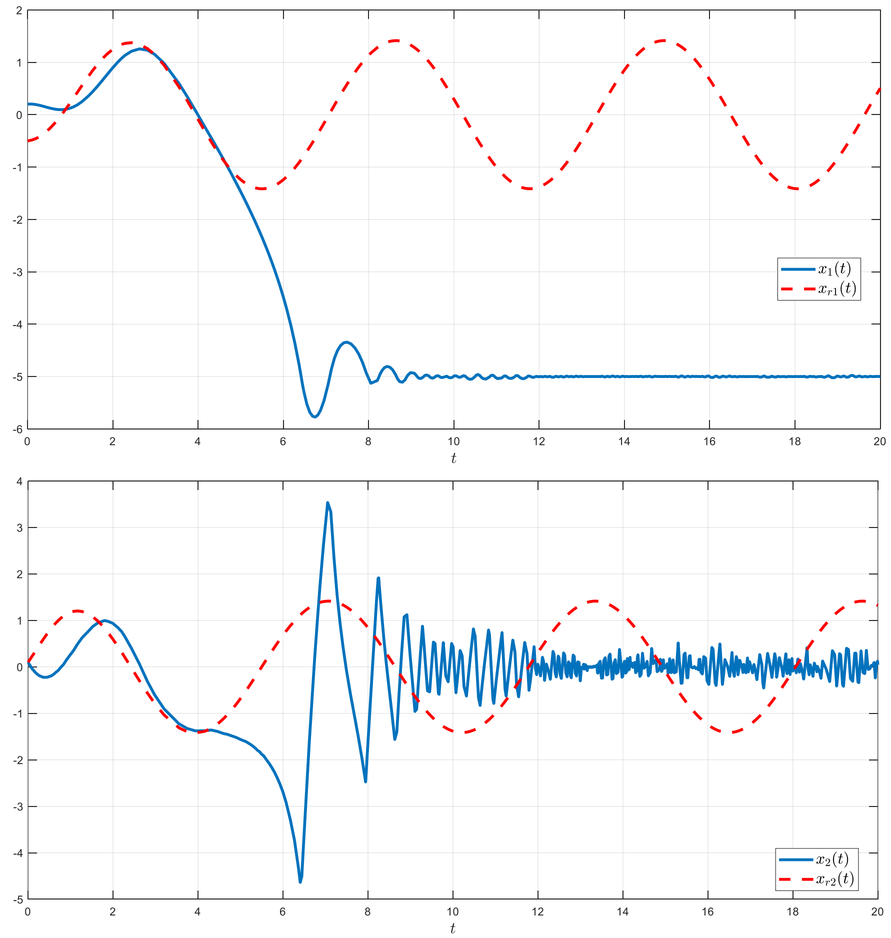

Fig. 3: State and reference responses using the method in Gu et al. (2017)

It is observed that, when the reliable control law (case 1) is applied the system dynamics are stabilized despite actuator failures, however, when the control laws are designed without taking into account the actuator failures as in case 2 and (Gu et al., 2017), and applied to the system with failures, it can be seen that the performances of the system are degraded. Thus, the simulations validate that the proposed control scheme is effective in accommodating actuator faults in the system.

6. Conclusions and future work

Under the L_2 −gain performance, the robust reliable tracking control design problem of the T-S fuzzy system with external disturbance, and actuator faults have been studied. With comprehensive consideration of non-linearity, parameter uncertainty, exogenous disturbance, and actuator failure, an effective reliable fuzzy controller has been designed. Among the main results, the main findings are as follows: (i) The model of the fault includes a non-linear part which is more general than the conventional actuator fault models has been investigated (ii) Using a non-quadratic Lyapunov function sufficient conditions with L_2-gain performance has been derived to deal with the reliable tracking control problem for the system under consideration (iii) A reliable robust controller has been also synthesized to cope with uncertainties in the T-S fuzzy model. The simulation studies on the Duffing forced-oscillation systems have confirmed that the proposed strategy is feasible and effective. As part of future research topics, the suggested developments will be extended to non-linear systems with event-triggered and saturation issues.

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Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

References

Chang YC and Yen HM (2005). Adaptive output feedback tracking control for a class of uncertain nonlinear systems using neural networks. IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), 35(6): 1311-1316. https://doi.org/10.1109/TSMCB.2005.850158 **PMid:16366255**

Chu X and Li M (2019). Observer-based model following sliding mode tracking control of discrete-time linear networked systems with two-channel event-triggered schemes and quantizations. Applied Mathematics and Computation, 355: 428-448. https://doi.org/10.1016/j.amc.2019.03.010

Fu L, Lam HK, Liu F, Zhou H, and Zhong Z (2022). Robust tracking control of interval type-2 positive Takagi-Sugeno fuzzy systems with external disturbance. IEEE Transactions on Fuzzy Systems, 30(10): 4057-4068. https://doi.org/10.1109/TFUZZ.2021.3136926

Gu Z, Yue D, Liu J, and Ding Z (2017). H∞ tracking control of nonlinear networked systems with a novel adaptive event-triggered communication scheme. Journal of the Franklin Institute, 354(8): 3540-3553. https://doi.org/10.1016/j.jfranklin.2017.02.020

Kamal E, Aitouche A, and Oueidat M (2013). Fuzzy fault-tolerant control of wind-diesel hybrid systems subject to sensor faults. IEEE Transactions on Sustainable Energy, 4(4): 857-866. https://doi.org/10.1109/TSTE.2013.2253138

