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Estimation of reliability function based on the upper record values for generalized gamma Lindley stress–strength model: Case study COVID-19

M. O. Mohamed *

Mathematics Department, Faculty of Science, Zagazig University, Zagazig, Egypt

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ABSTRACT

In this paper, the problem of estimation when X and Y are two independent upper record values from gamma Lindley distribution is considered. Maximum likelihood and the Bayesian estimator methods were used to set the best-estimated reliability function. The importance of this research is because this model, when applied, can obtain reliability values that depend on upper record values, which is an interesting problem in many real-life applications. Also, based on WHO data on the COVID-19 pandemic, a stress-strength model was applied based on the upper recorded values for Mont-Carlo simulation data.

1. Introduction

Record statistics are very important because they are widely used in many real-life applications that include data related to weather, sports, disease, economics, and life test studies. Many researchers focused on studying records such as Chandler (1952), Ahsanullah (2004), Arnold et al. (1992), and Teimouriri and Gupta (2012) who studied record values with an explanation and their properties especially to illustrate their importance and applications.

Based on the importance of upper records in many applications especially in statistics, many authors have analyzed them based on various models and applications to show how to apply them and also how we identically from it in many fields. In this study, let $X_1, X_2, \ldots$ be an infinite sequence of independent and identical distribution random variables with probability density function (x), and cumulative distribution function $F(x)$. An observation X_i said to be upper previous if it is exceeding all the previous observations, i.e. $X_i > X_j$ for $i < j$.

The model of stress-strength in reliability probability is very important to model in such applications as industry lifetime tests, weather, and diseases. The stress-strength reliability of a system defines the probability that the system will function properly until the strength exceeds the stress. The basic underlying philosophy in reliability studies is to examine whether a part of a product can sustain a certain amount of stress under some conditions so that it can survive for a longer period. For more explanation, there are a lot of researchers who studied this model. Jamal et al. (2019) studied the reliability function based on the model of stress-strength with the Pareto distribution function in a multi-component system. The stress-strength model of reliability with truncated hazard rate was studied by Bai et al. (2019). Baklizi (2008a; 2014; 2008b) studied the different situation for reliability under the stress-strength model. Hassan et al. (2015) researched the exponential inverted Weibull distribution and applied it for lower record data. Abd-elfattah and Mohamed (2011) explained the Weibull distribution on the stress-strength model. In addition, Poisson–exponential distribution and its application to reliability with the stress-strength model were studied by Mohamed (2015).

The applicability of reliability in the upper stress-strength (USS) model is based on samples of upper record values strongly in age tests, as most patients die when exposed to high levels of anti-drug drugs.

For example, if a sample of patients is made up of eight patients, they were given a gentle amount of anti-drug drugs (of low concentration) at the beginning of the treatment. Then the concentration of the anti-inflammatory drugs is gradually increased until we reach high levels of anti-inflammatory drugs. We will find that some patients have a breakdown due to the increase in concentration and dose together. But the few patients can tolerate the doses given, So the first

* Corresponding Author.
Email Address: mo11577.mm@gmail.com
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Corresponding author's ORCID profile:
https://orcid.org/0000-0003-0792-3919

patient who was able to tolerate the higher doses is registered as the first upper value. Then the next one in time is recorded as a second note, and so on to get a sample of the highest record values.

The organization of the paper is as follows: Section 2 presents the build of the reliability model. Section 3 presents the estimated of θ_1 and θ_2 by using the MLE method for reliability. Section 4 presents the estimated of θ_1 and θ_2 by using the Bayesian method under SE and LINEX loss function. Section 5 provides the steps of MC for simulated data and their application on estimation values of parameters and reliability function including their tables and figures. Section 6 gives the real example of COVID -19 for a model of reliability and outlined their data outputs including their tables and figures. Section 7 presents the conclusion.

2. Building a system of reliability

The strength (X) and stress (Y) are independent random variables. The reliability of a component with strength (X) and stress (Y) imposed on it is given by:

$$R = P(Y < X) = \int_0^\infty F_y(x) f_x(x)dx. \tag{1}$$

The function that has received interest to be studied in the stress-strength model (SS) is a new distribution that focuses on life data description where produced by Laribi et al. (2021), called Generalized Gamma-Lindley (GGL) distribution with probability density function:

$$f_x(x) = \frac{\theta^{\alpha+1}x^{\alpha-1}e^{-\theta x}((\gamma+\gamma\theta-\theta)x+1)}{\gamma(1+\theta)}, x, \theta, \alpha, \gamma > 0$$

we will study a special case of it when $\alpha = 1$, then,

$$f_x(x) = \frac{\theta^2 e^{-\theta x}((\gamma+\gamma\theta-\theta)x+1)}{\gamma(1+\theta)}, x, \theta, \gamma > 0$$

which is called Gamma Lindley (GL) distribution.

