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On the memory-dependent derivative electric-thermoelastic wave characteristics in the presence of a continuous line heat source

Magdy M. Amin [1, 2], Mohamed H. Hendy [1, 2, *], Magdy A. Ezzat [3]

[1]Department of Mathematics, Faculty of Science, Northern Border University, Arar, Saudi Arabia
[2]Department of Mathematics, Faculty of Science, Al Arish University, Al Arish, Egypt
[3]Department of Mathematics, College of Science and Arts, Qassim University, Al Bukairiyah, Saudi Arabia

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ABSTRACT

In the present work, the definition of memory-dependent derivative (MDD) heat transfer in a solid body was used to investigate the problem of wave characteristics in an unbounded electric-thermoelastic solid due to a continuous line heat source in the presence of a uniform magnetic field. Both Laplace and Hankel's transform strategies are used to acquire the widespread answer in a closed-form. Analytical findings were obtained for the distribution within the medium of various fields such as temperature, displacement, and stresses. For the inversion of the Laplace transformations, a computational approach is used. The distributions of the numerical consequences of the non-dimensional considered bodily variables are represented graphically. Detailed comparative evaluation is represented thru the numerical outcomes to estimate the results of the kernels, time-delay, figure-of-merit, and magnetic number on the behavior of all variables. The effect offers a concept to research main electric-thermoelastic materials as any other type of pertinent materials.

1. Introduction

Building and structural scientists, as well as designers, developers, and makers, are all fascinated with heat transfer. Traditional applications, such as general power frameworks and heat exchangers, have been investigated extensively by Ismael (2017), Faisal (2020), and Slayi and Ashmawy (2019).

A few endeavors have been made to beat the downsides of the Fourier constitutive law of heat conduction and these endeavors offered to ascend to the theory of generalized thermoelasticity. Parallel research work is additionally being sought after amid the most recent couple of decades in the field of thermoelasticity to give significant improvements in the territory of "thermoelasticity", representing non-Fourier heat conduction in elastic materials.

Load and Shulman (1967) supplanted the Fourier law with the recipe created by Cattaneo (1958) and Vernotte (1961) in coupled thermoelasticity presented by Biot (1956) and proposed the broadened theory of thermoelasticity with one relaxation time for homogeneous elastic media. The works of Ezzat et al. (1996; 2001; 2003), Sherief et al. (2011), El-Karamany and Ezzat (2013), and El Sherief et al. (2020) are contributions to the field.

Fractional calculus extends ordinary calculus. Fractional calculus is a useful mathematical tool for a wide range of difficulties in science and engineering because it can more easily and accurately describe mechanical and physical processes with historical memory and spatial non-local correlation. The physical meaning of the fractional derivative's parameters is basic and exact, and the fractional derivative is easy to express. Fractional calculus is currently widely used in a broad range of fields, including mathematical physics, classical and quantum mechanics, control theory, nonlinear dynamics, signal and image processing, thermodynamics, and biological engineering. Povstenko (2016) investigated new thermoelasticity models that use fractional derivatives. The fractional order theory of thermoelasticity was derived by Sherief et al. (2010) and Youssef (2010). Ezzat et al. (2015) introduced a new model of thermoelasticity theory in the context of a new consideration of heat conduction with fractional order. Ezzat and El-Bary (2016) studied the effects of variable thermal conductivity and fractional order of heat transfer on a perfect conducting infinitely long hollow cylinder

* Corresponding Author.
Email Address: hendy442003@yahoo.com (M. H. Hendy)
https://doi.org/10.21833/ijaas.2022.08.001
Corresponding author's ORCID profile:
https://orcid.org/0000-0003-1919-1647

and Hendy et al. (2019) solved a two-dimensional problem for thermoviscoelastic materials with fractional order heat transfer, while Khamis et al. (2020) solved some problems in fractional thermoelasticity theory.

The memory-dependent derivatives presented by Wang and Li (2011) proved to be a useful mathematical tool that filled a gap in many practical scenarios. The integer ordered differential operator is a local operator whereas the fractional ordered differential operator is non-local. The non-local nature of the fractional derivative establishes its somewhat memory-dependent nature, which is a much more realistic fitting to real world physical problems. Thus in some recent studies fractional ordered derivatives come into account more frequently than integer ordered derivatives in various physical problems. Parallel to fractional ordered derivatives, memory-dependent derivatives serve as an important mathematical tool in describing many real world phenomena. One can refer to Yu et al. (2014), Ezzat et al. (2014; 2016a; 2016b), Ezzat and El-Bary (2015), and Ezzat (2020) for an overview of utilizations of memory-dependent derivative analytics.

