International Journal of Advanced and Applied Sciences, 9(9) 2022, Pages: 25-32

Contents lists available at Science-Gate

International Journal of Advanced and Applied Sciences

Journal homepage: http://www.science-gate.com/IJAAS.html

Fractional solution of helical motion of a charged particle under the influence of Lorentz force

Moaz Mohammad Altarawneh *

Department of Physics, Mutah University, Mutah, Jordan

ARTICLE INFO

Article history:
Received 7 March 2022
Received in revised form
22 May 2022
Accepted 31 May 2022

Keywords:
Helical trajectory
Fractional operator
Laplace transform
Generalized solution
Mittag-Leffler function

ABSTRACT

In this study, a generalized solution for the helical motion of a charged particle in uniform electric and magnetic fields is obtained using a powerful fractional derivative approach. Using this approach, the differential equations that describe the helical motion of a charged particle in the fields were obtained. The solution for the fractional differential equations is presented in great detail in terms of a series solution using the Mittag-Leffler function. The Laplace transform technique was used to solve the differential equations in the regular form and in the fractional form (with fractional parameter γ). Two and three-dimensional plots were presented for the trajectory of the particle before and after introducing the fractional operator for different values of γ. Features of delay in the motion and dissipation in the medium have been observed in the fractional solution too. The importance of our work stems from the two- and three-dimensional visualization of the obtained generalized helical trajectories that can be applied to similar types of motions in nature and the universe.

1. Introduction

When studying the motion of an object under the influence of a net force, usually Newton's second law is used to find the position, the velocity, and the acceleration at any time. To use Newton's second law, a set of second-order differential equations along with initial conditions must be established. After solving the set of differential equations, the exact trajectory of the moving object can be found in great detail (Morin, 2008). Recently, there is an increasing interest in employing fractional calculus when solving differential equations that describe a wide range of motions (Bokhari et al., 2022; Elzahar et al., 2020; Rosales et al., 2014). The motivation for using fractional calculus is not limited to building mathematical simulations for dynamical systems only but also looking for new potential applications (Sun et al., 2018; Lorenzo and Hartley, 2016; Shishkina and Sitnik, 2020). However, while differential equations with integer orders provide exact solutions for motions, fractional differential equations do not provide meaningful solutions that describe motions in most cases so far (Baleanu et al., 2012). In fact, fractional differential equations provide general solutions and exact solutions can be obtained only as a special case of the general fractional solutions (Morales-Delgado et al., 2017).

The helical motion is a common type of motion that can be witnessed in many physical systems, for example, the motion of charged particles emitted from the sun (solar wind) into the earth's magnetic field (Uddin et al., 2021; Somov, 2013). When the radius of the helical trajectory is increasing or decreasingwith time the trajectory can be developed into a helical cone trajectory. A helical cone can be observed in weather phenomena systems too (e. g., wind motion in hurricanes and tornadoes) and in two dimensions it can be observed in galactic motion in the universe (Abdelhady et al., 2021; Bryant and Krabbe, 2021).

When attempting to module such motion and to study how its solution evolves with time, fractional calculus provides a powerful tool to achieve this task. In general, solving fractional differential equations for any kind of motion enables us to observe the time evolution of the solution, delays, and what resembles a dissipation behavior in the medium where the particle is moving (Gómez-Aguilar et al., 2015; Koksal, 2019; Martínez et al., 2018; Nasrolahpour, 2013; Pskhu and Rekhviashvili, 2018). In the helical motion particularly, in a three-dimension (3D) picture, we expectthe circular part of the solution to be vulnerable to delays and

* Corresponding Author.
Email Address: moaz@mutah.edu.jo
https://doi.org/10.21833/ijaas.2022.09.004
Corresponding author's ORCID profile:
https://orcid.org/0000-0001-9278-9907

dissipations when fractional differentiation is introduced. Also, one would expect delays and dissipation in the direction perpendicular to the circular part of the trajectory too.

The main goal of this work is to obtain generalized solutions to the differential equations that describe the motion of a charged particle in uniform electric and magnetic fields using a simple fractional operator. Finding the generalized solutions of the differential equations enables us to build a 3D picture of the motion's trajectories and visualize them numerically, which were not considered in previous works (Morales-Delgado et al., 2017). Building such 3D pictures helps in modeling similar types of motions in nature and the universe that are driven by different types of forces (Lorenzo and Hartley, 2016). This supports the ongoing effort to bridge the gap between the real systems and generalized solutions obtained for different fractional orders.

In this work, the differential equations that describe the helical motion of a charged particle in electric and magnetic fields will be introduced and solved exactly using the Laplace transform, the final solution will be plotted in 3D. To introduce the fractional approach for the helical motion, a fractional operator will be applied to the differential equations describing the motion of the charged particle in a uniform electric field parallel to a uniform magnetic field along the z-axis. The new set of fractional differential equations will be solved using the Laplace transform and the final solutions will be plotted for different orders of fractional differentiation (γ). At the end of this work, a conclusion that summarizes our findings will be presented.

2. The helical trajectory equations

To get the helical trajectory for a charged particle of mass m and charge q, an electric field should be applied so that $\vec{E} = E_r\hat{r} + E_z\hat{z}$ in cylindrical coordinates with a magnetic field pointing along the z-axis as $\vec{B} = B\hat{z}$ too. In this case, the magnetic field is responsible for the circular path of the trajectory, and the electric field component along the $\hat{r}$ direction is responsible for increasing or decreasing the radius of the trajectory (depending on charge) and the E_z component is responsible about the drift along the z-axis.

