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Robust fuzzy control for non-linear systems with uncertainties: A Takagi-Sugeno model approach

Faisal Alsaket *, Mourad Kchaw, Ahmed Al-Shammari

College of Engineering, University of Hail, Hail, Saudi Arabia

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ABSTRACT

This article studies the problem of robust control design for a class of uncertain nonlinear systems using the Takagi–Sugeno (TS) fuzzy models. The objective of this study is to design state feedback and an observer-based controller such that the closed-loop system is asymptotically stable. For this purpose, sufficient conditions are derived, and the corresponding controllers are designed by solving a set of linear matrix inequalities (LMIs). The effectiveness of the proposed design approach is provided via numerical simulations for a permanent magnet synchronous motor (PMSM).

1. Introduction

It is well known that in the real world the physical systems and processes are generally nonlinear. Very recently, the Takagi–Sugano (TS) fuzzy models have known a huge reputation as an important approach to deal with nonlinear systems. The (TS) fuzzy models are qualified to describe a nonlinear system by a set of fuzzy IF-THEN rules in the form of local linear or affine models which are smoothly connected by fuzzy membership functions (Tanaka et al., 1996). Based on the sector non-linearity methodology (Kuppusamy and Joo, 2019; Lo and Lin, 2004) a nonlinear model can be exactly represented by its equivalent (TS) one. Note that the Parallel Distributed Compensation (PDC)method combined with quadratic Lyapunov functions, provides a basis for the analysis and control design of (TS) fuzzy systems in view of the powerful conventional control theory (Kim and Lee, 2000; Cao and Frank, 2000).

On the other hand, robustness is considered the most important requirement that should be achieved by the control system. Thus, the problem of robust control of uncertain systems has received a great deal of attention. The (TS) fuzzy models used to describe non-linear systems may be affected by uncertainties which can be provided from the modeling procedure or also from the inherent uncertainties in the real system (Ding et al., 2006).

Accordingly, the standard approach to cope with stability and stabilization problems for (TS) fuzzy systems consists in finding common quadratic Lyapunov functions that satisfy sufficient conditions, guaranteeing stability. These conditions are frequently expressed as Linear Matrix Inequality (LMI) constraints solvable through convex optimization techniques. Within this framework, very effective strategies have been suggested to overcome mathematical and numerical difficulties, promoting less-conservative conditions (Fang et al., 2006; Tuan et al., 2004; 2001).

On the other hand, since state variables are usually not completely available in practical control systems, output feedback or observer-based control is more feasible. Especially, the observer-based controller is widely accepted for its simple structure and explicit physical meaning. In this regard, the observer-based control output feedback control is probably well suited for feedback control, while the problem of designing observers for nonlinear systems has also been investigated by a number of scholars (Yoneyama, 2006; Lin et al., 2005; Takagi and Sugeno, 1985).

Motivated by the aforementioned concerns, the objective of this paper is two folds. First, we will address the issue of robust stabilization for (TS) fuzzy model with norm bounded uncertainty. Sufficient conditions will be derived such the closed-loop system is robust against the norm bounded uncertainty. Second, for the system with a partially measurable state, a robust observer-based controller will be designed. Moreover, for the task of control of a permanent magnet synchronous motor (PMSM)

* Corresponding Author.
Email Address: faisall6666@hotmail.com (F. Alsaket)
https://doi.org/10.21833/ijaas.2022.05.004
Corresponding author's ORCID profile:
https://orcid.org/0000-0002-6857-1948

the effectiveness of the proposed method will be verified by simulation studies.