So, both strength and stress have the (GL) density functions with parameters θ_1 and θ_2, the parameter γ is a common parameter for two functions, as follows:

$$f_x(x) = \frac{\theta_1^2 e^{-\theta_1 x}((\gamma+\gamma\theta_1-\theta_1)x+1)}{\gamma(1+\theta_1)}, x, \theta_1, \gamma > 0$$

and

$$f_y(y) = \frac{\theta_2^2 e^{-\theta_2 x}((\gamma+\gamma\theta_2-\theta_2)x+1)}{\gamma(1+\theta_2)}, y, \theta_2, \gamma > 0 \tag{2}$$

According to Eq. 1, the reliability system is:

$$R = \int_0^\infty \int_0^y \frac{[\gamma+\gamma\theta_1-\theta_1 x+1]e^{-\theta_1 x}\theta_2^2}{\gamma(1+\theta_1)} \frac{[\gamma+\gamma\theta_2-y\theta_2+1]e^{-\theta_2 y}}{\gamma(1+\theta_2)} \, dx\, dy.$$

The mathematical formulation of reliability after completing integrals is as follows:

$$R = \frac{\theta_2^2}{\gamma^2(1+\theta_1)(1+\theta_2)}[2\theta_1(1+\theta_2) + 2\theta_2^2 + \gamma\theta_1^2 + \gamma\theta_2 + (\gamma-1)\theta_1^3 + (\gamma-1)\theta_1\theta_2 + (\gamma-1)\theta_1^2\theta_2 + (\gamma-1)\theta_2^2] \tag{3}$$

3. Maximum likelihood function method (MLE)

In order to estimate θ_1 and θ_2, let $\underline{r} = (r_0, r_1, \dots r_n)$ be a set of first observed (n+1) upper record values from (GL) distribution with parameters(θ_1, γ), and $\underline{s} = (s_0, s_1, \dots s_m)$ be an independent set of an observed first (m+1) upper record values from (GL) distribution with parameters(θ_2, γ), where γ is a known parameter.

Therefore, the likelihood function of the observed $\underline{r}$ and $\underline{s}$, respectively is given by:

$$L(\theta_1, \gamma|\underline{r}) = f(r_n)\prod_{i=1}^{n-1}\frac{f(r_i)}{1-F(r_i)} =$$
$$= \theta_1^2[\gamma(+\gamma\theta_1-\theta_1)r_n + 1]\, e^{-\theta_1 r_n}\prod_{i=1}^{n-1}\frac{\theta_1^2[(\gamma+\gamma\theta_1-\theta_1)r_i+1]e^{-\theta_1 r_i}(1+\theta_1)}{(\gamma\theta_1+\gamma-\theta_1)(\theta_1 r_i+1)+\theta_1)e^{-\theta_1 r_i}} \tag{4}$$

and

$$L(\theta_2, \gamma|\underline{s}) = f(s_m)\prod_{j=1}^{m-1}\frac{g(s_j)}{1-G(s_j)} =$$
$$= \theta_2^2[\gamma(+\gamma\theta_2-\theta_2)s_m + 1]\, e^{-\theta_2 s_m}\prod_{j=1}^{m-1}\frac{\theta_2^2[(\gamma+\gamma\theta_2-\theta_2)s_j+1]e^{-\theta_1 s_j}(1+\theta_2)}{(\gamma\theta_2+\gamma-\theta_2)(\theta_2 s_j+1)+\theta_2)e^{-\theta_2 s_j}} \tag{5}$$

Therefore, the joint likelihood function of the observed $\underline{r}$ and $\underline{s}$ is given by:

$$L(\theta_1, \theta_2, \gamma|\underline{r}, \underline{s}) = \theta_1^2[\gamma(+\gamma\theta_1-\theta_1)r_n + 1]\, e^{-\theta_1 r_n}\theta_2^2[\gamma(+\gamma\theta_2-\theta_2)s_m + 1]\, e^{-\theta_2 s_m}\prod_{i=1}^{n-1}\frac{\theta_1^2[(\gamma+\gamma\theta_1-\theta_1)r_i+1]e^{-\theta_1 r_i}(1+\theta_1)}{(\gamma\theta_1+\gamma-\theta_1)(\theta_1 r_i+1)+\theta_1)e^{-\theta_1 r_i}}\prod_{j=1}^{m-1}\frac{\theta_2^2[(\gamma+\gamma\theta_2-\theta_2)s_j+1]e^{-\theta_1 s_j}(1+\theta_2)}{(\gamma\theta_2+\gamma-\theta_2)(\theta_2 s_j+1)+\theta_2)e^{-\theta_2 s_j}}. \tag{6}$$

We can get the estimated values of both θ_1 and θ_2, would be written as $\hat{\theta}_{ML1}$ and $\hat{\theta}_{ML2}$.

By imposing Eq. 6 with zero, and defined mathematically as:

$$\hat{\theta}_{ML1} = \frac{\partial\, l_n L}{\partial\theta_1} = \frac{(2+2(n-1))}{\theta_1} + \frac{(n-1)}{(1+\theta_1)} r_n ln[(\gamma+\gamma\theta_1+\theta_1)r_n+1] - \frac{\theta_1 r_n^2(\gamma-1)}{(\gamma+\gamma\theta_1-\theta_1)r_n+1} - \frac{(\gamma-1)}{(\gamma+\gamma\theta_1-\theta_1)}\frac{2(n-1)(\gamma+\gamma\theta_1-\theta_1)^{n-2}(\gamma-1)^{n-2}\prod_{i=1}^{n-1}(\theta_1(r_i+1))}{(\gamma+\gamma\theta_1-\theta_1)^{n-1}\prod_{i=1}^{n-1}(\theta_1(r_i+1))+\theta_1^{n-1}}$$

$$\frac{-2(\gamma+\gamma\theta_1-\theta_1)^{n-1} n\,\theta_1^{n-1} \prod_{i=1}^{n-1}(r_i+1)+\theta_1^{n-2}(n-1)}{(\gamma+\gamma\,\theta_1-\theta_1)^{n-1}\prod_{i=1}^{n-1}(\theta_1(r_i+1))+\theta_1^{n-1}} = 0.$$