For simplicity, in this study, we will consider an electric field with a component along the z-direction only so that $\vec{E} = E\hat{z}$. Such consideration leads to a trajectory with a fixed radius (helical path). Also, assuming the particle has an initial velocity along the x-axis equal to v_0 as in Fig. 1.

Starting from the electromagnetic force acting on a charged particle placed in an electric and magnetic field $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ (Griffiths, 2005), the differential equations that describe the motion along the x, y and z can be written as following:

$$m\ddot{x}(t) = qB\dot{y}(t) \quad (1)$$
$$m\ddot{y}(t) = -qB\dot{x}(t) \quad (2)$$
$$m\ddot{z}(t) = qE \quad (3)$$

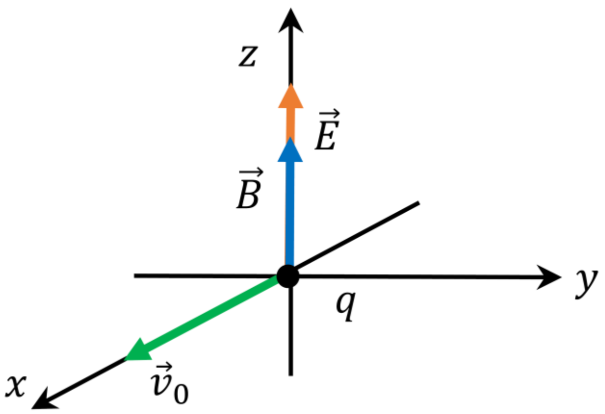

Fig. 1: A particle of mass m and charge q with an initial velocity $\vec{v}_0$ in electric field $\vec{E}$ and magnetic field vector $\vec{B}$ at $t = 0$

Rewriting Eqs. 1, 2, and 3 as third-order differential equations in terms of time and by setting $\omega_0 = \frac{qB}{m}$, Eqs. 1, 2, and 3 are rewritten as follows:

$$\dddot{x}(t) + \omega_0^2\,\dot{x}(t) = 0 \quad (4)$$
$$\dddot{y}(t) + \omega_0^2\dot{y}(t) = 0 \quad (5)$$
$$\ddot{z}(t) - \omega_0\frac{E}{B} = 0 \quad (6)$$

Starting with the motion along the x-axis and by applying the Laplace transform for Eq. 4 as:

$$L\{\dddot{x}(t)\} + L\{\omega_0^2\dot{x}(t)\} = 0 \quad (7)$$

where the terms $L\{\dddot{x}(t)\}$ and $L\{\omega_0^2\dot{x}(t)\}$ are defined as (Kimeu, 2009; Arfken et al., 2013):

$$L\{\dddot{x}(t)\} = s^3X(s) - s^2x(t)|_{t=0} - s\dot{x}(t)|_{t=0} - \ddot{x}(t)|_{t=0} \quad (8)$$
$$L\{\omega_0^2\,\dot{x}\} = \omega_0^2(sX(s) - x(t)|_{t=0}) \quad (9)$$

Using Eq. 8 and Eq. 9, then Eq. 7 is rewritten as:

$$s^3X(s) - s^2x(t)|_{t=0} - s\dot{x}(t)|_{t=0} - \ddot{x}(t)|_{t=0} + \omega_0^2(sX(s) - x(t)|_{t=0}) = 0 \quad (10)$$

By applying the initial conditions suggested for this motion along the x-axis as $x(0) = 0$, $\dot{x}(0) = v_0$ and $\ddot{x}(0) = 0$, Eq. 10 can be:

$$X(s) = \frac{v_0}{s^2+\omega_0^2} \quad (11)$$

Applying the inverse of Laplace transform so that:

$$x(t) = L^{-1}\{X(s)\} \quad (12)$$
$$x(t) = v_0\,L^{-1}\left\{\frac{1}{s^2+\omega_0^2}\right\} \quad (13)$$

Then the solution along the x-axis is written as (Kimeu, 2009; Arfken et al., 2013):

$$x(t) = \frac{v_0}{\omega_0}\sin(\omega_0 t) \quad (14)$$

In the Mittag-Leffler function format the above solution can be written as:

$$x(t) = v_0 t\, E_{2,2}(-\omega_0^2 t^2) \quad (15)$$

where $E_{2,2}(-\omega_0^2 t^2)$ is the Mittag-Leffler function which has the general form (Duan and Chen, 2018):

$$E_{\alpha,\beta}(-\omega_0^2 t^2) = \sum_{k=0}^{\infty} \frac{(-\omega_0^2 t^2)^k}{\Gamma(\alpha k+\beta)} \quad \alpha > 0, \beta > 0 \quad (16)$$

In the same way, applying the Laplace transform for Eq. 5 is as follows:

$$L\{\dddot{y}(t)\} + L\{\omega_0^2 \dot{y}(t)\} = 0 \quad (17)$$

Using the definition of Laplace transform for the $\dddot{y}(t)$ and for $\omega_0^2 \dot{y}(t)$ as:

$$L\{\dddot{y}(t)\} = s^3 Y(s) - s^2 y(t)|_{t=0} - s\dot{y}(t)|_{t=0} - \ddot{y}(t)|_{t=0} \quad (18)$$
$$L\{\omega_0^2\, \dot{y}\} = \omega_0^2 (sY(s) - y(t)|_{t=0}) \quad (19)$$