2. System descriptions and preliminaries

The (TS) fuzzy dynamic model is described by fuzzy IFTHEN rules, which locally represent linear input-output relations of nonlinear systems. A continuous fuzzy model with parameter uncertainties can be described by,

$\mathbf{R}_i$: If θ_1 is F_i^1 and If θ_2 is $F_i^2 \cdots$ If θ_s is F_i^s, Then

$$\begin{cases} \dot{x}(t) = A_i(t)x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^q$ are the state vector, the input vector, and the output vector, respectively. A_i, B_i, and C_i are constant real matrices with appropriate dimensions; r is the number of model rules; $\theta(t) = [\theta_{1,t}, \theta_{2,t}, \dots, \theta_{p,t}]$ is the premise variable vector and $h_i(\theta(t))$ denotes the normalized membership function which satisfies $h_i(\theta(t)) \geqslant 0, i \in \mathbb{S} \triangleq \{1,2,\dots,r\}$ and $\sum_i^r h(\theta(t)) = 1$ for all t.

Assume that $A_i(t) = A_i + \Delta A_i$, is a time-varying system matrix. A_i, B_i, and C_i are constant matrices with appropriate dimensions. Parameter uncertainty ΔA_i is assumed to be of the form,

$$\Delta A_i = M_i \Delta N_i \tag{2}$$

where M_i, N_i and N_{ui} are known real constant matrices and Δ is an unknown time-varying matrix function satisfying,

$$\Delta^T \Delta \leqslant I \tag{3}$$

The overall fuzzy model is inferred as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r h_i(\theta(t))\{A_i(t)x(t) + B_{2i}u(t)\} \\ y(t) = \sum_{i=1}^r h_i(\theta(t))C_{2i}x(t) \end{cases} \tag{4}$$

where $h_i(\theta)$ are the normalized weight functions defined by:

$$h_i(\theta) = \frac{\prod_{j=1}^s F_j^i(\theta_j)}{\sum_{i=1}^r \prod_{j=1}^s F_j^i(\theta_j)}, \; i = 1,2,\cdots,r$$

and $F_j^i(\theta_j)$ represents the membership degrees of θ_j in the fuzzy set F_j^i . Note that the normalized weights $h_i(\theta)$ satisfy,

$$h_i(\theta) \geqslant 0, \; i = 1,2,\dots,r \; \sum_{i=1}^r h_i h_i(\theta) = 1. \tag{5}$$

Lemma 2.1: (Petersen, 1987) Given matrices M, N, and P of appropriate dimensions and with P symmetrical, then,

$$P + M\Delta N + N^T F^T(t) M^T < 0 \tag{6}$$

for any Δ satisfying $\Delta^T \Delta \leq I$, if and only if there exists a scalar $\epsilon > 0$ such that,

$$P + \epsilon M M^T + \epsilon^{-1} N^T N < 0 \tag{7}$$

Lemma 2.2: For given matrices S > 0, P and R of appropriate dimensions the following two inequalities are equivalent,

$$-P + R^T S^{-1} R < 0 \Leftrightarrow \begin{bmatrix} -P & R^T \\ * & -S \end{bmatrix} < 0 \tag{8}$$

Lemma 2.3: (Xiaodong and Qingling, 2003) The following inequality holds:

$$\sum_{i=1}^r \sum_{j=1}^r h_i h_j \Upsilon_{ij} < 0 \tag{9}$$

if

$$\Upsilon_{ii} < 0, i = 1,2,\cdots,r \tag{10}$$

$$\frac{2}{r-1}\Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} < 0, \; j > i \tag{11}$$

For the purpose of control design, we consider different feedback schemes including fuzzy state feedback and fuzzy observer-based state feedback.

3. State-feedback controller

In this section, given a (TS) fuzzy model of a nonlinear plant in the form of 4, we address the design of a state feedback controller using the PDC approach.

$$u(t) = \sum_{i=1}^r h_i(t) K_i x(t) \tag{12}$$

The closed-loop system of 4 and 12 is given by,

$$\{\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j \{\mathcal{A}_{ij}(t)x(t)\} \tag{13}$$

where

$$\begin{aligned} \mathcal{A}_{ij}(t) &= \mathcal{A}_{ij} + \Delta\mathcal{A}_{ij}(t), \; \mathcal{A}_{ij} = A_i + B_i K_j \\ \Delta\mathcal{A}_{ij}(t) &= M_i \Delta N_i \end{aligned} \tag{14}$$