The establishment of magnetoelasticity was displayed by Knopoff (1955) and created by Kaliski and Petykiewicz (1959). Among the authors who considered the generalized magnetothermoelasticity equations are Hendy et al. (2018), Khamis et al. (2021), and Noshad and Kolahchi (2015).

Direct conversion of energy and heat using thermoelectric materials has gotten a lot of interest because of its prospective use in Peltier coolers and thermoelectric power generators see Rowe (1995). The contributions of Shercliff (1979) and Ezzat and Youssef (2010) to continuum mechanics of thermoelectric materials are important.

The main objective of this work is to look at how the kernel function with different types, time-delay parameter, and magnetic number as well as a figure-of-merit effect on all obtaining functions in a one-dimensional problem. We employ the potential function approach in combination with the Laplace and Hankel transform methodology to generate solutions in the transformed domain. Analytically, Hankel inversion is performed and the exact solution of the considered problem is obtained in Laplace transform domain. The numerical computation and graphical plots of the distribution of the field variables for copper material are used to demonstrate the analytical conclusions.

2. Mathematical model

In the absence of an external electric field E, a steady magnetic field of strength H pervades the medium. The system of governing equations of the linear electro-thermoelasticity theory with memory-dependent derivative consists of the following equations:

1. The figure-of-merit ZT_o at some reference temperature T_o (Rowe, 1995),

$$ZT_o = \frac{\sigma s_o^2}{k} T_o \tag{1}$$

2. The first Thomson relation at T_o (Shercliff, 1979),

$$\pi_o = s_o T_o \tag{2}$$

3. Modified Fourier's heat conduction law (Ezzat and Youssef, 2010),

$$q_i = -kT_{,j} + \pi_o J_i \tag{3}$$

4. Modified Ohm's law is defined as (Ezzat and Youssef, 2010),

$$J_i = \sigma[E_i + \mu_o(\dot{u}_k \Lambda H_j) - s_o T_{,i} \tag{4}$$

5. Displacement equation, taking into account the Lorentz forces,

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} + \mu_o (J\Lambda H)_i \tag{5}$$

6. The constitutive equation,

$$\sigma_{ij} = \lambda e_{kk} + 2\mu e_{ij} - \gamma(T - T_o) \tag{6}$$

7. The energy equation in the presence of heat sources (Ezzat et al., 2014)

$$(1 + \omega D_\omega)\left(\rho C_E \frac{\partial T}{\partial t} + \gamma T_o \frac{\partial e}{\partial t}\right) = k\nabla^2 T - \pi_o J_{,j} + (1 + \omega D_\omega) Q \tag{7}$$

where,

$$D_\omega f(t) = \frac{1}{\omega} \int_{t-\omega}^{t} K(t - \xi)\, f'(\xi) d\xi$$

8. The kinematic relations:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{8}$$

Together with the preceding equations, they provide a full system of generalized electro-thermoelasticity with a memory-dependent derivative for an electrically conducting material in the presence of a continuous line heat source. A comma signifies material derivatives in the above equations. The summarization convention is employed.

3. Formulation of the problem

We consider a homogeneous isotropic unbounded thermoelectric elastic solid in the presence of a continuous line heat source under axisymmetric conditions. Let (r, ψ, z) be cylindrical coordinates, with the z-axis serving as the axis of symmetry. Assume also that the initial magnetic field H acts in the direction of the z-axis and has the components $(0, 0, H_o)$. As a result, electro-thermoelastic interactions in nature are symmetrical around the axis, with temperature and displacement determined by distance r from the axis and time t.

