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Magneto-thermoelasticity Green–Naghdi theory with memory-dependent derivative in the presence of a moving heat source

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ABSTRACT

In the present work, a mathematical model of the Green-Naghdi thermoelasticity theory of type III (GN-III) with memory-dependent derivative (MDD) heat transfer for a perfectly conducting isotropic media has been constructed. The state-space and Laplace transform techniques are adopted for the solution of a half-space problem in the presence of a moving heat source with constant velocity. The inversion of the Laplace transforms is carried out using a numerical approach. Numerical results for all fields are given and illustrated graphically. Comparison is made with the results predicted by coupled thermoelasticity (DCT). The influences of MDD parameters and heat source speed on all fields are examined.

1. Introduction

Load and Shulman (1967) were among the first to expand Biot's concept of connected thermoelasticity (Biot, 1956). Wave propagation speeds are guaranteed to be limited according to this theory. Contributions to the subject are represented by the works of Ezzat and Youssef (2010) and Ezzat et al. (2015). Green and Naghdi (1991) developed an enhanced thermoelasticity theory that includes the "thermal displacement gradient" as an independent constitutive element. Notable works in this field were the works of Chandrasekharaiah (1996), El-Karamany and Ezzat (2013; 2016; 2015), and Khamis et al. (2021). Povstenko (2009) investigated new thermoelasticity models that use fractional derivatives. The fractional order theory of thermoelasticity was derived by Sherief et al. (2010) and Ezzat (2011). Yu et al. (2013), Hendy et al. (2019), and Khamis et al. (2020) solved some problems in fractional order generalized thermoelasticity. Parallel to fractional ordered derivatives, memory-dependent derivatives serve as an important mathematical tool in describing many real-world phenomena. One can refer to Yu et al. (2014), and Ezzat et al. (2014; 2015; 2016) for an overview of utilizations of memory-dependent derivative analytics.

The goal of this work is to look at how MDD parameters and both electric and magnetic fields affect temperature, displacement, and stress distributions in one-dimensional problems. The answer is discovered using a state-space method. The resulting formulation is used for a variety of problems using the Laplace transform approach. Ezzat (2008) introduced a good review of the state-space approach in thermoelasticity and MHD theories.

2. Mathematical model

We shall start with a perfect conductivity thermoelastic material that fills half of the space and is pierced by an initial magnetic field H. As a result of the magnetic field's impact, induced magnetic field h and induced electric field E to arise in the conducting medium. We suppose that h and E are both small in magnitude, in accordance with the assumptions of the linear theory of thermoelasticity. Also, there arises a force F (the Lorentz Force). Due to the effect of the force, points of the medium undergo a displacement vector u, which gives rise to a temperature. The governing equations, in the presence of heat source, are given as:

(i) Linearized equations of electromagnetism for slowly moving perfect conducting media Ezzat (1997):

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$curl\ h = J + \varepsilon_o \frac{\partial E}{\partial t}$ (1)
$curl\ E = -\mu_o \frac{\partial h}{\partial t}$ (2)
$E = -\mu_o (\frac{\partial u}{\partial t} \Lambda H)$ (3)
$div\ h = 0$ (4)

(ii) Displacement equation, taking into account the Lorentz force is:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} + \mu_o (J \Lambda H)_i \quad (5)$$

(iii) Constitutive equation:

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - \gamma (T - T_o) \delta_{ij} \quad (6)$$

(iv) Strain-displacement relation:

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (7)$$

(v) Heat equation with MDD (Ezzat at el., 2014):

$$\left(n_1 k \frac{\partial}{\partial t} + n_2 k^*\right) \theta_{,ii} = \rho C_E \frac{\partial^2 \theta}{\partial t^2} + \gamma T_o \frac{\partial^2 e}{\partial t^2} - \frac{\partial Q}{\partial t} + \int_{t-\omega}^{t} K(t-\xi)(\rho C_E \frac{\partial^3 \vartheta(x,\xi)}{\partial \xi^3} + \gamma T_o \frac{\partial^3 e(x,\xi)}{\partial \xi^3} - \frac{\partial^2 Q}{\partial \xi^2})\, d\xi \quad (8)$$

where the kernel function $K(t-\omega)$ can be picked unreservedly as:

$$K(t - \xi) = 1 - \frac{2\mathrm{n}}{\omega}(t-\xi) + \frac{\mathrm{m}^2 (t-\xi)^2}{\omega^2} = \begin{cases} 1 & if\ \ m = n = 0 \\ 1 - \frac{(t-\xi)}{\omega} & if\ \ m = 0, \quad n = \frac{1}{2} \\ (1 - \frac{t-\xi}{\omega})^2 & if\ \ m = \mathrm{n} = 1 \end{cases}$$

and $\theta = |T - T_o|$ and $\frac{\theta}{T_o} \ll 1$ together with the previous equations, constitute a complete system of generalized magneto-thermoelasticity based on GN-III theory with a memory-dependent derivative for a medium with perfect electric conductivity.