By using the above definitions, Eq. 17 can be written as:

$$s^3 Y(s) - s^2 y(t)|_{t=0} - s\dot{y}(t)|_{t=0} - \ddot{y}(t)|_{t=0} + \omega_0^2 (sY(s) - y(t)|_{t=0}) = 0 \quad (20)$$

After applying the initial conditions $(y(0) = \dot{y}(0) = 0,\ \ddot{y}(0) = -\omega_0 v_0)$, Eq. 17 is resulted as:

$$s^3 Y(s) + \omega_0^2\, sY(s) + \omega_0 v_0 = 0 \quad (21)$$
$$Y(s) = \frac{-\omega_0 v_0}{s(s^2+\omega_0^2)} \quad (22)$$

Rewriting Eq. 22 as a sum of two fractions, we found the following:

$$Y(s) = -\omega_0 v_0 \left(\frac{\frac{1}{\omega_0^2}}{s} - \frac{\frac{1}{\omega_0^2} s}{s^2+\omega_0^2} \right) \quad (23)$$
$$Y(s) = \frac{-v_0}{\omega_0} \left(\frac{1}{s} - \frac{s}{s^2+\omega_0^2} \right) \quad (24)$$

Now taking the inverse of $Y(s)$ Laplace transform as:

$$L^{-1}\{Y(s)\} = -\frac{v_0}{\omega_0} L^{-1} \left\{ \frac{1}{s} - \frac{s}{s^2+\omega_0^2} \right\} \quad (25)$$

The solution on the y-axis is found to be (Duan and Chen, 2018):

$$y(t) = -\frac{v_0}{\omega_0}(1 - \cos \omega_0 t) \quad (26)$$

In the Mittag-Leffler function format the above solution can be written as:

$$y(t) = -\frac{v_0}{\omega_0}\left(1 - E_{2,1}(-\omega_0^2 t^2)\right) \quad (27)$$

For the motion along the z-axis by applying the Laplace transform for Eq. 6, it can be found that:

$$L\{\ddot{z}(t)\} - L\left\{\omega_0 \frac{E}{B}\right\} = 0 \quad (28)$$

Writing the definition of Laplace transform for the $\ddot{z}(t)$ and for $\omega_0 \frac{E}{B}$ as (Kimeu, 2009; Arfken et al., 2013):

$$L\{\ddot{z}\} = s^2 Z(s) - sz(t)|_{t=0} - s\dot{z}(t)|_{t=0} \quad (29)$$
$$L\left\{\omega_0 \frac{E}{B}\right\} = \omega_0 \frac{E}{Bs} \quad (30)$$

Using the initial conditions along the z-axis $z(t) = \dot{z}(0) = 0$, Eq. 28 is written as:

$$s^2 Z(s) - \omega_0 \frac{E}{Bs} = 0 \quad (31)$$
$$Z(s) = \omega_0 \frac{E}{B} \frac{1}{s^3} \quad (32)$$

Using $L^{-1}\left\{\frac{1}{s^\alpha}\right\} = \frac{t^{\alpha-1}}{\Gamma(\alpha)}$ the solution along the $z-$axis is:

$$z(t) = \omega_0 \frac{E}{B} \frac{t^2}{2} \quad (33)$$

Plotting the final solutions for $x, y,$ and z in Eq. 15, Eq. 27, and Eq. 33, the motion trajectory is found to be a helical trajectory as in Fig. 2.

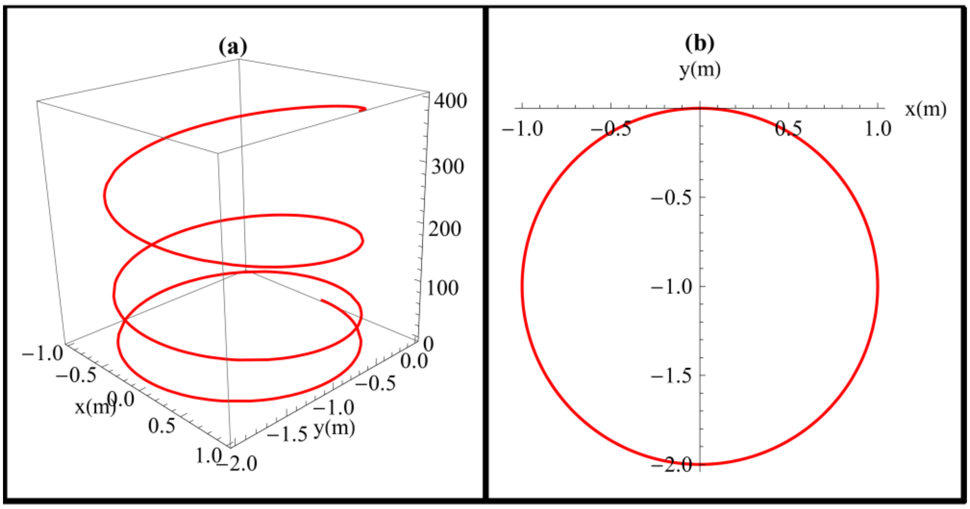

Fig. 2: a) The trajectory of the charged particle in 3D and b) 2D in the XY-plane as the values of ω_0, E, B and v_0 are set to 1 unit