The fundamental governing equations are given as:

i. The components of the displacement vector will be taken the form,

$$u_r = u(r,t), u_\psi = 0,\ \ u_z = 0 \tag{9}$$

From Eq. 8 we can obtain the strain components,

$$e = e_{rr} + e_{\psi\psi} + e_{zz} = \frac{1}{r^2}\frac{\partial}{\partial x}(r^2\, u) \tag{10}$$

ii. The equation of motion in the presence of a constant magnetic field is,

$$\rho\frac{\partial^2 u}{\partial t^2} = \mu\nabla^2 u - \frac{\mu}{r^2}u + (\lambda+2\mu)\frac{\partial e}{\partial r} - \sigma\mu_o^2 H_o^2\frac{\partial u}{\partial t} - \gamma\frac{\partial T}{\partial r} \tag{11}$$

iii. The heat transfer equation in the presence of a continuous line source is,

$$\mathrm{k}(1+\mathrm{ZT_o})\nabla^2\mathrm{T} = (1+\omega\mathrm{D}_\omega)\left(\rho\mathrm{C_E}\frac{\partial\mathrm{T}}{\partial\mathrm{t}} + \gamma\mathrm{T_o}\frac{\partial\mathrm{e}}{\partial\mathrm{t}} - \mathrm{Q}\right) \tag{12}$$

From now on, the Kernel function form $K(t-\xi)$can be chosen freely as:

$$K(t-\xi) = 1 - \frac{2\mathrm{n}}{\omega}(t-\xi) + \frac{\mathrm{m}^2(t-\xi)^2}{\omega^2} = \begin{cases} 1 & if\ \ m = n = 0 \\ 1 - \dfrac{(t-\xi)}{\omega} & if\ \ m = 0,\quad n = \dfrac{1}{2} \\ (1 - \dfrac{t-\xi}{\omega})^2 & if\ \ m = \mathrm{n} = 1 \end{cases}$$

iv. The normal stress components are

$$\sigma_{rr} = 2\mu\frac{\partial u}{\partial r} + \lambda e - \gamma(T - T_o) \tag{13}$$

$$\sigma_{\psi\psi} = \mu\frac{u}{r} + \lambda e - \gamma(T - T_o) \tag{14}$$

where, e is the cubical dilatation $e = \frac{\partial u}{\partial r} + \frac{u}{r} = \frac{1}{r}\frac{\partial ru}{\partial r}$, and ∇^2 is the one-dimensional Laplace's operator in cylindrical coordinates, namely $\nabla^2 = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$.

We shall use the following non-dimensional variables:

$$r' = c_o\eta_o r,\ u' = c_o\eta_o u,\ t' = c_o^2\eta_o t,\ \tau_o' = c_o^2\eta_o\tau,\ \sigma_{ij}' = \frac{\sigma_{ij}}{\lambda+2\mu},\ \ \theta = \frac{\gamma}{\lambda+2\mu}(T - T_o)$$

$$\varphi' = c_o^2\eta^2\varphi,\ Q' = \frac{J}{k\rho c_o^4\eta^2}Q,\ q' = \frac{\gamma}{k\rho c_o^3\eta_o}\ q,\ \eta = \frac{\rho C_E}{k}, h' = \frac{h}{H_o},\ \ J' = \frac{J}{H_o c_o\eta}$$

Eqs. 9-14 take the following forms (dropping the primes for convenience):

$$\nabla^2\varphi - \alpha\frac{\partial^2\varphi}{\partial t^2} - \alpha M\frac{\partial\varphi}{\partial t} = \alpha\theta \tag{15}$$

$$\nabla^2\theta = \frac{(1+\omega\mathrm{D}_\omega)}{1+\mathrm{ZT_o}}\left(\frac{\partial\theta}{\partial\mathrm{t}} + \varepsilon\frac{\partial}{\partial\mathrm{t}}\nabla^2\varphi - \mathrm{Q}\right) \tag{16}$$

$$\sigma_{rr} = \frac{\partial u}{\partial r} + \beta\frac{u}{r} - \theta, \tag{17}$$

$$\sigma_{\psi\psi} = \beta\frac{\partial u}{\partial r} + \frac{u}{r} - \theta, \tag{18}$$

where, φ is the thermoelastic potential function is given by,

$$u = \frac{\partial\varphi}{\partial r} \tag{19}$$

and $\beta = \frac{2\lambda}{\lambda+2\mu}$, $\varepsilon = \frac{\gamma^2 T_o}{\rho C_E(\lambda+2\mu)}$, $c_o = \sqrt{\frac{(\lambda+2\mu)}{\rho}}$, $M = \frac{\sigma\mu_o^2 H_o^2}{\rho c_o^2\eta}$.