Limiting cases: Eq. 16 when $\omega \longrightarrow 0$ a, so that $|D_\omega(f(x,t)| \leq \left|\frac{\partial f(x,t)}{\partial t}\right| = \left|\lim_{\omega} \frac{f(x,t+\omega) - f(x,t)}{\omega}\right|$ leads to the Fourier law for the following theories:

(1) Biot theory (Biot, 1956), $n_1 = 1, \ \ n_2 = 0$
(2) Green-Naghdi of type III theory with energy dissipation (Green and Naghdi, 1991), $n_1 = 1, \ n_2 = 1$
(3) Green-Naghdi of type II without energy dissipation (Green and Naghdi, 1993), $n_1 = 0, \ \ n_2 = 1$

3. Physical problem

We shall consider a solid occupying the region $x \geq 0$, where the x-axis is taken perpendicular to the bonding plane of half-space pointing inwards. Assume also that the initial conditions are homogeneous and the initial magnetic field has components $(0, 0, H_o)$. The induced magnetic field h will have one component h in the z-direction, while the induced electric field E will have one component E in the y-direction. For the one-dimensional problems, all the considered functions will depend only on the space variables x and t.

The displacement components,

$$u_x = u(x,t), \ \ u_y = u_z = 0 \quad (9)$$

The strain-displacement relation,

$$e = \frac{\partial u}{\partial x} \quad (10)$$

From Eq. 1, it follows that the electric current density J will have one component only J in the y-direction, given by,

$$J = -(\frac{\partial h}{\partial x} + \varepsilon_o \mu_o H_o \frac{\partial^2 u}{\partial t^2}) \quad (11)$$

The vector Eqs. 2 and 3, reduce to the following scalar equations:

$h = -H_o \frac{\partial u}{\partial x}$ (12)
$E = \mu_o H_o \frac{\partial u}{\partial t}$ (13)

By using the previous equations in the displacement Eq. 5, we arrive at:

$$\boldsymbol{\alpha \frac{\partial^2 u}{\partial t^2} = c_o^2 \frac{\partial^2 u}{\partial x^2} - \frac{\gamma}{\rho} \frac{\partial \theta}{\partial x}} \quad (14)$$

where $\alpha = 1 + \alpha_o^2/c^2$, c is the speed of light given by $c = (1/\mu_o \varepsilon_0)^{1/2}$, α_o is Alfven velocity, and $c_o = \sqrt{v_o^2 + \alpha_o^2}$ is the speed of propagation of longitudinal waves.

The energy equation in GN-III theory with memory-dependent derivative in the presence of heat sources:

$$k\left(n_1 \frac{\partial}{\partial t} + n_2 \kappa\right) \theta_{,ii} = \rho C_E \frac{\partial^2 \theta}{\partial t^2} + \gamma T_o \frac{\partial^2 e}{\partial t^2} - \frac{\partial Q}{\partial t} + \int_{t-\omega}^{t} K(t-\xi)(\rho C_E \frac{\partial^3 \vartheta(x,\xi)}{\partial \xi^3} + \gamma T_o \frac{\partial^3 e(x,\xi)}{\partial \xi^3} - \frac{\partial^2 Q}{\partial \xi^2})\, d\xi \quad (15)$$

Let us introduce the following non-dimensional variables:

$$x^* = c_o \eta_o x\,, u^* = c_o \eta_o u\,, t^* = c_o^2 \eta_o t\,, \theta^* = \frac{\gamma \theta}{\rho c_o^2}\,, \sigma^* = \frac{\sigma}{\rho c_o^2}\,, h^* = \frac{h}{H_o},$$
$$E^* = \frac{E}{\mu_o H_o c_o}\,, J^* = \frac{J}{\eta_o H_o c_o}\,, \kappa^* = \frac{\kappa}{\eta_o c_o^2}\,, Q^* = \frac{\gamma}{k \rho c_o^4 \eta_o^2} Q,$$
$$T_o = \frac{\delta_o \rho c_o^2}{\gamma}$$

The Eqs. 10-15 in non-dimensional form reduce to:

$J = -(\frac{\partial h}{\partial x} + V^2 \frac{\partial^2 u}{\partial t^2})$ (16)
$h = -\frac{\partial u}{\partial x}$ (17)
$E = \frac{\partial u}{\partial t}$ (18)

$$\frac{\partial^2\sigma}{\partial x^2} = \alpha\frac{\partial^2 e}{\partial t^2} \quad (19)$$
$$\left(n_1\frac{\partial}{\partial t} + n_2\kappa\right)\frac{\partial^2\theta}{\partial x^2} = \frac{\partial^2}{\partial t^2}(1+\omega D_\omega)(\theta+\varepsilon e) - \frac{\partial}{\partial t}(1+\omega D_\omega)Q \quad (20)$$
$$\sigma = \frac{\partial u}{\partial x} - \theta \quad (21)$$

where $\varepsilon = T_o\gamma^2/[(\lambda+2\mu)k\eta]$, $\eta = \rho C_E/k$ and $V = c_o/c$.