Kang HB and Lee HJ (2018). Sampled-data static output-feedback control for nonlinear systems in T–S form via descriptor redundancy. Neurocomputing, 318: 1-6. https://doi.org/10.1016/j.neucom.2018.06.048

Kaviarasan B, Sakthivel R, and Kwon OM (2016). Robust fault-tolerant control for power systems against mixed actuator failures. Nonlinear Analysis: Hybrid Systems, 22: 249-261. https://doi.org/10.1016/j.nahs.2016.05.003

Kchaou M, Gassara H, and El-Hajjaji A (2018). Adaptive sliding mode control for fuzzy singular systems with time delay and input nonlinearity. International Journal of Adaptive Control and Signal Processing, 32(3): 464-479. https://doi.org/10.1002/acs.2854

Kchaou M, Jerbi H, Abassi R, VijiPriya J, Hmidi F, and Kouzou A (2021). Passivity-based asynchronous fault-tolerant control for nonlinear discrete-time singular Markovian jump systems: A sliding-mode approach. European Journal of Control, 60: 95-113. https://doi.org/10.1016/j.ejcon.2021.04.004

Kchaou M, Toumi A, and Souissi M (2011). Delay-dependent H∞ resilient output fuzzy control for nonlinear discrete-time systems with time-delay. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 19(2): 229-250. https://doi.org/10.1142/S0218488511006988

Latrach C, Kchaou M, Rabhi A, and El Hajjaji A (2015). Decentralized networked control system design using Takagi-Sugeno (TS) fuzzy approach. International Journal of Automation and Computing, 12(2): 125-133. https://doi.org/10.1007/s11633-015-0879-9

Latrech C, Kchaou M, and Guéguen H (2018). Networked non-fragile H∞ static output feedback control design for vehicle dynamics stability: A descriptor approach. European Journal of Control, 40: 13-26. https://doi.org/10.1016/j.ejcon.2017.10.005

Li A, Liu M, Cao X, and Liu R (2022). Adaptive quantized sliding mode attitude tracking control for flexible spacecraft with input dead-zone via Takagi-Sugeno fuzzy approach. Information Sciences, 587: 746-773. https://doi.org/10.1016/j.ins.2021.11.002

Makni S, Bouattour M, El Hajjaji A, and Chaabane M (2019). Robust observer based fault tolerant tracking control for T–S uncertain systems subject to sensor and actuator faults. ISA Transactions, 88: 1-11. https://doi.org/10.1016/j.isatra.2018.11.022 **PMid:30545772**

Shi H, Li P, Cao J, Su C, and Yu J (2020). Robust fuzzy predictive control for discrete-time systems with interval time-varying delays and unknown disturbances. IEEE Transactions on Fuzzy Systems, 28(7): 1504-1516. https://doi.org/10.1109/TFUZZ.2019.2959539

Takagi T and Sugeno M (1985). Fuzzy identification of systems and its applications to modeling and control. IEEE Transactions on Systems, Man, and Cybernetics, 15(1): 116-132. https://doi.org/10.1109/TSMC.1985.6313399

Tao J, Lu R, Su H, Shi P, and Wu ZG (2018). Asynchronous filtering of nonlinear Markov jump systems with randomly occurred quantization via T–S fuzzy models. IEEE Transactions on Fuzzy Systems, 26(4): 1866-1877. https://doi.org/10.1109/TFUZZ.2017.2754999

Tuan HD, Apkarian P, Narikiyo T, and Yamamoto Y (2001). Parameterized linear matrix inequality techniques in fuzzy control system design. IEEE Transactions on Fuzzy Systems, 9(2): 324-332. https://doi.org/10.1109/91.919253

Wang S, Jiang Y, Li Y, and Liu D (2015). Reliable observer-based H∞ control for discrete-time fuzzy systems with time-varying delays and stochastic actuator faults via scaled small gain theorem. Neurocomputing, 147: 251-259. https://doi.org/10.1016/j.neucom.2014.06.069

Wang Y, Shi P, and Yan H (2018). Reliable control of fuzzy singularly perturbed systems and its application to electronic circuits. IEEE Transactions on Circuits and Systems I: Regular Papers, 65(10): 3519-3528. https://doi.org/10.1109/TCSI.2018.2834481

Xie L (1996). Output feedback H∞ control of systems with parameter uncertainty. International Journal of Control, 63(4): 741-750. https://doi.org/10.1080/00207179608921866

Yan H, Wang J, Wang F, Wang Z, and Zou S (2019). Observer-based reliable passive control for uncertain T–S fuzzy systems with time-delay. International Journal of Systems Science, 50(5): 905-918. https://doi.org/10.1080/00207721.2019.1585996

Zhang D, Han QL, and Jia X (2015). Network-based output tracking control for a class of TS fuzzy systems that can not be stabilized by nondelayed output feedback controllers. IEEE Transactions on Cybernetics, 45(8): 1511-1524. https://doi.org/10.1109/TCYB.2014.2354421 **PMid:25222965**

Zhang D, Han QL, and Jia XC (2014). Observer-based H∞ output tracking control for networked control systems. International Journal of Robust and Nonlinear Control, 24(17): 2741-2760. https://doi.org/10.1002/rnc.3021