Assuming that the heat source operating on the current medium is of the continuous line type, we may write it as follows:

$$Q(r,t) = \frac{1}{2\pi r}Q_o\delta(r)H(t) \tag{20}$$

where, Q_o is constant, $\delta(r)$ is the Dirac delta function and $H(t)$ is the Heaviside unit step function.

We assume that all field variables vanish at $r \longrightarrow \infty$. Mathematically we can write:

$$u,\theta,\sigma_{rr}\ and\ \sigma_{\theta\theta} \longrightarrow 0\ as\ r \longrightarrow \infty \tag{21}$$

The initial conditions of the problem are given by,

$$u(r,0) = \dot{u}(r,0) = \theta(r,0) = \dot{\theta}(r,0) = \sigma_{rr}(r,0) = \sigma_{\dot{\psi}\psi}(r,0) = \sigma_{\psi\psi}(r,0) = 0 \tag{22}$$

4. The solution in the Laplace transform domain

Applying the Laplace transform defined by the formula,

$$L\{g(t) = \bar{g}(s) = \int_0^\infty e^{-st}g(t)dt$$

to both sides of Eqs. 15-21, and using the initial conditions 22, we arrive at:

$$\nabla^2\bar{\varphi} = \beta(s\bar{\theta} + \varepsilon s(s+M)\bar{\varphi} = \alpha\bar{\theta} \tag{23}$$

$$\nabla^2\bar{\theta} = \beta(s\bar{\theta} + \varepsilon s\nabla^2\bar{\varphi} - \bar{Q}) \tag{24}$$

$$\bar{\sigma}_{rr} = \left[\left(\frac{\lambda_o - 1}{r}\right)\frac{\partial}{\partial r} + \alpha s(s+M)\right]\bar{\varphi} \tag{25}$$

$$\bar{\sigma}_{\psi\psi} = \left[\left(\frac{\lambda_o - 1}{r}\right)\frac{\partial^2}{\partial r^2} + \alpha s(s+M)\right]\bar{\varphi} \tag{26}$$

where,

$$\bar{Q} = \frac{1}{2\pi rs}Q_o\delta(r) \tag{27}$$

$$G(s) = (1 - e^{-s\omega})\left(1 - \frac{2n}{\omega s} + \frac{2m^2}{\omega^2 s^2}\right) - (m^2 - 2n + \frac{2m^2}{\omega s})e^{-s\omega} \tag{28}$$

and,

$\beta = (1 - G)/\,(1 + \mathrm{ZT_o})$ and $\lambda_o = 2\lambda/\,(\lambda + 2\mu)$.

Eliminating $\bar{\theta}$ from Eqs. 23 and 24, we obtain:

$$\{\nabla^4 - [\alpha s(s+M) + s\beta(1+\alpha\varepsilon)]\nabla^2 + \alpha s^3\beta\}\bar{\varphi} = -\frac{Q_o\beta}{2\pi rs}\delta(r) \tag{29}$$

Eq. 29 can be factorized as:

$$(\nabla^2 - k_1^2)(\nabla^2 - k_2^2)\bar{\varphi} = -\frac{Q_o\beta}{2\pi rs}\delta(r) \tag{30}$$

where, k_1^2 *and* k_2^2 are the roots of the characteristic equation,

$$k^4 - [\alpha s(s + M) + s\beta(1 + \alpha\varepsilon)]k^2 + \alpha\beta s^3 = 0 \tag{31}$$

and satisfy the following two relations:

$$k_1^2 + k_2^2 = \alpha s(s + M) + s\beta(1 + \alpha\varepsilon)$$
$$k_1^2\ k_2^2 = \alpha\beta s^3 \tag{32}$$

Applying Hankel transform which can be defined as:

$$\hat{\bar{f}}(\varsigma, s) = \int_0^\infty r\, J_o(\varsigma r)\bar{f}(r, s)dr \tag{33}$$

on Eq. 30, we have

$$(\varsigma_1^2 + k_1^2)(\varsigma_2^2 - k_2^2)\hat{\bar{\varphi}} = -\frac{Q_o\beta}{2\pi rs} \tag{34}$$

where, J_o is the Bessel function of the first kind of order zero.