Theorem 1: Closed-loop fuzzy system 13 is robustly stable, if there exist matrices P > 0 such that:

$$\Upsilon_{ii} < 0, i \in \mathbb{S} \tag{15}$$

$$\frac{2}{r-1}\Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} < 0, \; j > i \tag{16}$$

where

$$\Upsilon_{ij} = \begin{pmatrix} P\mathcal{A}_{ij} + \mathcal{A}_{ij}^T P \epsilon P M_i & & N_i \\ * & -\epsilon I & 0 \\ * & * & -\epsilon I \end{pmatrix} \tag{17}$$

Proof: Under the conditions of the theorem, we first establish the stability of the system in 13. In the sequel, we choose a Lyapunov function candidate for system 13 as follows:

$$V(x(t)) = x^T(t) P x(t) \tag{18}$$

Then, the derivative of Lyapunov function 18 gives:

$$\dot{V}(x(t)) = 2x^T(t) P \dot{x}(t) \tag{19}$$

Considering 13, we get,

$$\dot{V}(x(t)) = 2x^T(t) P \sum_{i=1}^r \sum_{j=1}^r h_i h_j (\mathcal{A}_{ij} + \Delta\mathcal{A}_{ij}(t)) \tag{20}$$

Equivalently, the following inequality holds using Lemma 2.1,

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \big(P\mathcal{A}_{ij} + \mathcal{A}_{ij}^T P \quad (21)$$
$$+\epsilon(PM_i)(PM_i)^T + \epsilon^{-1}N_i^T N_i\big) < 0 \quad (22)$$

Then, according to the Schur complement, it is easy to see,

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \Upsilon_{ij} \quad (23)$$
$$= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \begin{pmatrix} P\mathcal{A}_{ij} + \mathcal{A}_{ij}^T P & \epsilon P M_i & N_i \\ * & -\epsilon I & 0 \\ * & * & -\epsilon I \end{pmatrix} < 0 \quad (24)$$

Thus, the inequalities are verified according to Lemma 2.3.

Theorem 2 Consider the fuzzy system 4 and the PDC fuzzy controller 12. If there exist matrices $P > 0$ and Y_i such that the following LMI is verified,

$$\Upsilon_{ii} < 0, i \in \mathbb{S} \quad (25)$$
$$\frac{2}{r-1}\bar{\Upsilon}_{ii} + \bar{\Upsilon}_{ij} + \bar{\Upsilon}_{ji} < 0,\ j > i \quad (26)$$

where

$$\bar{\Upsilon}_{ij} = \begin{pmatrix} A_i X + B_{2i} Y_i + (A_i X + B_{2i} Y_i)^T & \epsilon M_i & X N_i^T \\ * & -\epsilon I & 0 \\ * & * & -\epsilon I \end{pmatrix} \quad (27)$$

then, the closed-loop system 4 is robustly stable, and the feedback gains are given by $K_i = Y_i X^{-1}$.

Proof: Let $X = P^{-1}$. By performing the congruence transformation to 15, and 16 by $\mathrm{diag}(X, I, I)$, inequalities 25 and 26 hold by setting $Y_i = K_i X$.

4. Observer-based controller

The establishment of a PDC control law requires the measurement of the state vector. As this condition is rarely verified in practice, the use of a fuzzy observer is necessary for this case. The observer shares the same fuzzy sets as the model taken into account. The fuzzy observer is given by the following model,

$$\begin{cases} \dot{x}_c(t) & = \sum_{i=1}^{r} h_i(\theta(t))\{A_i(t)x_c(t) + B_{2i}u(t) \\ & \quad + L_i(y(t) - y_c(t))\} \\ y_c(t) & = \sum_{i=1}^{r} h_i(\theta(t)) C_{2i} x_c(t) \end{cases} \quad (28)$$