4. Laplace transform and state-space approach

Using the Laplace transform with the parameters s defined by the expression:

$$\left.\begin{aligned} L\{g(t) = \bar{g}(s) = \int_0^\infty e^{-st}g(t)dt \\ L\{D^n g(t)\} = s^n\, L\{g(t)\} \end{aligned}\right\} s > 0$$

to both sides of Eqs. 16-21, we get a coupled system of the following equations:

$$\bar{J} = -\left(\frac{\partial\bar{h}}{\partial x} + V^2\, s^2\,\bar{u}\right) \quad (22)$$
$$\bar{h} = -\frac{\partial\bar{u}}{\partial x} \quad (23)$$
$$\bar{E} = s\bar{u} \quad (24)$$
$$D^2\bar{\theta} = s\varpi\bar{\theta} + s\varpi\varepsilon\bar{e} - \varpi\bar{Q} \quad (25)$$
$$D^2\bar{\sigma} = \alpha s^2 e \quad (26)$$
$$\bar{\sigma} = \bar{e} - \bar{\theta} \quad (27)$$

where,

$$L\{\omega D_\omega f(t)\} = F(s)\begin{cases} [(1-e^{-s\omega})], & if\ m=n=0 \\ \left[1-\frac{1}{\omega s}(1-e^{-s\omega})\right], & if\ m=0,\quad n=\frac{1}{2} \\ \left[\left(1-\frac{2}{\omega s}\right)+\frac{2}{\omega^2 s^2}\ (1-e^{-s\omega})\right], & if\ m=n=1 \end{cases} \quad (28)$$

$$F(s) = L\{\frac{\partial^2\theta}{\partial t^2} + \varepsilon\frac{\partial^2 e}{\partial t^2} - \frac{\partial Q}{\partial t} = s^2(\bar{\theta}+\varepsilon\bar{e}) - s\bar{Q},\ D = \frac{d}{dx},\ \varpi(s) = \frac{s(1+W)}{n_1 s + n_2\kappa}$$

$$W(s) = (1-e^{-s\omega})\left(1-\frac{2n}{\omega s}+\frac{2m^2}{\omega^2 s^2}\right) - \left(m^2 - 2n + \frac{2m^2}{\omega s}\right)e^{-s\omega}$$

and all the initial functions are equal to zero.

We regard the medium to be exposed to a moving heat source of consistent quality, releasing its vitality indefinitely while traveling down the x-axis in a positive direction at a constant speed v. The non-dimensional geometry of the moving heat source is assumed to be the case:

$$Q(x,t) = Q_o\delta(x - vt) \quad (29)$$

where Q_o is a constant heat. Taking Laplace transform, we obtain:

$$\bar{Q}(x,s) = \ell\ exp(-hx) \quad (30)$$

where $\ell = \frac{Q_o}{v}$ $and\ h = s/v$.

Eliminating $\bar{e}$ and $\bar{\theta}$ from Eqs. 25–27, we have:

$$D^2\bar{\theta} = L_1\bar{\theta} + L_2\bar{\sigma} - L_3\exp(-hx) \quad (31)$$

where,

$L_1 = s\varpi(1+\varepsilon), L_2 = s\varpi\varepsilon, L_3 = \ell\varpi,$

and,

$$D^2\bar{\sigma} = M_1(\bar{\theta}+\bar{\sigma}) \quad (32)$$

where $M_1 = \alpha s^2$.

Using the temperature and the stress component in the x-direction as state components, Eqs. 31 and 32 may be combined in the framework form as follows:

$$D^2\bar{G}(x,s) = A(s)\bar{G}(x,s) + F(s)\exp(-hx) \quad (33)$$

where,

$\bar{G}(x,s) = \begin{bmatrix}\bar{\theta}(x,s)\\ \bar{\sigma}(x,s)\end{bmatrix}, A(s) = \begin{bmatrix}L_1 & L_2\\ M_1 & M_2\end{bmatrix}$ and $F(s) = \begin{bmatrix}-\ell\varpi\\ 0\end{bmatrix}$.

Solutions of Eq. 33 that stay bounded for large x can be written as:

$$\bar{G}(x,s) = \exp\left[-\sqrt{A(s)}\ x\right]\bar{G}_o(s) + D(s)exp(-hx) \quad (34)$$

where,

$$\bar{G}_o(s) = \begin{bmatrix}G_1(s)\\ G_2(s)\end{bmatrix}, \quad D(s) = \begin{bmatrix}D_1(s)\\ D_2(s)\end{bmatrix} = [h^2 I - A(s)]^{-1}F(s)\ \ and\ I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