By applying the inverse Hankel transform of Eq. 34 we achieve,

$$\bar{\varphi}(r, s) = \frac{Q_o\beta}{2\pi s(k_1^2 - k_2^2)}\sum_{i=1}^{2}(-1)^{i-1}K_o(k_i r) \tag{35}$$

where, $K_o(k_i r)$ is the modified Bessel function of the second kind of order zero.

The displacement and temperature distributions can be obtained by substituting Eq. 35 into Eqs. 19, and 23, respectively as:

$$\bar{u}(r, s) = \frac{Q_o\beta}{2\pi s(k_1^2 - k_2^2)}\sum_{i=1}^{2}(-1)^{i}k_i K_o(k_i r) \tag{36}$$

$$\bar{\theta}(r, s) = \frac{Q_o\beta}{2\pi s\alpha(k_1^2 - k_2^2)}\sum_{i=1}^{2}(-1)^{i}(k_I - \alpha s(s + M)K_o(k_i r) \tag{37}$$

Substituting from Eqs. 36 and 37 into Eqs. 25 and 26, we obtain the following solutions for the components of normal stress:

$$\bar{\sigma}_{rr}(r, s) = \frac{Q_o\beta\left(\sum_{i=1}^{2}(-1)^{i-1}[(1-\lambda_o)k_i K_1(k_i r) + \alpha rs(s+M)K_1(k_o r)]\right)}{2\pi s(k_1^2 - k_2^2)} \tag{38}$$

$$\bar{\sigma}_{\theta\theta}(r, s) = \frac{Q_o\beta\left(\sum_{i=1}^{2}(-1)^{i-1}[r(\lambda_o - 1)k_i^2 + \alpha rs(s+M)K_o(k_o r) + (\lambda_o - 1)k_i K_1(k_i r)]\right)}{2\pi s(k_1^2 - k_2^2)} \tag{39}$$

where, $K_1(k_i r)$ is the modified Bessel function of the second kind of order one.

This completes the solution in the Laplace transform domain. In the physical domain, the solution may be derived by reversing the Laplace transforms involved in Eqs. 36-39. Due to the involvement of the intricate expressions given above on the Laplace transform parameter s, it is a difficult task to obtain the inverse Laplace transform of Eqs. 35-39 analytically all values of the time. As a result, we get numerical results, which are discussed in the next section, in order to evaluate the influence of various kernel functions, time delays, and magnetic numbers on the nature of all physical fields, such as temperature, displacement, and radial and circumferential stresses.

5. Numerical results and discussion

In order to invert the Laplace transforms in Eqs. 36-39, we adopt a numerical inversion method based on Fourier series expansion (Hoing, 1984). For this purpose, we used the Fortran 77 programming language on a personal computer with an I7 processor. The analysis is conducted for a copper material. The values of physical constants are taken as Ezzat et al. (2016a):

$\rho = 8954\, kg/m^3$, $k = 0.55$ J/m. sec. K, $\mu = 3.86(10)^{10}$ N/ m^2, $\varepsilon = 0.0168$
$C_E = 381.1$ J/kg. K, $\lambda = 7.76(10)^{10}$ N/ m^2, $T_o = 293K$, and $\mu_o H_o = 1$ Tesla

Using the solutions supplied by Eqs. 36, 37, 38, and 39, we compute the numerical values of temperature, displacement, radial stress, and circumferential stress at non-dimensional time value, namely ($t = 0.1$). In this part, we aim to show how the kernel function, time delay, and magnetic number affect the nature of all physical fields including temperature, displacement, and both radial and circumferential stresses. The outcomes are represented graphically in Figs. 1-6, for different values of r and temperature, θ.