where $x_c(t)$ is the state estimation of $x(t)$, y_c is the observer output, $L_i \in \mathbb{R}^{n\times q}$ and are the observer gain matrices. Suppose the following control law is used:

$$u(t) = \sum_{i=1}^{r} h_i(t) K_i x_c(t) \quad (29)$$

The closed-loop system of 4 and 29 is shown as follows:

$$\{\dot{\tilde{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \{\tilde{\mathcal{A}}_{ij}(t)\tilde{x}(t)\} \quad (30)$$

where

$$\tilde{x}(t) = [x(t)e(t)]^T$$
$$\tilde{\mathcal{A}}_{ij}(t) = \tilde{\mathcal{A}}_{ij} + \tilde{\mathcal{M}}_{ij}\Delta\tilde{\mathcal{N}}_{ij}$$
$$\tilde{\mathcal{A}}_{ij} = \begin{bmatrix} A_i + B_{2i}K_j & -B_{2i}K_j \\ 0 & A_i - L_i C_{2i} \end{bmatrix}$$
$$\tilde{\mathcal{M}}_i = \begin{bmatrix} Mi \\ 0 \end{bmatrix}, \tilde{\mathcal{N}}_i = [Ni0]$$

Theorem 3 Closed-loop fuzzy system (30) is robustly stable, if there exist matrices $P > 0$ such that:

$$\tilde{\Upsilon}_{ii} < 0, i \in \mathbb{S} \quad (31)$$
$$\frac{2}{r-1}\tilde{\Upsilon}_{ii} + \tilde{\Upsilon}_{ij} + \tilde{\Upsilon}_{ji} < 0,\ j > i \quad (32)$$

where

$$\tilde{\Upsilon}_{ij} = \begin{pmatrix} \mathbf{A}_{ij} + \mathbf{A}_{ij}^T & \epsilon \mathbf{M}_i & \tilde{\mathcal{N}}_i^T \\ * & -\epsilon I & 0 \\ * & * & -\epsilon I \end{pmatrix} \quad (33)$$

where

$$\begin{aligned} \mathbf{A}_{ij} &= \begin{pmatrix} Z_{ij}^1 & * \\ -P_2(B_{2i}K_j)^T & Z_{ij}^2 \end{pmatrix} \\ Z_{ij}^1 &= (A_i + B_{2i}K_j)^T + P_1(A_i + B_{2i}K_j) \\ Z_{ij}^2 &= (P_2 A_i - F_i C_{2j})^T + P_2 A_i - F_i C_{2j} \\ \mathbf{M}_i &= \begin{pmatrix} P_1 M_i \\ 0 \end{pmatrix} \end{aligned} \quad (34)$$

Proof: By following the same lines to prove Theorem 1, it is easy to verify that,

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \big(\tilde{P}\tilde{\mathcal{A}}_{ij} + \tilde{\mathcal{A}}_{ij}^T \tilde{P} \quad (35)$$
$$+\epsilon(\tilde{P}\tilde{\mathcal{M}}_i)(\tilde{P}\tilde{\mathcal{M}}_i)^T + \epsilon^{-1}\tilde{\mathcal{N}}_i^T\tilde{\mathcal{N}}_i\big) < 0 \quad (36)$$

By setting $\tilde{P} = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$, inequalities 31-32 hold using Lemma 2.3.

Note that, there is no effective algorithm for solving the parameters $P_1 P_2$, K_i, and L_i in Theorem 3. However, we can use the two-step procedure to solve them.