We shall use the well-known Cayley-Hamilton theorem to find the form of the matrix. $\exp\left[\sqrt{A(s)}\ x\right]$The characteristic equation of the matrix $A(s)$ can be written as:

$$k^2 - (L_1+M_1)k + M_1(L_1-L_2) = 0 \quad (35)$$

The roots of this equation, namely, k_1 and k_2, satisfy the following relations:

$$k_1 + k_2 = L_1 + M_1 \quad (36)$$
$$k_1 k_2 = M_1(L_1 - L_2) \quad (37)$$

The Taylor series expansion of the matrix exponential in Eq. 34 has the form,

$$\exp\left[-\sqrt{A(s)}\ x\right] = \sum_{n=0}^{\infty}\frac{\left[-\sqrt{A(s)}\ x\right]^n}{n!} \quad (38)$$

We can express and higher powers of the matrix in terms of and, where I is the unit matrix of the second request, using the Cayley–Hamilton hypothesis. As a result, the infinite series in Eq. 38 may be reduced to:

$$\exp\left[-\sqrt{A(s)}\ x\right] = a_o(x,s)I + a_1(x,s)A(s)$$

where, $a_o$and a_1 are coefficients relying upon x and s.

By the Cayley–Hamilton hypothesis, the trademark roots k_1 *and* k_2 of the matrix A must satisfy,

$$\exp[-\sqrt{k_1}\ x] = a_o I + a_1 k_1, \exp[-\sqrt{k_2}\ x] = a_o I + a_1 k_2 \quad (39)$$

The solution to the above two-linear equation system is given by,

$$a_o = \frac{k_1 e^{-\sqrt{k_2}x} - k_2 e^{-\sqrt{k_1}x}}{k_1 - k_2}, \text{ and } a_1 = \frac{e^{-\sqrt{k_1}x} - e^{-\sqrt{k_2}x}}{k_1 - k_2}$$

hence the entries of the matrix $\exp\left[-\sqrt{A(s)}\ x\right] = L_{ij}(x,s), i, j = 1, 2$, are given by,

$$L_{11} = \frac{(k_1 - L_1)e^{-\sqrt{k_2}x} - (k_2 - L_1)e^{-\sqrt{k_1}x}}{k_1 - k_2}, L_{12} = \frac{L_2\left(e^{-\sqrt{k_1}x} - e^{-\sqrt{k_2}x}\right)}{k_1 - k_2},$$
$$L_{22} = \frac{(k_1 - M_2)e^{-\sqrt{k_2}x} - (k_2 - M_2)e^{-\sqrt{k_1}x}}{k_1 - k_2}, L_{21} = \frac{M_1(e^{-\sqrt{k_1}x} - e^{-\sqrt{k_2}x})}{k_1 - k_2} \quad (40)$$

additionally,

$$D_1 = \frac{\ell^2 \varpi (M_1 - h^2)}{(h^2 - k_1)(h^2 - k_2)}, \qquad D_2 = \frac{\ell^2 \varpi_3 M_1}{(h^2 - k_1)(h^2 - k_2)}$$

we may form solution 33 into a shape.

$$\begin{bmatrix} \bar{\theta}(x,s) \\ \bar{\sigma}(x,s) \end{bmatrix} = \begin{bmatrix} L_{11}(x,s) & L_{12}(x,s) \\ L_{21}(x,s) & L_{22}(x,s) \end{bmatrix} \begin{bmatrix} G_1(s) \\ G_2(s) \end{bmatrix} + \begin{bmatrix} D_1(s) \\ D_2(s) \end{bmatrix} \exp(-hx) \quad (41)$$

To get $G_1(s)$ and $G_2(s)$ we set $x = 0$ on Eq. 41, and we obtain:

$$\begin{bmatrix} \bar{\theta}(0,s) \\ \bar{\sigma}(0,s) \end{bmatrix} = \begin{bmatrix} L_{11}(0,s) & L_{12}(0,s) \\ L_{21}(0,s) & L_{22}(0,s) \end{bmatrix} \begin{bmatrix} G_1(s) \\ G_2(s) \end{bmatrix} + \begin{bmatrix} D_1(s) \\ D_2(s) \end{bmatrix}$$

which implies to,

$$\begin{bmatrix} G_1(s) \\ G_2(s) \end{bmatrix} = \begin{bmatrix} \bar{\theta}(0,s) \\ \bar{\sigma}(0,s) \end{bmatrix} - \begin{bmatrix} D_1(s) \\ D_2(s) \end{bmatrix} \quad (42)$$

As a result, the accurate solution in the Laplace domain for every combination of boundary conditions is supplied by,

$$\bar{\theta}(x,s) = [\bar{\theta}(0,s) - D_1(s)]L_{11}(x,s) + [\bar{\sigma}(0,s) - D_2(s)]L_{12}(x,s) + D_1(s)\exp(-hx) \quad (43)$$
$$\bar{\sigma}(x,s) = [\bar{\theta}(0,s) - D_1(s)]L_{21}(x,s) + [\bar{\sigma}(0,s) - D_2(s)]L_{22}(x,s) + D_2(s)\exp(-hx) \quad (44)$$

In the absence of a magnetic field, the analogous formulae for Green-Naghdi of type-III with memory-dependent derivative thermoelasticity can be obtained by plugging $\alpha_o = 0$ in Eq. 41.