The temperature field has been shown to fluctuate in two ways: First, when the kernel functions are the same but the time-delay parameter is different, and second when the kernel functions are the same but the time-delay parameter is different. Fig. 1 indicates the variation in temperature for different values of time-delay $\omega(0.0005, 0.005, 0.05)$ and $K(t, \xi) = 1$. We can observe that the temperature behavior is significantly influenced by the time delay and the region of influence increases with a decrease in the time-delay parameter. It is observed that the effect of time delay is more significant at a lower time of interaction. Fig. 2 shows the distribution of the temperature for different forms of the kernel function, namely as $K(t, \xi) = 1, 1 - (t - \xi)/\omega$ and $[1 - (t - \xi)/\omega]^2$ for a time-delay of $\omega = 0.15$ The temperature's influence zone is explicitly declared to be confined in all scenarios. It is noted that the temperature field has the maximum value at the initial points, i.e. when the distance $r = 0$. Furthermore, the temperature field is observed to have one local lowest value and then one local highest value before completely vanishing at a specific distance. As the distance rises, this field's value decreases. We also find that when the non-linear form of kernel function is used as $K(t, \xi) = [1 - (t - \xi)/\omega]^2$, the temperature reaches its maximum value and the lowest value for the constant kernel function $K(t, \xi) = 1$. As a result, we uncover that the kernel function has a significant impact on temperature fluctuation, with the kernel's influence being more prominent near extreme points at all times.

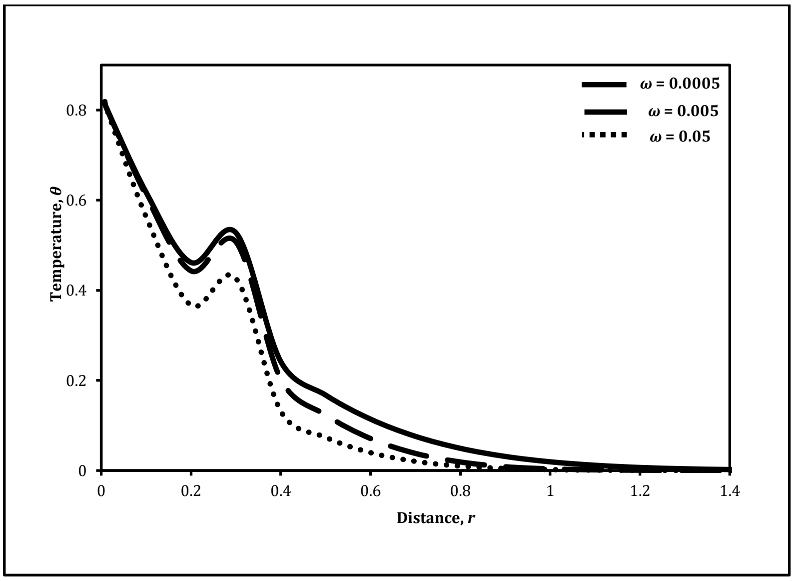

Fig. 1: The variation of temperature for different values of time-delay ω and kernel function $K(t,\xi) = 1$

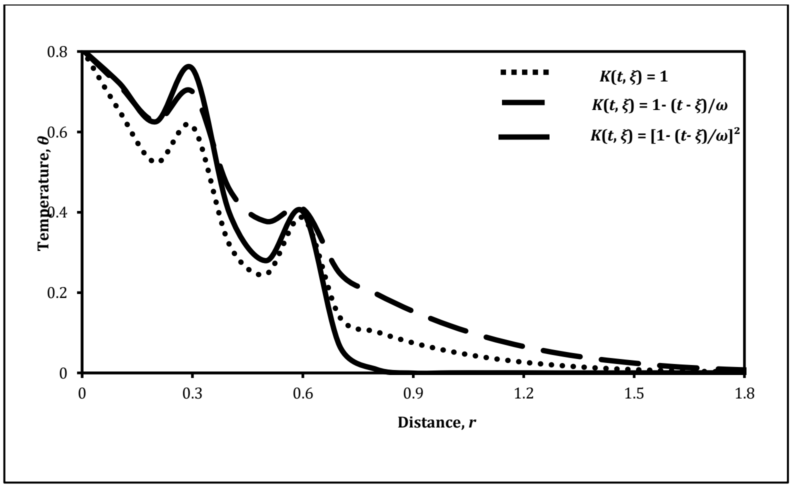

Fig. 2: The variation of temperature for different kernel functions at time-delay $\omega = 0.15$

Fig. 3 depicts temperature as a function of figure-of-merit for several thermoelectric materials. We can see that the figure-of-merit of thermoelectric material is proportional to temperature. In addition, we learned from this Fig. 3 that the value of the figure-of-merit increases as time delay increases.