$$\hat{\theta}_{ML2} = \frac{\partial\, lnL}{\partial\theta_2} = \frac{(2+2(m-1))}{\theta_2} + \frac{(m-1)}{(1+\theta_2)} s_m \ln[(\gamma+\gamma\theta_2+\theta_2)s_m+1] \frac{\theta_2 s_m^2(\delta-1)}{(\gamma+\gamma\theta_2-\theta_2)s_m+1} + \frac{(\gamma-1)}{(\gamma+\gamma\theta_1-\theta_1)}$$

$$\frac{-2(m-1)(\gamma+\gamma\theta_2-\theta_2)^{m-2}(\gamma-1)\prod_{j=1}^{m-1}\left(\theta_2(s_j+1)\right)+(\gamma+\gamma\,\theta_2-\theta_2)^{m-1}\theta_2^{m-2}}{(\gamma+\gamma\,\theta_2-\theta_2)^{m-1}\prod_{j=1}^{m-1}\left(\theta_2(s_j+1)\right)+\theta_2^{m-1}} = 0. \quad (7)$$

The estimated value of R, written as $\hat{R}_{ML}$, mathematically as follows:

$$\hat{R}_{ML} = \frac{\hat{\theta}_{ML2}^2}{\gamma^2(1+\hat{\theta}_{ML1})(1+\hat{\theta}_{ML2})}[2\theta_1(1+\hat{\theta}_{ML2})+2\,\hat{\theta}_{ML2}^2+\gamma\hat{\theta}_{ML1}^2+\gamma\hat{\theta}_{ML2}+(\gamma-1)\hat{\theta}_{ML1}^3+(\gamma-1)\;\hat{\theta}_{ML1}\hat{\theta}_{ML2}+(\gamma-1)\,\hat{\theta}_{ML1}^2\;\hat{\theta}_{ML2}+(\gamma-1)\,\hat{\theta}_{ML2}^2]. \quad (8)$$

In order to get the different estimated values of the reliability function using the MLE method. We will apply the Monte-Carlo simulation (MC) method to find data generated based on the USS model

4. Bayesian method for estimation

To find the estimated value of R by using the Bayes method, two unknown parameters θ_1, θ_2 should be estimated at first after that using Eq. 3, would find the estimated value of R.

Assuming the prior's distribution function for θ_1 and θ_2, are gamma functions with parameters (α_1, β_1) and (α_2, β_2), respectively.

The joint prior of the parameters density function is mathematically written as:

$$\pi(\theta_1,\theta_2) = \frac{1}{\Gamma(\alpha_1)\Gamma(\alpha_2)}\theta_1^{\alpha_1-1}\theta_2^{\alpha_2-1}e^{-\beta_1\theta_1-\beta_2\theta_2}, \quad \alpha_1,\beta_1,\theta_1,\alpha_2,\beta_2,\theta_2 > 0. \quad (9)$$

Based on the observed samples, the joint density function of θ_1 and θ_2 is:

$$\pi(\theta_1,\theta_2,\underline{s},\underline{r}) = \frac{\theta_1^{\alpha_1-1}\theta_2^{\alpha_2-1}e^{-\beta_1\theta_1-\beta_2\theta_2}}{\Gamma(\alpha_1)\Gamma(\alpha_2)}[\gamma(+\gamma\,\theta_1-\theta_1)r_n+1]\,e^{-\theta_1 r_n}[\gamma(+\gamma\,\theta_2-\theta_2)s_m+1]\,e^{-\theta_2 s_m}\prod_{i=1}^{n-1}\frac{\theta_1^2[(\gamma+\gamma\theta_1-\theta_1)r_i+1]e^{-\theta_1 r_i}(1+\theta_1)}{(\gamma\theta_1+\gamma-\theta_1)(\theta_1 r_i+1)+\theta_1)e^{-\theta_1 r_i}}\prod_{j=1}^{m-1}\frac{\theta_2^2[(\gamma+\gamma\theta_2-\theta_2)s_j+1]e^{-\theta_1 s_j}(1+\theta_2)}{(\gamma\theta_2+\gamma-\theta_2)(\theta_2 s_j+1)+\theta_2)e^{-\theta_2 s_j}}. \quad (10)$$

The joint posterior function of θ_1 and θ_2, is defined mathematically as:

$$\pi^*(\theta_1\theta_2|\underline{s},\underline{r}) = \frac{\pi(\theta_1,\theta_2,\underline{s},\underline{r})}{\int_0^\infty\int_0^\infty \pi(\theta_1,\theta_2,\underline{s},\underline{r})d\theta_1 d\theta_2} \quad (11)$$

Assuming that the Bayesian estimator value of R, based on the SE loss function is written as $\hat{R}_{SE}$, can be defined mathematically as:

$$\hat{R}_{SE} = E(R/\underline{s},\underline{r}) = \int_0^\infty\int_0^\infty R\,\pi^*(\theta_1\theta_2/(\underline{s},\underline{r}|\quad)d\theta_1 d\theta_2. \quad (12)$$