3. Fractional approach

To obtain the fractional solution for the equations Eq. 4-Eq. 6, the ordinary time derivative operator will be modified to a fractional time derivative operator as follows (Gómez-Aguilar et al., 2015; Lorenzo and Hartley, 2016; Martínez et al., 2018; Pskhu and Rekhviashvili, 2018):

$$\frac{d}{dt} \xrightarrow{PPPP} \frac{1}{\sigma^{1-\gamma}} \frac{d^\gamma}{dt^\gamma} \qquad 1 < \gamma \leq n \tag{34}$$

where, the order of the time derivative is γ (an arbitrary parameter). The parameter σ is equal to T_0 (where T_0 is the periodic time of the motion, $T_0 = 1/\omega_0$) and it is introduced so that the dimension of the differential equations is consistent (Shishkina and Sitnik, 2020). As can be seen below for $\gamma = 1$, the fractional operator in Eq. 34 converts into an ordinary derivative in time as:

$$\frac{1}{\sigma^{1-\gamma}} \frac{d^\gamma}{dt^\gamma}\bigg|_{\gamma=1} = \frac{d}{dt} \tag{35}$$

There are many definitions that could be used to find the fractional derivatives e.g., the Riemann–Liouville fractional derivative definition and the Caputo fractional derivative definition. However, in this work, the Caputo definition of the fractional derivative will be used since it has the value of zero when $f(t)$ is constant and since its Laplace Transform can be expressed in terms of the initial values. The Caputo definition has the following integral form (Gómez-Aguilar et al., 2015):

$$\frac{d^q}{dt^q} f(t) = \frac{1}{\Gamma(n-q)} \int_0^t \frac{f^{(n)}(\eta)}{(t-\eta)^{q-n+1}} d\eta \tag{36}$$

where, $n = 1, 2, \ldots \in N$ and $n - 1 < q \leq n$, q is the order of derivative and can have non-integer values. Since $n = 3$, the value of γ is limited by the condition $2 < 3\gamma < 3$ or $2/3 < \gamma \leq 1$.

The motion of the charged particle on the x-axis (represented by Eq. 4) can be rewritten into the fractional differential form as:

$$D^{3\gamma}x(t) + \sigma^{2(1-\gamma)}\omega_0^2 D^\gamma x(t) = 0 \tag{37}$$

where, $D^{3\gamma} = \frac{d^{3\gamma}}{dt^{3\gamma}}$, $D^\gamma = \frac{d^\gamma}{dt^\gamma}$, and $\omega^2 = \sigma^{2(1-\gamma)}\omega_0^2$. Taking the Laplace transform of Eq. 37, we get:

$$L\{D^{3\gamma}x(t)\} + L\{\omega^2 D^\gamma x(t)\} = 0 \tag{38}$$

The Laplace transform for the first and second terms in Eq. 38 can be written respectively as (Kimeu, 2009):

$$L\{D^{3\gamma}x(t)\} = s^{3\gamma}X(s) - \sum_{k=0}^{n-1} s^{n-k-1} D^{k-(n-3\gamma)}x(0) \tag{39}$$

$$L\{D^\gamma x(t)\} = s^\gamma X(s) - \sum_{k=0}^{n-1} s^{n-k-1} D^{k-(n-\gamma)}x(0) \tag{40}$$

That can be simplified as follow:

$$L\{D^{3\gamma}x(t)\} = s^{3\gamma}X(s) - s^2 D^{3(\gamma-1)}x(0) - sD^{3\gamma-2}x(0) - D^{3\gamma-1}x(0) \tag{41}$$

$$L\{D^\gamma x(t)\} = s^\gamma X(s) - D^{\gamma-1}x(0) \tag{42}$$

The initial conditions along the x-axis on the fractional form are written as:

$$\begin{aligned} D^{3\gamma-3}x(0) &= 0 \\ D^{3\gamma-1}x(0) &= 0 \\ D^{3\gamma-2}x(0) &= c \\ D^{\gamma-1}x(0) &= 0 \end{aligned} \tag{43}$$

where c is a constant that is reduced to v_0 for $\gamma = 1$.

After inserting the initial conditions for the fractional form, Eq. 38 can be rewritten as:

$$s^{3\gamma}X(s) - c\,s + \omega^2 s^\gamma X(s) = 0 \tag{44}$$

Solving for $X(s)$as:

$$X(s) = \frac{cs}{s^{3\gamma} + \omega^2 s^\gamma} \tag{45}$$

Rewriting the Eq. 45 in a form that can be found in Laplace transform tables, Eq. 45 can have the form (Kimeu, 2009):

$$X(s) = \frac{cs^{1-\gamma}}{s^{2\gamma} + \omega^2} \tag{46}$$

Taking the Laplace inverse transform of Eq. 46, we get:

$$L^{-1}\{X(s)\} = cL^{-1}\left\{\frac{s^{1-\gamma}}{s^{2\gamma} + \omega^2}\right\} \tag{47}$$

The constant c easily can be verified to be equal to $\omega_0^{(3-3\gamma)} v_0$. The solution of the above inverse Laplace transform in terms of the Mittag-Leffler function can be written as:

$$x(t) = \omega_0^{(3-3\gamma)} v_0 t^{3\gamma-2} E_{2\gamma,3\gamma-1}(-\omega^2 t^{2\gamma}) \tag{48}$$

where, $2/3 < \gamma \leq 1$. Eq. 48 represents a generalized solution on the x −axis. Also, it can be verified that the fractional solution obtained in Eq. 48 for $\gamma = 1$ is equivalent to Eq. 15 which has been obtained by ordinary differential derivatives.