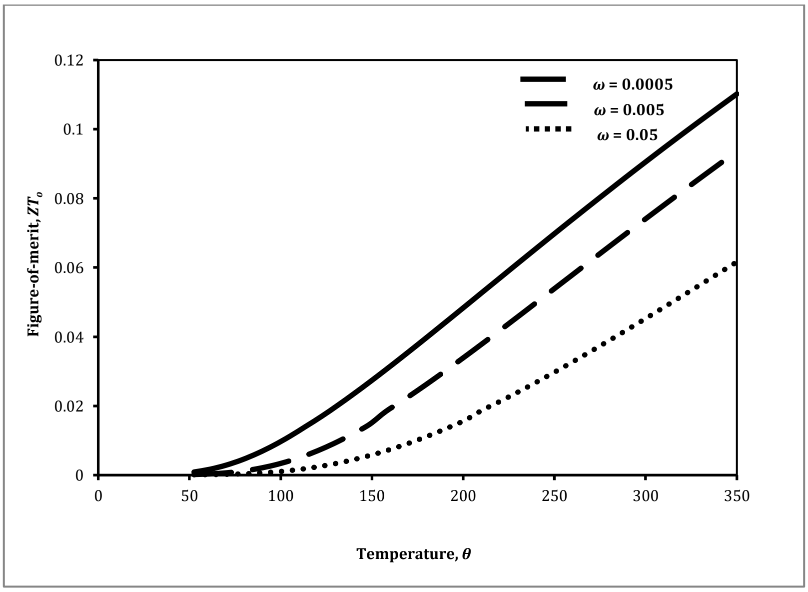

Fig. 3: The dimensionless figure-of-merit ZT_0 is plotted as a function of temperature for several values of time-delay ω and kernel function $K(t,\xi) = 1 - (t - \xi)/\omega$