Moreover, for the LINEX loss function, the estimated Bayesian estimator of R, written as $\hat{R}_{LE}$ based on the LINEX loss function which can be defined mathematically as:

$$\hat{R}_{LE} = \frac{-1}{\vartheta}\ln E\left(e^{-\vartheta R}\right) = \frac{-1}{\vartheta} ln \int_0^\infty\int_0^\infty e^{-\vartheta R}\,\pi^*(\theta_1\theta_2|\underline{s},\underline{r})d\theta_1 d\theta_2. \quad (13)$$

Since the posterior function is very complex to find its value and to find a close form of Bayesian estimation of R which can be found based on values of $\hat{R}_{SE}$ and $\hat{R}_{LE}$ we will be based on the research of Lindley (1980), which has studied in detail the approximate values of both $\hat{R}_{SE}$ and $\hat{R}_{LE}$, the mathematical formulation of them is as follows:

$$\hat{R}_{SE} = \hat{R}_{ML} + \frac{1}{2}[R_{11}\sigma_{11} + R_{22}\sigma_{22} + L_{111}\sigma_{11}{}^2 R_1 + L_{222}\sigma_{22}{}^2 R_2]. \quad (14)$$

Also, for the LINEX loss function the mathematical formula:

$$\hat{R}_{LE} = \frac{1}{\vartheta}\ln\left[e^{-\vartheta\hat{R}_{ML}} + \frac{1}{2}[R_{11}\sigma_{11} + R_{22}\sigma_{22} + L_{111}\sigma_{11}{}^2 R_1 + L_{222}\sigma_{22}{}^2 R_2]\right]. \quad (15)$$

Using the Mont-Carlo technique for simulation (MC) to generate simulated data and finding the estimated values of Reliability function by Bayes method for simulation and calculated MSEs values for $\hat{R}_{SE}$ and $\hat{R}_{LE}$ which represented SE and LINEX loss function.

5. Mont-Carlo (MC) method for simulation

To study the behavior of $\hat{R}_{ML}$, $\hat{R}_{SE}$ and $\hat{R}_{LE}$, the estimated values of reliability by using the MLE and Bayes procedure based on the estimated values of θ_1 and θ_2. Presented below are the Monte Carlo simulation model steps:

1. Generate a sample of the upper values using some steps and some restricted-on parameters.

2. The sets of parameter values are considered $(\theta_1, \theta_2, \gamma)$=(1,1,3), (1.5,1,3), (3,0.5,3) and (0.3,0.9,3).
3. The true values of reliability R in USS model with given values of θ_1 and θ_2, in mathematical Eq. 3, are: (0.722), (0.906), (0.126) and (0.306).
4. The sample of upper record values of stress and strength random variables (n, m) are chosen to be: (10,10), (10,15), (15,10), (15,15), (15,20), (20,15), (20,20), (20,30), (30,20), (30,30).
5. The MLE of R is obtained by using estimated values of $\hat{\theta}_{ML1}$ and $\hat{\theta}_{ML2}$, in Eq. 8 to find $\hat{R}_{ML}$ value.
6. The Bayes estimator of R under SE and LINEX loss function, respectively. Based on the estimated values $\theta_1$and θ_2, where the prior of Bayes method was Gamma distribution, with the following values:

Prior I: (α_1, β_1), (α_2, β_2) = (5,2), (2,5)
Prior II: (α_1, β_1), (α_2, β_2)= (9,3), (3,1)
Prior III: (α_1, β_1), (α_2, β_2)= (5.5,2.5), (4,1.5) and $\vartheta = (-2,2)$.

7. To study the behavior of the estimated value of reliability by using different choices of sample sizes of upper records as (n, m)=(15,15), (15,20), (20,15), (20,20), (20,30), (30,20), (30,30).
8. All the results were repeated 5000 times.

6. Numerical results

Numerical results are reported in Tables 1-4, the following results can be observed in the estimated values of reliability by the Bayes method ($\hat{R}_{ML}$, $\hat{R}_{SE}$ and $\hat{R}_{LE}$) as follow.

Table 1: MSEs and bias results of $\hat{R}_{ML}$, $\hat{R}_{SE}$ and $\hat{R}_{LE}$ when $(\theta_1, \theta_2, \gamma) = (1,1,3)$

Exact R	n, m	MLE Method		Bayes Procedure		
		AB	MSE	Loss function	AB	MSE
0.722	(10,10)	0.08359	0.00802	SE	0.000204	0.0072
				LINEX($\vartheta = 2$)	0.000306	0.003
				LINEX($\vartheta = -2$)	0.000200	0.0076
	(10,15)	0.08360	0.00701	SE	0.000101	0.0065
				LINEX($\vartheta = 2$)	0.000610	0.0066
				LINEX($\vartheta = -2$)	0.000106	0.0070
	(15,10)	0.08365	0.00602	SE	0.000353	0.0042
				LINEX($\vartheta = 2$)	0.000235	0.0039
				LINEX($\vartheta = -2$)	0.000155	0.0123
	(15,15)	0.08378	0.00401	SE	0.000843	0.0031
				LINEX($\vartheta = 2$)	0.000245	0.0019
				LINEX($\vartheta = -2$)	0.000835	0.004
	(15,20)	0.08380	0.00288	SE	0.000252	0.0022
				LINEX($\vartheta = 2$)	0.000503	0.0021
				LINEX($\vartheta = -2$)	0.000425	0.0029
	(20,15)	0.08386	0.00278	SE	0.000255	0.0019
				LINEX($\vartheta = 2$)	0.000252	0.0022
				LINEX($\vartheta = -2$)	0.000247	0.0027
	(20,20)	0.08386	0.000267	SE	0.000655	0.0018
				LINEX($\vartheta = 2$)	0.000355	0.0009
				LINEX($\vartheta = -2$)	0.000445	0.0029
	(20,30)	0.08392	0.00258	SE	0.000261	0.0012
				LINEX($\vartheta = 2$)	0.000256	0.0017
				LINEX($\vartheta = -2$)	0.000435	0.0901
	(30,20)	0.08399	0.00236	SE	0.000263	0.0010
				LINEX($\vartheta = 2$)	0.000659	0.0014
				LINEX($\vartheta = -2$)	0.000350	0.0024
	(30,30)	0.08424	0.00213	SE	0.000604	0.0009
				LINEX($\vartheta = 2$)	0.000563	0.0004
				LINEX($\vartheta = -2$)	0.000255	0.0019