The same procedure above can be followed for the ordinary equation on the y-axis part of the motion. The initial conditions for the motion on the y-axis at $t = 0$ used earlier for the ordinary differential equations can be rewritten in the fractional form as:

$$\begin{aligned} D^{3\gamma-3}y(0) &= 0 \\ D^{3\gamma-2}y(0) &= 0 \\ D^{3\gamma-1}y(0) &= c_0 \\ D^{\gamma-1}y(0) &= 0 \end{aligned} \tag{49}$$

where, c_0 is a constant that is reduced to $\omega_0 v_0$for $\gamma = 1$.

Performing the same task and rewriting Eq. 22 in the fractional form and after inserting the initial conditions and using $\omega^2 = \sigma^{2(1-\gamma)}\omega_0^2$ the following equation can be found:

$$s^{3\gamma}Y(s) + \omega^2 s^{\gamma}Y(s) - c_0 = 0 \quad (50)$$

Solving for $Y(s)$ it can be found that $Y(s)$ can be written as:

$$Y(s) = \frac{c_0}{s^{3\gamma}+\omega^2 s^{\gamma}} \quad (51)$$
$$= \frac{c_1}{s^{\gamma}} + \frac{c_2}{s^{2\gamma}+\omega^2} \quad (52)$$

From Eqs. 51 and 52, it can be verified that $c_2 = -c_1 s^{\gamma}$ and $c_0 = c_1\omega^2$, which means Eq. 52 can be rewritten as:

$$Y(s) = \frac{c_1}{s^{\gamma}} - \frac{c_1 s^{\gamma}}{s^{2\gamma}+\omega^2} \quad (53)$$

Taking the inverse Laplace transform of Eq. 53 we get (Kimeu, 2009; Arfken et al., 2013):

$$L^{-1}\{Y(s)\} = c_1\, L^{-1}\left\{\frac{1}{s^{\gamma}}\right\} - c_1\, L^{-1}\left\{\frac{s^{\gamma}}{s^{2\gamma}+\omega^2}\right\} \quad (54)$$

The solution of the above inverse Laplace transform is:

$$y(t) = c_1 \frac{t^{\gamma-1}}{\Gamma(\gamma)} - c_1\, t^{\gamma-1} E_{2\gamma,\gamma}(-\,\omega^2 t^{2\gamma}) \quad (55)$$

Using $c_0 = c_1\omega^2$ and $\omega^2 = \sigma^{2(1-\gamma)}\omega_0^2$ and comparing with Eq. 27, the constant $c_1 = \sigma^{(1-\gamma)}E/B$. Finally, Eq. 55 can be written as:

$$y(t) = -{\omega_0}^{(\gamma-2)} v_0 \left(\frac{t^{\gamma-1}}{\Gamma(\gamma)} - t^{\gamma-1} E_{2\gamma,\gamma}(-\,\omega^2 t^{2\gamma})\right) \quad (56)$$

where again $2/3 < \gamma \leq 1$. Eq. 56 represents the general solution on the y-axis which is reduced to the solution presented in Eq. 27 for $\gamma = 1$.

Now, the differential equation that describes the motion along the z-axis (Eq. 6) can be rewritten in terms of the fractional derivative as follows:

$$D^{2\gamma}z(t) - \frac{\sigma^{2(1-\gamma)}\omega_0 E}{B} = 0 \quad (57)$$

Taking $\omega = \sigma^{2(1-\gamma)}\omega_0$ and taking the Laplace transform, Eq. 57 is rewritten as:

$$L\{D^{2\gamma}z(t)\} - L\left\{\omega\frac{E}{B}\right\} = 0 \quad (58)$$

Using the definition (Kimeu, 2009; Arfken et al., 2013):

$$L\{D^{2\gamma}z(t)\} = s^{2\gamma}Z(s) - \sum_{k=0}^{n-1} s^{n-k-1}\, D^{k-(n-2\gamma)}Z(0) \quad (59)$$
$$L\{D^{2\gamma}z(t)\} = s^{2\gamma}Z(s) - sD^{2\gamma-2}z(0) - D^{2\gamma-1}z(0) \quad (60)$$

Inserting the initial conditions in the fractional form so that $D^{2\gamma-2}z(0) = 0$ and $D^{2\gamma-1}z(0) = 0$ and by using $L\left\{\omega\frac{E}{B}\right\} = \omega\frac{E}{Bs}$, the result of applying the Laplace transform for Eq. 58. has the following form:

$$s^{2\gamma}Z(s) - \omega\frac{E}{Bs} = 0 \quad (61)$$

The function $Z(s)$ can be rewritten as:

$$Z(s) = \frac{\omega E}{B}\frac{1}{s^{2\gamma+1}} \quad (62)$$

The inverse Laplace transform for $Z(s)$ can be found as:

$$L^{-1}\{Z(s)\} = \omega\frac{E}{B}L^{-1}\left\{\frac{1}{s^{2\gamma+1}}\right\} \quad (63)$$

Using $L^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{\Gamma(n-1)}$ and $n = 2\gamma$, the time-dependent general solution along the z-axis can be obtained as:

$$z(t) = \omega\frac{E}{B}\frac{t^{2\gamma}}{\Gamma(2\gamma-1)} \quad (64)$$

Using $\omega = \omega_0^{2\gamma-1}$, the above solution will have the following form:

$$z(t) = \omega_0^{2\gamma-1}\frac{E}{B}\frac{t^{2\gamma}}{\Gamma(2\gamma-1)} \quad (65)$$

The general solution along the z-axis at $\gamma = 1$ is reduced to the solution obtained from the ordinary differential equations in Eq. 33.