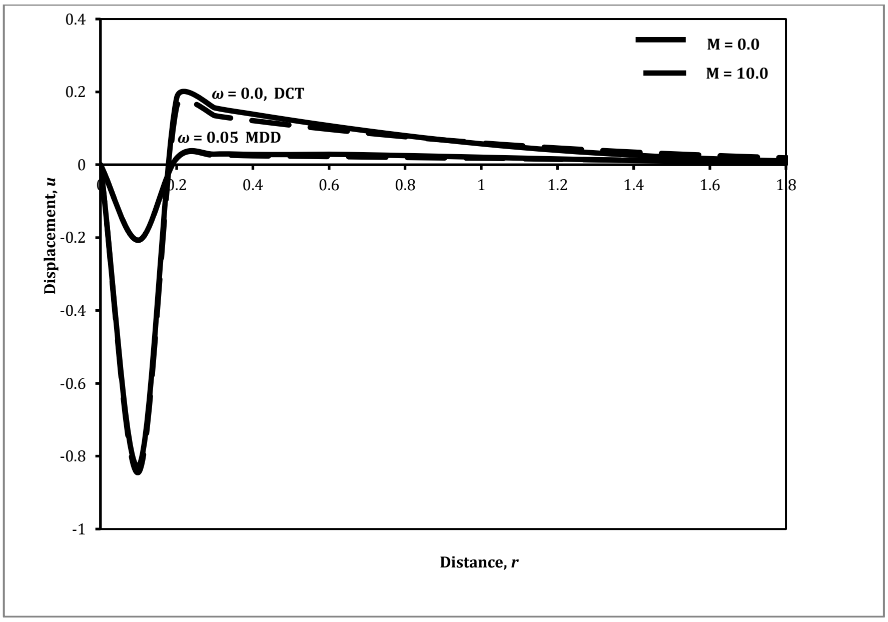

Fig. 4: The effect of magnetic number on displacement for different theories

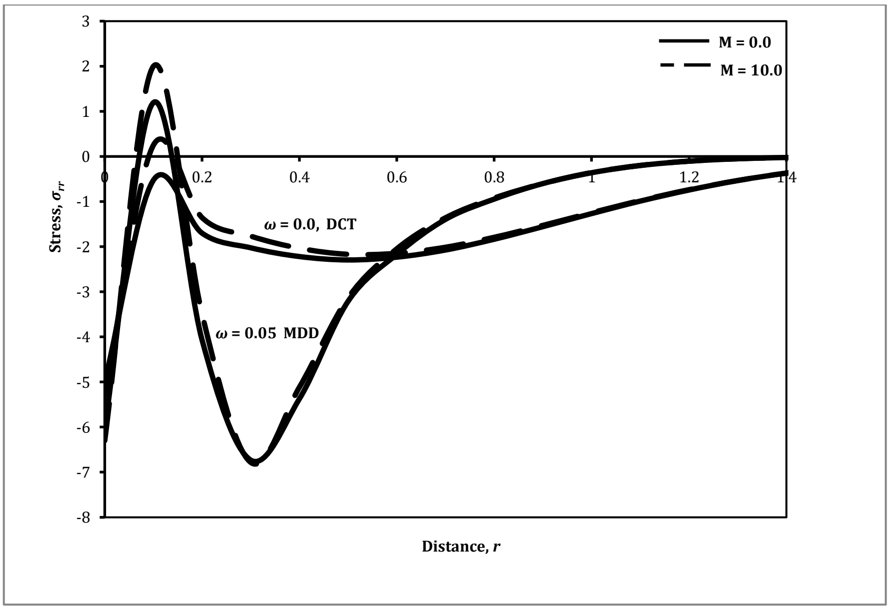

Fig. 5: The variation of stress for different values of time-delay ω and kernel function $K(t,\xi) = [1 - (t - \xi)/\omega]^2$

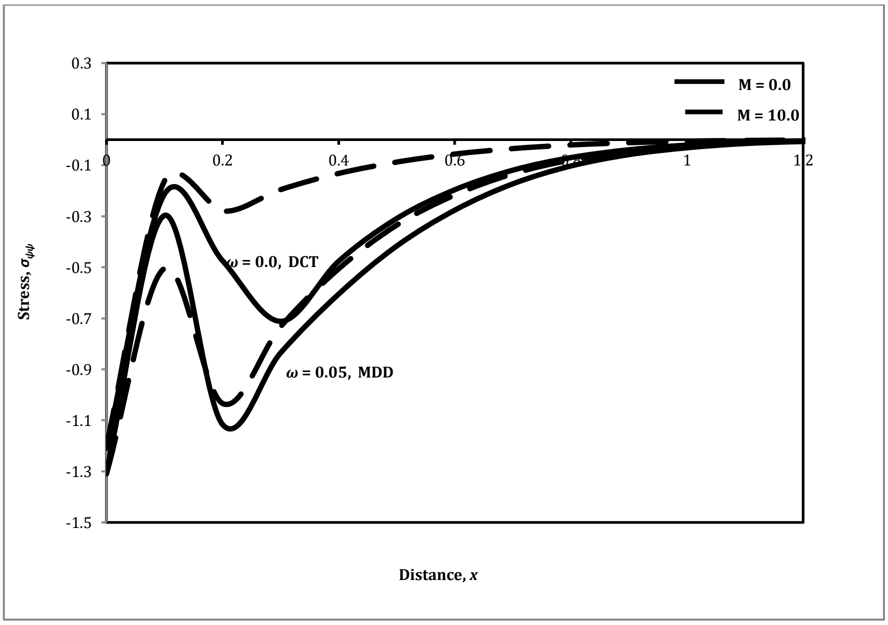

Fig. 6: The variation of temperature for different values of magnetic number M for different theories

Figs. 4, 5, and 6 depict the variation of the displacement, radial stress, and circumferential stress fields. The effects of magnetic number M are discussed in different theories. In Figs. 4, 5, and 6, dashed lines represent the solution obtained in the presence of a constant magnetic field, while solid lines are in the absence of this field. Figs. 4, 5, and 6 taught us that the magnetic field acts to reduce the fields. This is due to the fact that the magnetic field is related to a phrase that denotes a positive force that accelerates charge carriers.

6. Conclusion

The fundamental purpose of this research is to develop a new mathematical model for the Fourier law of heat conduction that incorporates memory-dependent derivatives and the thermoelectric figure-of-merit. This model allows us to increase the figure-of-merit performance of a thermoelectric material ZT.

The findings encourage more study into conducting thermoelectric materials, which are another type of thermoelectric material.

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Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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