5. Application

We investigate a semi-space with perfect conducting medium, and point of confinement conditions in the form:

(i) Thermal boundary condition:

We suppose that the bounding plane $x = 0$, is exposed to harmonic heating, i.e.

$$f(t) = \begin{cases} \sin\left(\frac{\pi i}{\beta}\right) & 0 \le t \le \beta \\ 0 & otherwise \end{cases} \text{ or } \bar{f}(s) = \frac{\pi\beta(1 - e^{-\beta s})}{\beta^2 s^2 + \pi^2} \quad (45)$$

(ii) Mechanical boundary condition:

The bounding plane $x = 0$ is taken to be traction-free, i.e.

$$\sigma(0,t) = 0 \text{ or } \bar{\sigma}(0,s) = 0. \quad (46)$$

Hence, we can use the conditions of 45 and 46 in Eqs. 43 and 44 to get the exact solution for the temperature and stress component in the Laplace transform domain in the following forms:

$$\bar{\theta}(x,s) = \theta_1(s)e^{-\sqrt{k_1}x} - \theta_2(s)e^{-\sqrt{k_2}x} + D_1(s)e^{-hx} \quad (47)$$
$$\bar{\sigma}(x,s) = \sigma_1(s)e^{-\sqrt{k_1}x} - \sigma_2(s)e^{-\sqrt{k_2}x} + D_2(s)e^{-hx} \quad (48)$$

where,

$$\theta_1(s) = \frac{1}{k_1 - k_2}\left[\left(\frac{\pi\beta(1 - e^{-\beta s})}{\beta^2 s^2 + \pi^2} - D_1\right)(k_1 - M_1) - L_2 D_2\right] \quad (49)$$
$$\theta_2(s) = \frac{1}{k_1 - k_2}\left[\left(\frac{\pi\beta(1 - e^{-\beta s})}{\beta^2 s^2 + \pi^2} - D_1\right)(k_2 - M_1) - L_2 D_2\right] \quad (50)$$
$$\sigma_1(s) = \frac{1}{k_1 - k_2}\left[\left(\frac{\pi\beta(1 - e^{-\beta s})}{\beta^2 s^2 + \pi^2} - D_1\right)M_1 - D_2(M_1 - k_1)\right] \quad (51)$$
$$\sigma_2(s) = \frac{1}{k_1 - k_2}\left[\left(\frac{\pi\beta(1 - e^{-\beta s})}{\beta^2 s^2 + \pi^2} - D_1\right)M_1 - D_2(M_1 - k_1)\right] \quad (52)$$

clearly, $\bar{\sigma}(0, s) = 0$, in agreement with Eq. 48.

From Eq. 27, the displacement field takes the form:

$$\bar{u}(x,s) = -\left[\frac{1}{\sqrt{k_1}}(\sigma_1 + \theta_1)e^{-\sqrt{k_1}x} - \frac{1}{\sqrt{k_2}}(\sigma_2 + \theta_2)e^{-\sqrt{k_2}x} + \frac{1}{h}(D_2 + D_1)e^{-hx}\right] \quad (53)$$

By substituting from Eq. 53 into Eqs. 23 and 24, we obtained the induced electric and magnetic fields,

$$\bar{E} = s\left[\frac{1}{\sqrt{k_2}}(\sigma_2 + \theta_2)e^{-\sqrt{k_2}x} - \frac{1}{\sqrt{k_1}}(\sigma_1 + \theta_1)e^{-\sqrt{k_1}x} - \frac{1}{h}(D_2 + D_1)e^{-hx}\right] \quad (54)$$
$$\bar{h}(x,s) = (\sigma_2 + \theta_2)e^{-\sqrt{k_2}x} - (\sigma_1 + \theta_1)e^{-\sqrt{k_1}x}(D_2 + D_1)e^{-hx} \quad (55)$$

Those complete the solution in the Laplace transform domain.

6. Numerical results and discussion

The technique dependent on a Fourier arrangement extension proposed by Hoing and

Hirdes (1984) and is created in detail in numerous writings, for example, the numerical code has been readied utilizing Fortran 77 programming language.

So as to translate the numerical calculations, we consider the material properties of copper material ($\varepsilon = 0.0168$), whose physical information is given in Table 1.

Table 1: Values of the constants (El Sherif et al., 2020)

$\rho = 8954\ kg/m^3$	$k = 386$ N/ Ks	$T_o = 293K$
$C_E = 383.1\ m^2/K$	$\lambda = 7.76(10)^{10}$ N/ m^2	$\mu = 3.86(10)^{10}$ N/ m^2
$\gamma = 210(10)^4$ N/ m^2K	$\eta_o = 3.36(10)^6$ sec/ m^2	$c_o = 4158$ m/s
$\mu_o = 1.256(10)^{-6}$ N s^2/C^2	$k^* = 124$ W / mKs	$\alpha_T = 1.78(10)^{-5}\ K^{-1}$
$\alpha_0 = 218(10)^{11}$ m/s	$c \approx 3(10)^6$ m/s	$v_o = 4158\ m/s$

Thinking about the above physical information, we have assessed the numerical estimations of the field amounts.