Now, having found the fractional solution (generalized solutions) of $x(t)$, $y(t)$, and $z(t)$ as in Eq. 48, Eq. 56, and Eq. 65, the trajectory for different values of γ can be plotted as in Fig. 3.

From Fig. 3, it can be seen that for $\gamma = 1$ the trajectory is a helical trajectory which is identical to what has been seen in Fig. 2 before introducing the fractional approach. Also, it's clear from Fig. 3 that as γ is reduced by 0.01 the helical trajectory started to be affected as a reduction in the radius of the motion with a small delay when compared to the path for $\gamma = 1$. As the γ value is decreased the dissipation and delay behaviors become clearer in the particle trajectory until $\gamma = 2/3$ at which the main features of helical trajectory totally disappeared. The resulted trajectory of the particle due to the application of the fractional approach on the ordinary differential equations of the helical trajectory is a cyclone trajectory. It is interesting to observe the time evolution of the cyclone motion as it developed to a helical motion as the γ value is increased from 2/3 to 1. The time evolution of the trajectory and the dissipation of the motion in the xy-plane can be too observed as in Fig. 4.

For different values of γ when the fractional approach is used. For the motion projection in XY-plane, the same phenomena of delay and dissipation are observed too, and they become clearer as the value of γ is decreased toward 2/3.It is important to draw the reader's attention to the point that the cyclone trajectory was mathematically the result of using the fractional approach to helical trajectory.

When considering the XY-trajectory of the motion, it can be noted that the trajectory of the motion is spiral. Such spiraltrajectory solution could be used to build mathematical modules for other natural phenomena e.g., hurricanes, tornados, and whirlpools (Lorenzo and Hartley, 2016).