4.1. Design procedure

1. From Theorem 2, solve the state feedback controller K_i
2. Substitute K_i into 31 and then solve the obtained LMIs to get $P_1 P_2$, and F_i
3. Design the observer gain by $L_i = P_2^{-1} F_i$

5. A simulation example

The following nonlinear equations represent the model of a permanent magnet synchronous motor (PMSM) in the synchronously rotating dq reference frame:

$$\begin{cases} \dot{\omega}(t) & = k_1 i_{qs}(t) - k_2\omega(t) - k_3 T_L \\ i_{qs}(t) & = -k_4 i_{qs}(t) - k_5\omega(t) - \omega(t) i_{ds}(t) + k_6 V_{qs}(t) \\ i_{ds}(t) & = -k_4 i_{ds}(t) + \omega(t) i_{qs}(t) + k_6 V_{ds}(t) \end{cases} \quad (37)$$

where ω is the electrical rotor angular speed, i_{qs} and V_{qs} are the quadrature current and voltage, respectively, i_{ds} and V_{ds} are the direct current and voltage, respectively. T_L denotes the load torque. The model parameters are defined,

as $k_1 = \frac{3p^2}{8J}\lambda_m, k_2 = \frac{B}{J}, k_3 = \frac{p}{2J}, k_4 = \frac{R_s}{L_s}, , k_5 = \frac{\lambda_m}{L_s}$ and $k_6 = \frac{1}{L_s}$.

Let $x(t) = [x_1(t)x_2(t)x_3(t)] = [i_d, i_q, v]$. As in Wang et al. (1996), the PMSM nonlinear model can be written as,

$$\begin{cases} \dot{x}_1(t) & = -x_1(t) + x_2(t)x_3(t) + u(t) \\ \dot{x}_2(t) & = x_2(t) + x_3(t)(1.1 - x_1(t)) + u(t) \\ \dot{x}_3(t) & = 5.46(x_2(t) - x_3(t)) \end{cases} \quad (38)$$

Assume that $x_3(t) \in [-d, d]$. The membership functions are $h_1(x_3(t)) = 0.5(1 + x_3(t)/d)$ and $h_2(x_3(t)) = 1 - h_1(x_3(t))$.

Then, based on the sector-nonlinear approach, the state equation of the PMSM can be represented by a TS fuzzy model with the following matrices:

$$A_1 = \begin{bmatrix} -1 & d & 0 \\ -d & -1 & 1.1 \\ 0 & 5.46 & -5.46 \end{bmatrix}, , A_2 = \begin{bmatrix} -1 & -d & 0 \\ d & -1 & 1.1 \\ 0 & 5.46 & -5.46 \end{bmatrix} \quad (39)$$

$$B_1 = B_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad (40)$$

Assume that only the states $x_1(t)$, and $x_3(t)$ are measurable. Thus, $C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and an observer should be designed according to the procedure described above. Assume the parameters uncertainties are set as $M_i = [0.100]^T$, and $M_i = [0.1, 00]$, for $i = 1,2$. The first step is achieved by solving the LMIs in Theorem 2, a feasible solution is obtained with the corresponding control gain matrices defined as,

$$K_1 = [-24.171\ 23.948 - 48.042] \quad (41)$$
$$K_2 = [5.7142 - 6.27168.6818] \quad (42)$$

Then, by substituting the gains K_i into 31-32 the obtained LMIs can be solved with the following parameters:

$$P_2 = \begin{bmatrix} 5.0459 & 0.0027814 & 0.76974 \\ 0.0027814 & 4.9042 & 0.0026528 \\ 0.76974 & 0.0026528 & 6.9155 \end{bmatrix} \quad (43)$$

$$L_1 = \begin{bmatrix} 1.9637 \\ 8.2402 \\ -3.5401 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.41606 \\ 6.1053 \\ -4.7555 \end{bmatrix} \quad (44)$$

The simulation result gives a potent verification of the effectiveness of the suggested control scheme and shows its robustness in spite of the uncertainties.

The simulation results are shown in Figs. 1-4 for an initial condition $x(0) = [1\ 5\ -4]^T$. Among them, Figs. 1-3 show the time responses of the system and observer states. The evolution of the control signal is plotted in Fig. 4. It can be seen that the system is stabilized regardless of uncertainties. Moreover, for the considered system with unmeasured states, the robust control problem can be achieved using the proposed control scheme, which is in concordance with the analysis in the paper.