The calculations were performed for an estimation of time, in particular $t = 0.1$. The numerical method laid out above was utilized to get the temperature, displacement, and stress appropriations just as the electric flow segments for various estimations of the thought-about parameters. The outcomes are shown graphically at various places of x as appeared in Figs. 1–6.

Fig. 1 speaks to the dimensionless estimation of temperature for the wide scope of outspread separation x ($0 \leq x \leq 1.4$) and for different values of time delay. In Fig. 1, strong lines speak to the arrangement got in the casing of Biot theory($\omega = 0$) and other lines speak to the arrangement relating to utilizing generalized magneto-thermoelasticity GN-III with MDD for time-delay $0.03, 0.3$ when the kernel function is taken as the form $[1 - (t - \xi)/\omega]^2$. Fig. 1 revealed that the new model's organization of any of the studied capabilities is limited to a certain location. The different kinds of these appropriations are less likely to occur after this point. This suggests that the configurations supporting the new generalized hypothesis demonstrate the behavior of restricted rates of wave spread.

Fig. 2 displays the temperature distribution at different values of heat source velocity v ($v = 2, 4, 6$) to show its effect, where we have noticed that the heat source velocity parameter v at $t = 0.1, \omega = 0.09$ and $K = [1 - (t - \xi)/\omega]$ has a significant effect on the temperature field. The peak value of the temperature is found at the points when x ($x = 2, 4, 6$) which mean that the heat source releases its maximum energy at the point $x = v\,t$ and just after this point the values of that fields decrease with high speed.

Figs. 3 and 4 depict the space variation of displacement and stress distributions. In Figs. 3 and 4, the effect of the heat source velocity v on these distributions is studied. We noticed that for different values of the heat source velocity parameter v $(2, 4, 6)$ at $t = 0.1, \omega = 0.07$, $K(t, \xi) = 1.0$ have a significant effect on all fields. We also learned from Figs. 3 and 4 that the increasing of the value of the parameter v causes increase in the magnitude of stress and displacement distributions.

The effects of time-delay parameter $\omega = 0.0, 0.05, 0.5$ on the induced magnetic and electric fields are shown in Figs. 5 and 6. We noticed that the time delay acts to diminish both fields. This is generally known as attractive damping.

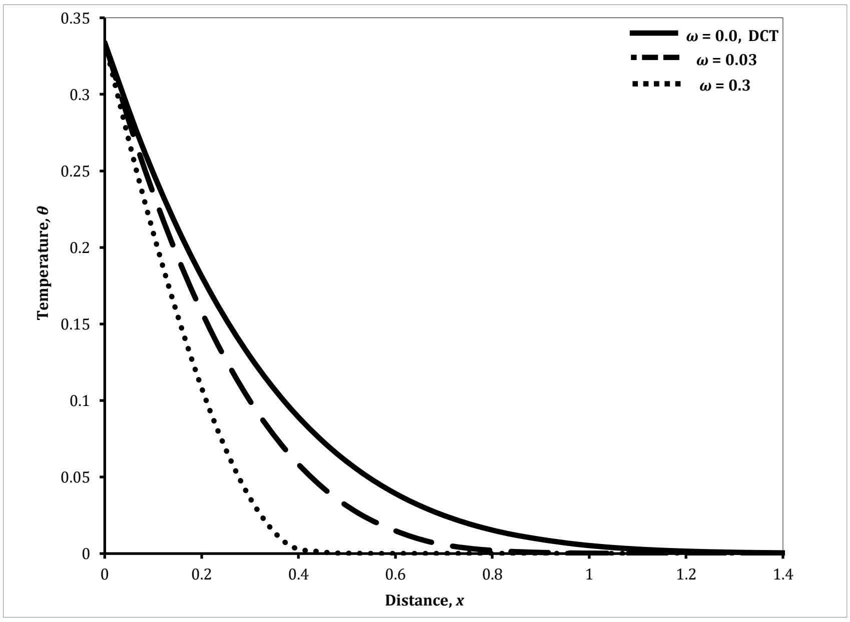

Fig. 1: The temperature distribution for different values of time-delay parameter ω