The results for the behavior of reliability functions which are estimated in the previous Tables 1-4, can be discussed below:

- For (n>m) and (n<m) the MSEs of $\hat{R}_{ML}$ tends to zero as n and m are increasing for different values of θ_1 and θ_2.
- For $\theta_1 > \theta_2$MSEs of $\hat{R}_{ML}$ have values smaller than the values for $\theta_1 < \theta_2$ for different values of n and m.
- In the case of MSEs for $\theta_1 > \theta_2$are tends to zero faster than the values for $\theta_1 < \theta_2$ for different values of n and m.
- For n=m the MSEs of $\hat{R}_{ML}$ at $\theta_1 > \theta_2$are smaller than the values at $\theta_1 < \theta_2$.
- The estimated value of reliability (R) under loss function SE and under loss function LINEX when $\vartheta = 2$ are smaller than The estimated value of R under loss function LINEX when $\vartheta = -2$ at $\theta_1 = \theta_2$ for different values of n and m.
- The estimated value of R under loss function SE and under loss function LINEX when $\vartheta = 2$ are smaller than The estimated value of R under loss function LINEX when $\vartheta = -2$ at $\theta_1 > \theta_2$ and $\theta_1 < \theta_2$ for different values of n and m. Take into account that the case of estimated values of R under loss function SE and under loss function LINEX in the case of $\theta_1 > \theta_2$ are Less than my peers in case of $\theta_1 < \theta_2$.

- The estimated value of R under loss function SE and under loss function LINEX when $\vartheta = 2$ are smaller than The estimated value of R under loss function LINEX when $\vartheta = -2$ at $\theta_1 > \theta_2$ for different values of n and m.
- For (n>m) and (n<m) the MSEs of $\hat{R}_{SE}$ and $\hat{R}_{LE}$ tends to zero as n and m are increasing for different values of θ_1 and θ_2.
- For $\theta_1 > \theta_2$ MSEs of $\hat{R}_{SE}$ and $\hat{R}_{LE}$ have values smaller than the values for $\theta_1 < \theta_2$ for different values of n and m.
- In the case of MSEs for $\theta_1 > \theta_2$ are tends to zero faster than the values for $\theta_1 < \theta_2$ for different values of n and m.
- For n=m the MSEs of $\hat{R}_{SE}$ and $\hat{R}_{LE}$ at $\theta_1 > \theta_2$ are smaller than the values at $\theta_1 < \theta_2$.

Table 2: MSEs and bias results of $\hat{R}_{ML}$, $\hat{R}_{SE}$ and $\hat{R}_{LE}$ when $(\theta_1, \theta_2, \gamma) = (1.5,1,3)$

Exact R	n, m	MLE Method		Bayes Procedure		
		AB	MSE	Loss function	AB	MSE
0.906	(10,10)	0.09420	0.00697	SE	0.000351	0.005
				LINEX($\vartheta = 2$)	0.000153	0.004
				LINEX($\vartheta = -2$)	0.000293	0.0097
	(10,15)	0.08391	0.00689	SE	0.000356	0.006
				LINEX($\vartheta = 2$)	0.000311	0.007
				LINEX($\vartheta = -2$)	0.000290	0.0099
	(15,10)	0.08393	0.00686	SE	0.000339	0.004
				LINEX($\vartheta = 2$)	0.000328	0.005
				LINEX($\vartheta = -2$)	0.000324	0.0001
	(15,15)	0.08396	0.00680	SE	0.000461	0.003
				LINEX($\vartheta = 2$)	0.000263	0.0009
				LINEX($\vartheta = -2$)	0.000919	0.0088
	(15,20)	0.08405	0.00666	SE	0.000764	0.001
				LINEX($\vartheta = 2$)	0.000564	0.003
				LINEX($\vartheta = -2$)	0.000229	0.0082
	(20,15)	0.08408	0.00661	SE	0.000365	0.0008
				LINEX($\vartheta = 2$)	0.000165	0.0005
				LINEX($\vartheta = -2$)	0.000234	0.0079
	(20,20)	0.08412	0.00663	SE	0.000165	0.0019
				LINEX($\vartheta = 2$)	0.000265	0.0009
				LINEX($\vartheta = -2$)	0.000656	0.0066
	(20,30)	0.08417	0.00646	SE	0.000365	0.0005
				LINEX($\vartheta = 2$)	0.000066	0.0001
				LINEX($\vartheta = -2$)	0.000466	0.0060
	(30,20)	0.08425	0.00632	SE	0.000166	0.0002
				LINEX($\vartheta = 2$)	0.000265	0.0003
				LINEX($\vartheta = -2$)	0.000272	0.0057
	(30,30)	0.08426	0.00530	SE	0.000366	0.0009
				LINEX($\vartheta = 2$)	0.000322	0.0004
				LINEX($\vartheta = -2$)	0.000285	0.0052