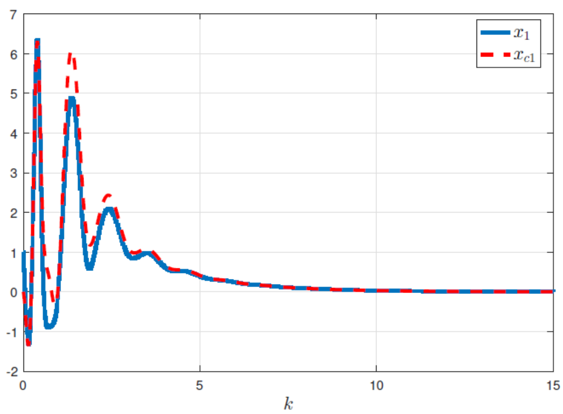

Fig .1: Trajectories of $x_1(t)$ and $\hat{x}_{c1}(t)$

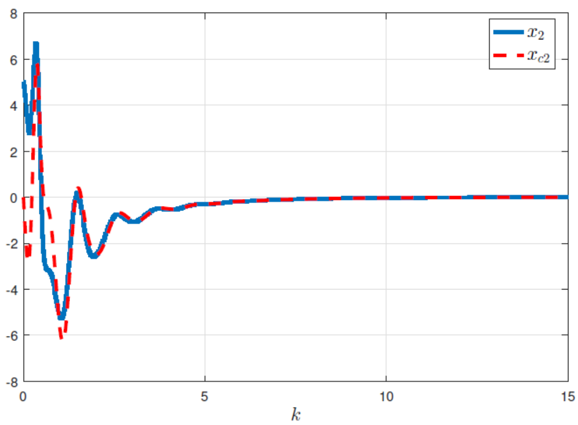

Fig. 2: Trajectories of $x_2(t)$ and $\hat{x}_{c2}(t)$

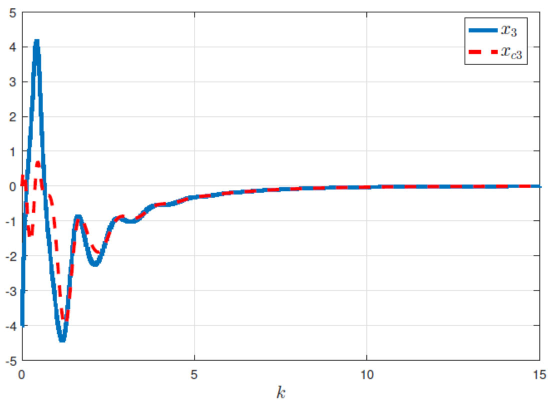

Fig. 3: Trajectories of $x_3(t)$ and $\hat{x}_{c3}(t)$

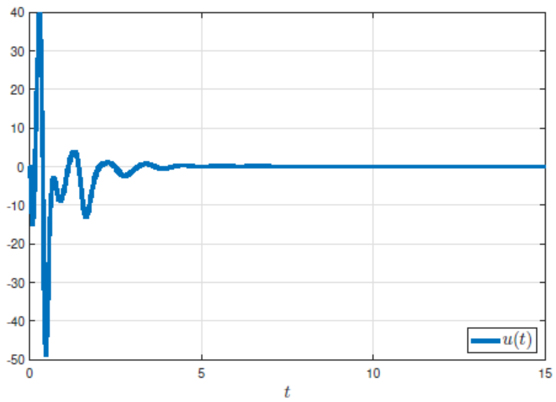

Fig. 4: Input trajectories

6. Conclusion

This paper is concerned with the robust control for nonlinear systems described by TS fuzzy models. On the basis of Lyapunov theory, the principal aspects of the proposed control scheme lie in the design of an observer to estimate the unmeasured states and the synthesis of a fuzzy controller for the nonlinear system. Moreover, sufficient conditions have been developed in terms of strict LMI, to guarantee the robust stability of the closed-loop system. Finally, the proposed design method is illustrated throughout a PMSM model.

Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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