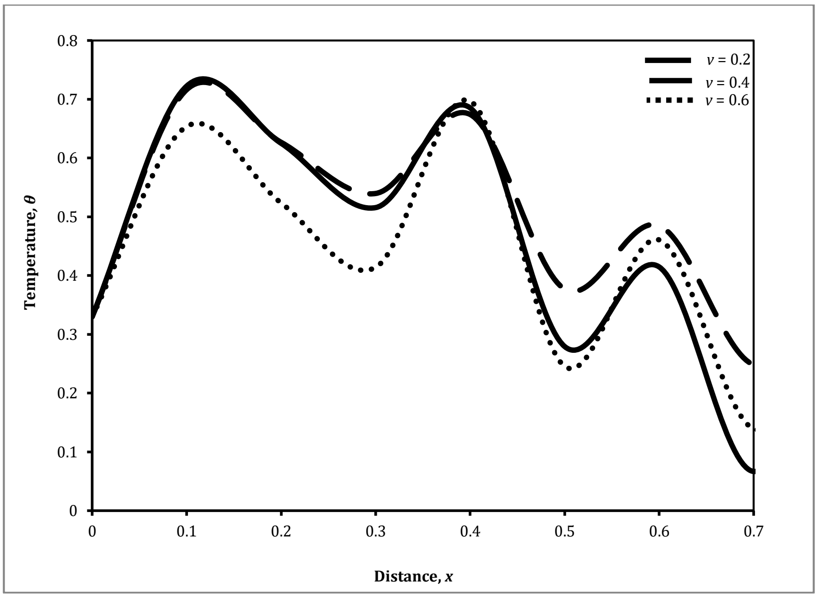

Fig. 2: The temperature distribution for different values of heat source velocity parameter v

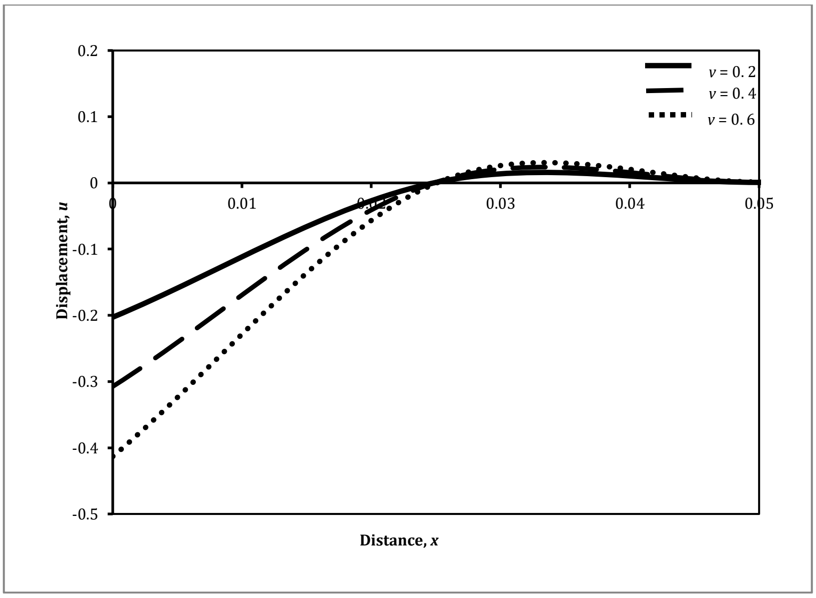

Fig. 3: Displacement distribution for different values of heat source velocity parameter v

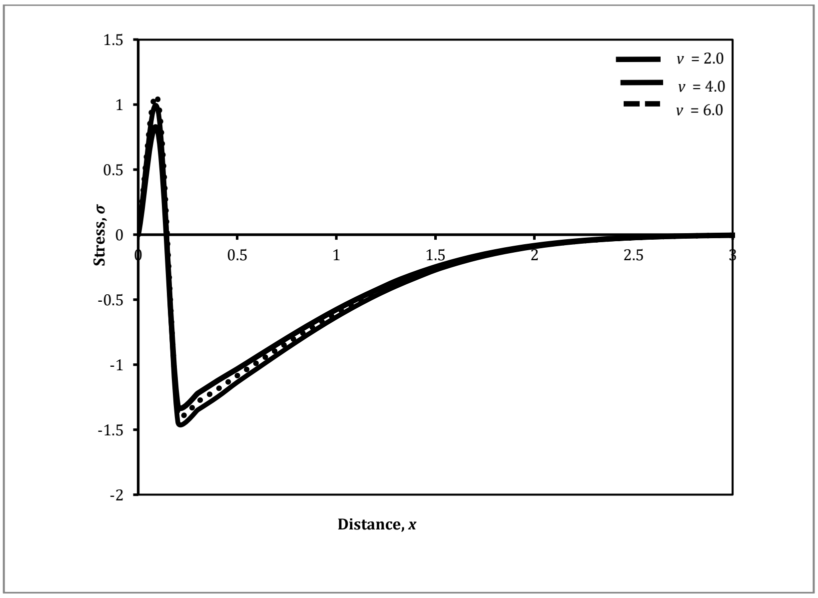

Fig. 4: Stress distribution for different values of the heat source velocity parameter v