Table 3: MSEs and bias results of $\hat{R}_{ML}$, $\hat{R}_{SE}$ and $\hat{R}_{LE}$ when $(\theta_1, \theta_2, \gamma) = (3,0.5,3)$

Exact R	n, m	MLE Method		Bayes Procedure		
		AB	MSE	Loss function	AB	MSE
0.126	(10,10)	0.03045	0.009333	SE	0.00076015	0.0092
				LINEX($\vartheta = 2$)	0.000344855	0.0058
				LINEX($\vartheta = -2$)	0.000052628	0.0090
	(10,15)	0.03097	0.009301	SE	0.000198773	0.0089
				LINEX($\vartheta = 2$)	0.000636834	0.0056
				LINEX($\vartheta = -2$)	0.000198773	0.0089
	(15,10)	0.03100	0.009299	SE	0.000673759	0.0072
				LINEX($\vartheta = 2$)	0.000656806	0.0049
				LINEX($\vartheta = -2$)	0.000074336	0.0083
	(15,15)	0.03111	0.009292	SE	0.000818793	0.0071
				LINEX($\vartheta = 2$)	0.000108679	0.0039
				LINEX($\vartheta = -2$)	0.000963766	0.0069
	(15,20)	0.03116	0.009289	SE	0.000121366	0.0062
				LINEX($\vartheta = 2$)	0.000265794	0.0031
				LINEX($\vartheta = -2$)	0.000656806	0.0079
	(20,15)	0.03129	0.009281	SE	0.000554469	0.0059
				LINEX($\vartheta = 2$)	0.000562944	0.0022
				LINEX($\vartheta = -2$)	0.000398321	0.0067
	(20,20)	0.03151	0.009267	SE	0.000137904	0.0048
				LINEX($\vartheta = 2$)	0.000994253	0.0019
				LINEX($\vartheta = -2$)	0.00061663	0.0098
	(20,30)	0.03164	0.009259	SE	0.000998567	0.0042
				LINEX($\vartheta = 2$)	0.000281495	0.0017
				LINEX($\vartheta = -2$)	0.000709953	0.0601
	(30,20)	0.03186	0.009245	SE	0.000284984	0.0040
				LINEX($\vartheta = 2$)	0.000998567	0.0012
				LINEX($\vartheta = -2$)	0.000832329	0.0064
	(30,30)	0.03195	0.009239	SE	0.000571166	0.0038
				LINEX($\vartheta = 2$)	0.000284984	0.0010
				LINEX($\vartheta = -2$)	0.000698716	0.0058

Table 4: MSEs and bias results of $\hat{R}_{ML}$, $\hat{R}_{SE}$ and $\hat{R}_{LE}$ when $(\theta_1, \theta_2, \gamma) = (0.3, 0.9, 3)$

Exact R	n, m	MLE Method		Bayes Procedure		
		AB	MSE	Loss function	AB	MSE
0.306	(10,10)	0.04883	0.0676	SE	0.003075	0.0522
				LINEX($\vartheta = 2$)	0.006583	0.0355
				LINEX($\vartheta = -2$)	0.004650	0.0701
	(10,15)	0.04905	0.0654	SE	0.005725	0.0591
				LINEX($\vartheta = 2$)	0.008295	0.0342
				LINEX($\vartheta = -2$)	0.004866	0.0599
	(15,10)	0.05126	0.0432	SE	0.002804	0.0529
				LINEX($\vartheta = 2$)	0.006340	0.0291
				LINEX($\vartheta = -2$)	0.001275	0.0539
	(15,15)	0.04514	0.0602	SE	0.003942	0.0518
				LINEX($\vartheta = 2$)	0.005901	0.0299
				LINEX($\vartheta = -2$)	0.004871	0.0709
	(15,20)	0.05149	0.0509	SE	0.003153	0.0528
				LINEX($\vartheta = 2$)	0.004065	0.0219
				LINEX($\vartheta = -2$)	0.006340	0.0791
	(20,15)	0.05176	0.0481	SE	0.006588	0.0594
				LINEX($\vartheta = 2$)	0.006085	0.0249
				LINEX($\vartheta = -2$)	0.008876	0.0670
	(20,20)	0.03151	0.0406	SE	0.007247	0.0518
				LINEX($\vartheta = 2$)	0.006063	0.0205
				LINEX($\vartheta = -2$)	0.003897	0.0709
	(20,30)	0.05197	0.0459	SE	0.003712	0.0421
				LINEX($\vartheta = 2$)	0.008822	0.0171
				LINEX($\vartheta = -2$)	0.005883	0.0601
	(30,20)	0.05211	0.0445	SE	0.005655	0.0401
				LINEX($\vartheta = 2$)	0.003712	0.0121
				LINEX($\vartheta = -2$)	0.001732	0.0642
	(30,30)	0.05496	0.0492	SE	0.007494	0.0382
				LINEX($\vartheta = 2$)	0.004976	0.0108
				LINEX($\vartheta = -2$)	0.007393	0.0586

Figs. 1-4 show comparing between MSES for different estimators of R in system USS (data from Tables 1-4).