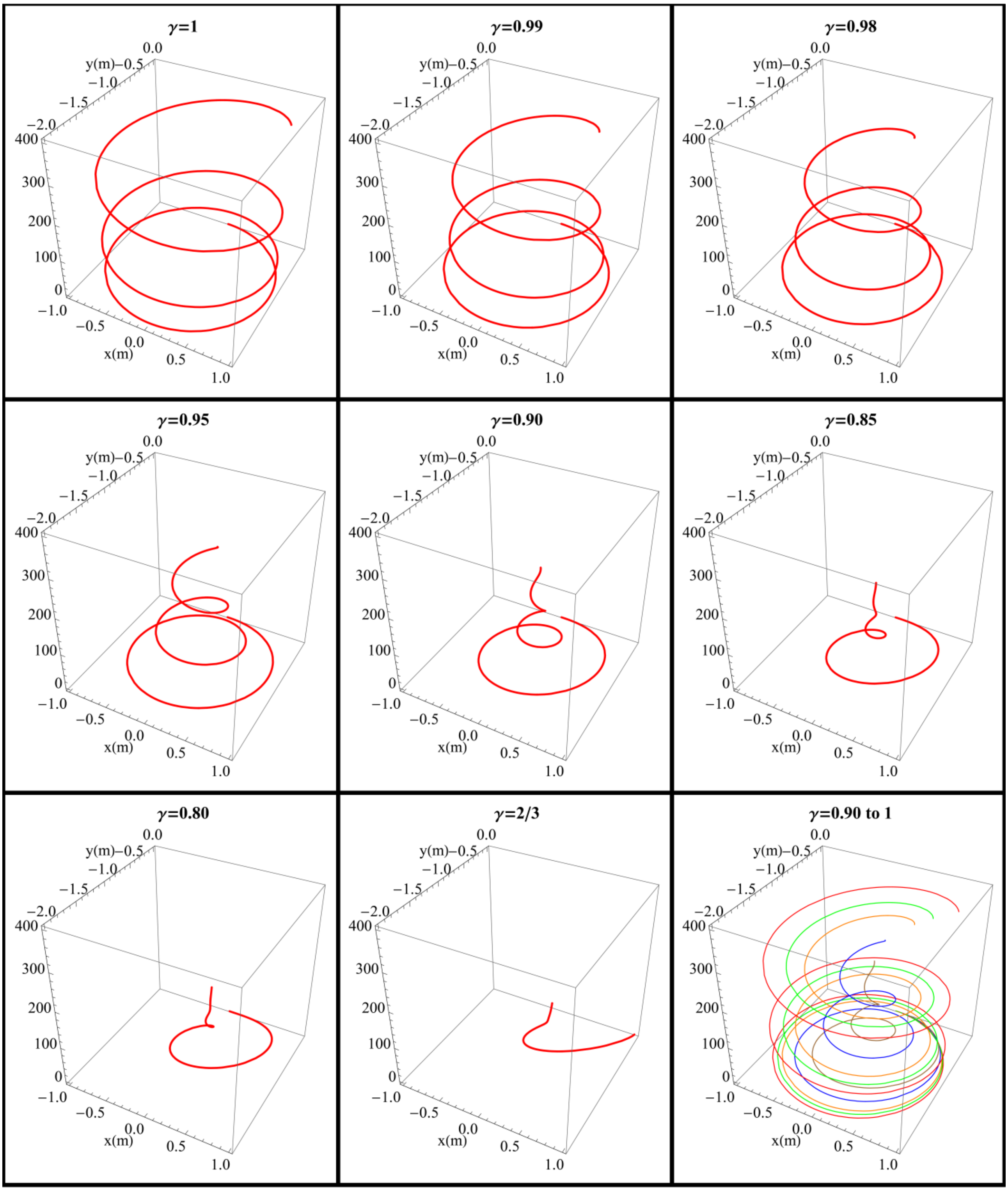

Fig. 3: The helical–cyclone trajectory of a charged particle moving in electric and magnetic fields pointing in the z-direction for different values of γ. For simplicity, the values of ω_0, E, B and v_0 are set to 1

4. Conclusion

The fractional derivative approach was applied for the differential equations that describe the motion of a charged particle in uniform electric and magnetic fields and a helical trajectory was obtained. After introducing the fractional operator in terms of γ (where $2/3 < \gamma \leq 1$), the obtained general solution describes a trajectory of a cyclone. The general solution which is written in terms of a series solution (Mittag-Leffler function) shows the behavior of delay in time and dissipation in the medium in an equivalent real system. A small change in the value of γ (e.g., from 1 down to 0.99) changes the trajectory from a helical to cyclone trajectory in 3D and to a spiral trajectory in 2D plots. The spiral trajectory obtained in this work can be generalized to build modules for similar scenario forces.

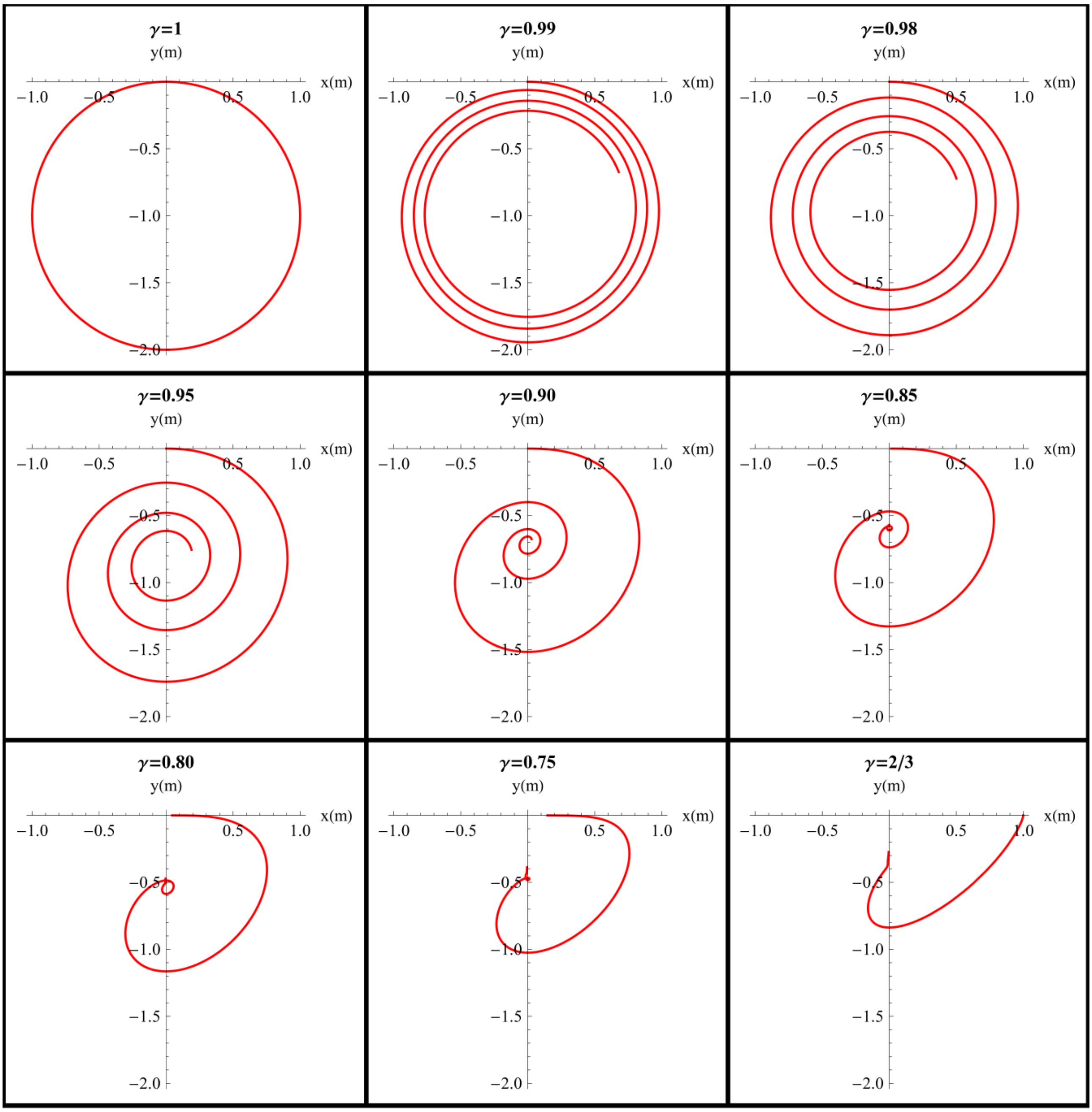

Fig. 4: The motion of the charged particle as observed in the XY-plane for different values of γ (all plots for the same time interval). For simplicity, the values of ω_0, E, B and v_0 are set to 1

Acknowledgment

This work was supported by the deanship of scientific research at Mutah University.

Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

References

Abdelhady AU, Spence SM, and McCormick J (2021). A three-dimensional six-degree-of-freedom windborne debris trajectory model for tornadoes. Journal of Wind Engineering and Industrial Aerodynamics, 214: 104622. https://doi.org/10.1016/j.jweia.2021.104622

Arfken GB, Weber HJ, and Harris FE (2013). Mathematical methods for physicists. 7th Edition, Elsevier, Waltham, USA.

Baleanu D, Diethelm K, Scalas E, and Trujillo JJ (2012). Fractional calculus: Models and numerical methods. Volume 3, World Scientific Publishing, Singapore, Singapore. https://doi.org/10.1142/8180

Bokhari A, Belgacem R, Kumar S, Baleanu D, and Djilali S (2022). Projectile motion using three parameter Mittag-Leffler function calculus. Mathematics and Computers in Simulation, 195: 22-30. https://doi.org/10.1016/j.matcom.2021.12.020

Bryant A and Krabbe A (2021). The episodic and multiscale Galactic Centre. New Astronomy Reviews, 93: 101630. https://doi.org/10.1016/j.newar.2021.101630

Duan J and Chen L (2018). Solution of fractional differential equation systems and computation of matrix Mittag–Leffler functions. Symmetry, 10(10): 503. https://doi.org/10.3390/sym10100503

Elzahar ER, Gaber AA, Aljohani AF, Machado JT, and Ebaid A (2020). Generalized Newtonian fractional model for the vertical motion of a particle. Applied Mathematical Modelling, 88: 652-660. https://doi.org/10.1016/j.apm.2020.06.054

Gómez-Aguilar JF, Yépez-Martínez H, Calderón-Ramón C, Cruz-Orduña I, Escobar-Jiménez RF, and Olivares-Peregrino VH (2015). Modeling of a mass-spring-damper system by fractional derivatives with and without a singular kernel. Entropy, 17(9): 6289-6303. https://doi.org/10.3390/e17096289

Griffiths DJ (2005). Introduction to electrodynamics. American Journal of Physics, 73(6): 574-576. https://doi.org/10.1119/1.4766311

Kimeu JM (2009). Fractional calculus: Definitions and applications. M.Sc. Thesis, Western Kentucky University, Bowling Green, USA.

Koksal ME (2019). Time and frequency responses of non-integer order RLC circuits. AIMS Mathematics, 4(1): 64-78. https://doi.org/10.3934/Math.2019.1.64

Lorenzo CF and Hartley TT (2016). The fractional trigonometry: With applications to fractional differential equations and science. John Wiley and Sons, Hoboken, USA. https://doi.org/10.1002/9781119139447

Martínez L, Rosales JJ, Carreño CA, and Lozano JM (2018). Electrical circuits described by fractional conformable derivative. International Journal of Circuit Theory and Applications, 46(5): 1091-1100. https://doi.org/10.1002/cta.2475

Morales-Delgado VF, Gómez-Aguilar JF, and Taneco-Hernandez MA (2017). Analytical solutions for the motion of a charged particle in electric and magnetic fields via non-singular fractional derivatives. The European Physical Journal Plus, 132: 527. https://doi.org/10.1140/epjp/i2017-11798-7

Morin D (2008). Introduction to classical mechanics: With problems and solutions. Cambridge University Press, Cambridge, UK.

Nasrolahpour H (2013). A note on fractional electrodynamics. Communications in Nonlinear Science and Numerical Simulation, 18(9): 2589-2593. https://doi.org/10.1016/j.cnsns.2013.01.005

Pskhu AV and Rekhviashvili SS (2018). Analysis of forced oscillations of a fractional oscillator. Technical Physics Letters, 44(12): 1218-1221. https://doi.org/10.1134/S1063785019010164

Rosales J, Guía M, Gómez F, Aguilar F, and Martínez J (2014). Two dimensional fractional projectile motion in a resisting medium. Open Physics, 12(7): 517-520. https://doi.org/10.2478/s11534-014-0473-8

Shishkina E and Sitnik S (2020). Transmutations, singular and fractional differential equations with applications to mathematical physics. Academic Press, Cambridge, USA. https://doi.org/10.1016/B978-0-12-819781-3.00017-3

Somov BV (2013). Motion of a charged particle in given fields. In: Somov BV (Ed.), Plasma astrophysics, part I: 87-113. Springer, New York, USA. https://doi.org/10.1007/978-1-4614-4283-7_5

Sun H, Zhang Y, Baleanu D, Chen W, and Chen Y (2018). A new collection of real world applications of fractional calculus in science and engineering. Communications in Nonlinear Science and Numerical Simulation, 64: 213-231. https://doi.org/10.1016/j.cnsns.2018.04.019

Uddin MJ, Nasrin ZM, and Li Y (2021). Effects of vertical wind shear and storm motion on tropical cyclone rainfall asymmetries over the North Indian Ocean. Dynamics of Atmospheres and Oceans, 93: 101196. https://doi.org/10.1016/j.dynatmoce.2020.101196