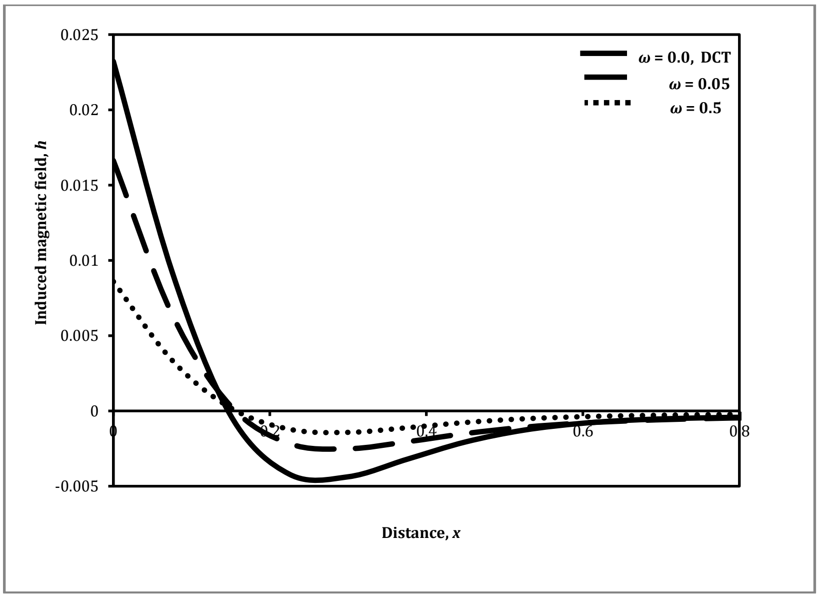

Fig. 5: The variation of induced electric field for different valuess of time-delay parameter ω

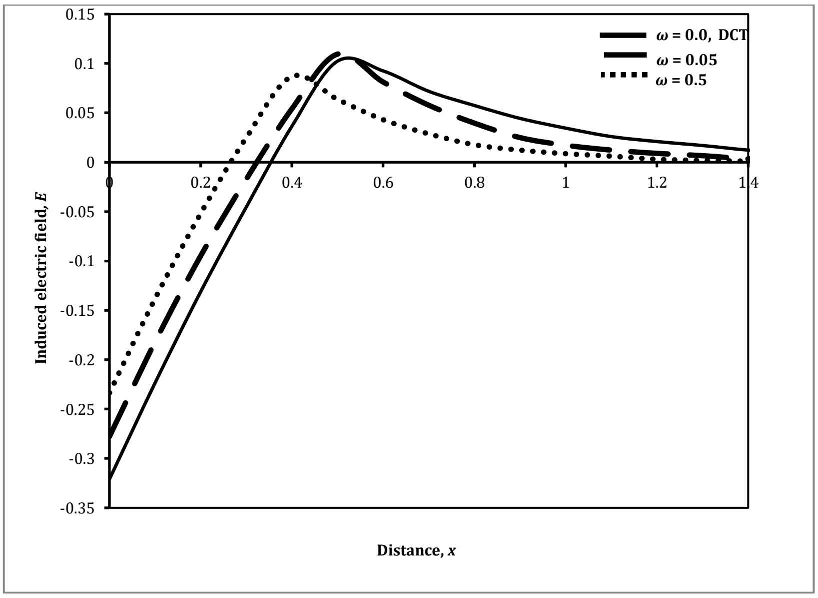

Fig. 6: The variation of the induced electric field at different values of time-delay parameter ω

7. Conclusion

The primary objective of this work is to take care of certain issues of thermal excitations in the hypothesis of coupled fields have a place with thermoelectric elastic materials. The expanding wide use in detecting and activation has pulled in much consideration towards hypotheses about materials displaying couplings between versatile, electric, attractive, and warm fields.

The conditions of wave hypothesis of thermoelectric materials exposed to MDD based on the change of the Fourier law was built on rough phenomenological conditions of thermo-electromagnetic versatility described by a limited speed of engendering of electromagnetic and flexible excitations.

As per the aftereffects of the work, we can see the nearness of MDD's parameters in Fourier law of heat conduction can assume a crucial job in expanding or diminishing the speed of the wave proliferation of all fields through the thermoelectric medium.

From the considered model we can set up some fundamental hypotheses on the straight coupled and generalized speculations of electro-thermo-viscoelasticity; for example the coupled hypothesis ($\omega = 0$) and the generalized case hypothesis($\omega > 0$).

List of symbols

λ, μ	Lame's constants
ρ	density
t	time
C_E	specific heat at constant strain
B_i	components of magnetic field strength
E_i	components of electric field vector
J_i	conduction electric density vector
H_i	magnetic field intensity
q_i	components of heat flux vector
H_o	constant component of magnetic field
μ_o	magnetic permeability

σ_{ij}	components of stress tensor
e_{ij}	components of strain tensor
u_i	components of displacement vector
θ	$T - T_o$
T_o	reference temperature chosen so that $\|T - T_o\|/T_o \ll 1$
α_T	coefficient of linear thermal expansion $\gamma = (3\lambda + 2\mu)\alpha_T$
$\varepsilon = \frac{\delta_o \gamma}{\rho C_E}$	thermoelastic parameter
$\eta = \frac{1}{\sigma_o \mu_o}$	magnetic diffusivity
η_o	$\frac{\rho C_E}{k}$
v_o^2	$\frac{\lambda + 2\mu}{\rho}$

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Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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