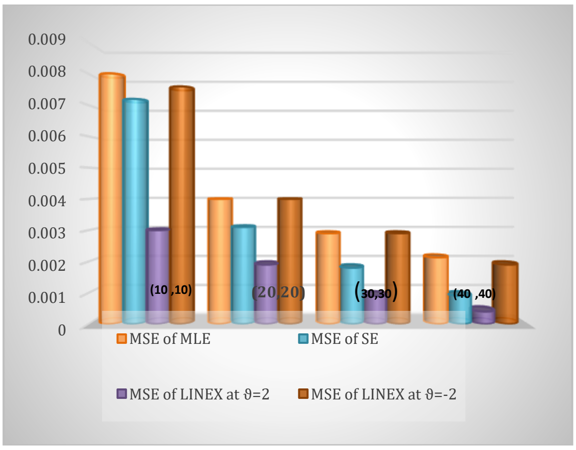

Fig. 1: Comparing MSES for different estimators of R in system USS (data from Table 1 at R=0.722)

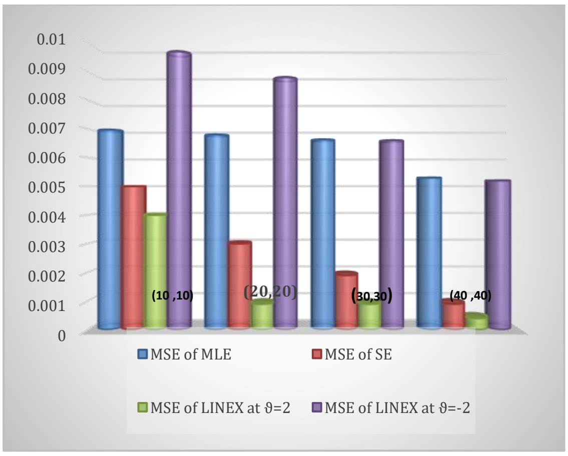

Fig. 2: Comparing MSES for different estimators of R in system USS (data from Table 2 at R=0.906)

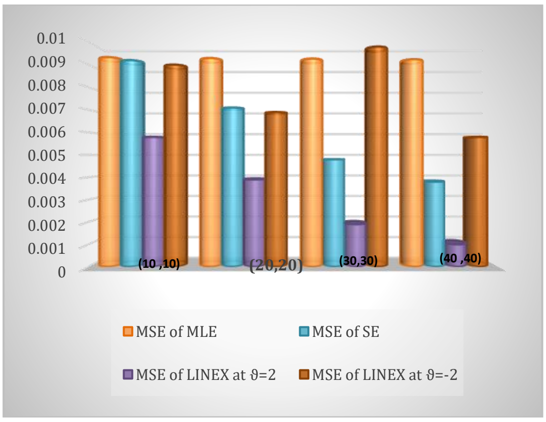

Fig. 3: Comparing MSES for different estimators of R in system USS (data from Table 3 at R=0.126)

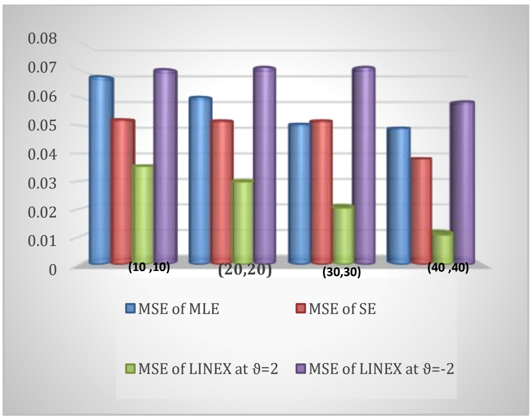

Fig. 4: Comparing MSES for different estimators of R in system USS (data from Table 4 at R=0.306)

Figs. 1-4 illustrate that the estimated value of R under loss function SE and under loss function LINEX when $\vartheta = 2$ are smaller than the estimated value of R under loss function LINEX when $\vartheta = -2$ at $\theta_1 > \theta_2$ for different values of n and m.

7. Real data application (COVID-19)

Laribi et al. (2021) studied the data from the World Health Organization in 2021 for COVID-19 belonging to the GGL distribution.

In this paper, World Health Organization data in 2021, has been studied through the stress-strength model using the upper values recorded for casualty numbers. Then we applied the study model so that the dead numbers represent stress and recovery numbers represent strength. We assessed reliability using methods demonstrated in previous sections for countries that were selected based on the assumption of higher values recorded in order to choose the best method for estimating the reliability function so that we could obtain clear readings for countries with the highest rates of overcoming COVID-19. The study includes many countries as may be seen in Table 5.

Table 5: Different countries with total numbers of deaths and total recover

Country	Number of deaths	New deaths	Total deaths	Total recovers
Philippines	9,257	4	9,261	448,258
Palestine	1,470	24	1,494	121,563
Pakistan	10,311	53	10,364	440,660
Oman	1,501	2	1,503	122,266
North Macedonia	2,530	8	2,538	62,929
Nepal	1,878	8	1,886	254,494
Namibia	215	2	217	21,055
Myanmar	2,728	17	2,745	109,548
Montenegro	689	5	694	39,347
Moldova	3,037	17	3,054	133,247
Mexico	126,851	344	127,195	1,090,905
Malaysia	494	11	505	97,218
Luxembourg	506	3	509	40,978
Lithuania	1,643	29	1,672	77,362
Libya	1,510	23	1,533	74,381
Lebanon	1,499	10	1,509	132,768
Afghanistan	2,230	9	2,239	42,405
Georgia	2,603	31	2,634	220,442
Estonia	244	3	247	19,323
El Salvador	1,358	7	1,365	41,787
DRC	596	1	597	14,716
Dominican Republic	2,418	2	2,420	132,935
Denmark	1,374	29	1,403	136,598
Czechia	11,960	74	12,034	612,214
Cuba	147	1	148	10,676
Croatia	4,072	56	4,128	202,442
Chile	16,767	43	16,810	584,457
Canada	15,740	25	15,765	497,258
Bolivia	9,186	11	9,197	133,013
Belgium	19,644	63	19,707	44,840
Belarus	1,451	9	1,460	182,630
Bangladesh	7,626	27	7,653	460,598
Armenia	2,850	14	2,864	144,091
Albania	1,193	3	1,196	34,648
Zambia	394	2	396	19,083
USA	358,830	145	358,975	12,364,189
Ukraine	18,854	123	18,977	728,865
UAE	679	5	684	189,709
Turkey	21,488	193	21,681	2,136,534
Switzerland	7,745	16	7,761	317,600
Slovenia	2,803	29	2,832	103,107
Slovakia	2,317	67	2,384	129,994
Serbia	3,325	37	3,362	31,536
Senegal	421	5	426	17,515
Saudi Arabia	6,246	7	6,253	354,443
S. Korea	962	20	982	44,507
Russia	58,506	504	59,010	2,618,882
Romania	15,979	60	16,039	574,897
Portugal	7,118	73	7,191	342,535
Poland	29,119	61	29,180	1,063,093
Austria	6,324	49	6,373	338,831
Latvia	680	12	692	30,501
Kyrgyzstan	1,359	1	1,360	76,563
Jordan	3,903	26	3,929	276,485
Japan	3,548	34	3,582	198,486
Jamaica	304	1	305	10,833
Italy	75,332	347	75,679	1,503,900
Israel	3,404	12	3,416	383,554
Iraq	12,834	5	12,839	543,720
Iran	55,540	102	55,642	1,013,018
Indonesia	22,734	179	22,913	631,937
India	149,656	185	149,841	9,943,332
Hungary	9,884	103	9,987	168,381
Honduras	3,173	13	3,186	57,348
Guatemala	4,833	6	4,839	127,450
Greece	4,957	36	4,993	9,989
Gibraltar	8	1	9	1,447
Germany	34,925	66	34,991	1,381,900

Here, assuming different choices for (n, m) and upper record values (r, s) of data in Table 5, and applying for USS model for finding estimating values of reliability, by using the ML method and Bayes method with SE and LINEX loss function, which represented with $\hat{R}_{ML}$, $\hat{R}_{SE}$ and $\hat{R}_{LE}$, respectively. The sample of stress and strength random variables (n, m) were chosen to be: (10,10), (15,15), and (20,20). The sample of upper values (r, s) were chosen to be (5,8). To analyze the data from the Bayesian procedure, the values of priors were selected as follows: Prior I: (α_1, β_1), (α_2, β_2) = (5,2), (2,5), Prior II: (α_1, β_1), (α_2, β_2) = (9,3), (3,1), Proir III: (α_1, β_1), (α_2, β_2) = (5.5,2.5), (4,1.5).

Table 6 shows The analysis of estimated reliability: $\hat{R}_{ML}$, $\hat{R}_{SE}$ and $\hat{R}_{LE}$ based on real data, when $\theta_1, \theta_2, \gamma = 1,1,3$.

Table 6: The analysis of estimated reliability: $\hat{R}_{ML}$, $\hat{R}_{SE}$ and $\hat{R}_{LE}$ based on real data ($\theta_1, \theta_2, \gamma = 1,1,3$)

n, m	r, s	$\hat{R}_{ML}$	priors	$\hat{R}_{SE}$	$\hat{R}_{LE}$ with $\vartheta = 2$	$\hat{R}_{LE}$ with $\vartheta = -2$
			Prior I	0.9888	0.9932	0.9901
10,10	5,8	0.9876	Prior II	0.9902	0.9955	0.9908
			Prior III	0.9895	0.9928	0.9899
			Prior I	0.9899	0.9945	0.9907
15,15	5,8	0,9866	Prior II	0.9934	0.9968	0.9956
			Prior III	0.9902	0.9934	0.9903
			Prior I	0.9902	0.9977	0.9910
20,20	5,8	0,9854	Prior II	0.9945	0.9984	0.9977
			Prior III	0.9909	0.9955	0.9923

8. Conclusion

It is observed that the Bayes estimated of R based on prior I, prior II, and prior III are close to MLE of R at some points of n, m. Based on the data in Table 6, we find that the values of the Bayes estimated of R are the best among the values of the whole Table 6, for more analyzing the values of the estimator of R based on Bayes using the prior distribution II is the best and largest in the values and the closest to the correct one. Therefore, we recommend using the Bayes method of estimation shown on a LINEX loss function with ϑ=2.

This paper is concerned with finding the estimated values of reliability based on the stress-strength model with upper record values. Both stress and strength are independent with Gamma Linedly distribution with different scale parameters. The system of USS was estimated by using the maximum likelihood estimation method and Bayes method with different loss functions as square error (SE) and LINEX (LE) loss function. MSEs of different estimators are obtained and tabulated in Tables 1-4.

The results obtained in COVID-19 can be generalized when choosing more upper record values in order to obtain the countries that can be constantly able to cope with the pandemic.

Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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