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A Takagi-Sugeno model approach for robust fuzzy control design for trajectory tracking of non-linear systems

Sulaiman Alkaik*, Mourad Kchaw, Ahmed Al-Shammari

College of Engineering, University of Hail, Hail, Saudi Arabia

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ABSTRACT

This article investigates the robust fuzzy tracking control design for a class of uncertain nonlinear systems using the Takagi–Sugeno (TS) fuzzy models. The main purpose of this study is to design state feedback and observer-based controllers such that the closed-loop system is asymptotically stable. Based on the Lyapunov theory, sufficient conditions are derived such that the closed-loop system is robustly stable. The linear matrix inequality LMI approach is used to obtain the state-feedback and observer gains. The effectiveness of the proposed design approach is provided via numerical simulations for a pendulum system.

1. Introduction

All recently, the analysis and synthesis of nonlinear systems have known increasing interests in academic theory as well as in industrial applications. However, the research on real physical systems and processes becomes quite difficult. Due to its excellent ability to express the nonlinear systems using the fuzzy logic and linear control theories, the Takagi-Sugeno (TS) fuzzy model has received a great deal of attention in the last few decades (Takagi and Sugeno, 1985). Based on the sector non-linearity methodology (Wang et al., 1996; Tanaka et al., 1996) a nonlinear model can be exactly represented by its equivalent (TS) one. The control design is carried out using the so-called parallel distributed compensation (PDC) scheme which consists to design a linear feedback controller for each local linear model. Note that the (PDC) method combined with quadratic Lyapunov functions, provides a basis for the analysis and control design of (TS) fuzzy systems in view of powerful conventional control theory (Tuan et al., 2004; Yoneyama, 2006).

On the other hand, robustness is considered the most important requirement that should be achieved by the control system. Thus the problem of robust control of uncertain systems has received a great deal of attention. The (TS) fuzzy models used to describe non-linear systems may be affected by uncertainties that can be provided from the modeling procedure or also from the inherent uncertainties in the real system (Cao and Frank, 2000).

Accordingly, the standard approach to cope with stability and stabilization problems for (TS) fuzzy systems consists in finding common quadratic Lyapunov functions that satisfy sufficient conditions, guaranteeing stability. These conditions are frequently expressed as Linear Matrix Inequality (LMI) constraints solvable through convex optimization techniques. Within this framework, very effective strategies have been suggested to overcome mathematical and numerical difficulties, promoting less conservative conditions (Ding et al., 2006; Fang et al., 2006; Kim and Lee, 2000).

On a different research front, in practice, the full system states are generally unavailable for measurement and the observers have been introduced as an interesting approach to estimate the state variables for controller design purposes. In this regard, the observer-based control output feedback control is probably well suited for feedback control, while the problem for designing observers for nonlinear systems described by (TS) fuzzy models has drawn considerable research attention with some remarkable results can be found in Lin et al. (2005), Lo and Lin (2004), and Xiaodong and Qingling (2003).

Besides, the tracking control problem is an important issue to be considered for many practical applications such as missile tracking control, robotic tracking control, and attitude tracking control of

* Corresponding Author.
Email Address: 1salkaik@gmail.com (S. Alkaik)
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Corresponding author's ORCID profile:
https://orcid.org/0000-0002-9040-0694

aircraft. However, the tracking control design is more difficult than the stabilization control design. Thus, some results about this control problem can be found in Tseng et al. (2001) and Wang and Tong (2006).

Based on the (TS) fuzzy model, the tracking control design is studied in this paper. The non-linear system is firstly described by the equivalent (TS) fuzzy model. Then, the H∞ tracking error performance is formulated, and a fuzzy observer-based controller is developed in order to reduce the tracking error. The considered fuzzy tracking control problem is cast in terms of a set of linear matrix inequality which can be solved effectively by a convex optimization technique.

2. System descriptions and preliminaries

The (TS) fuzzy dynamic model is described by fuzzy IF-THEN rules, which locally represent linear input-output relations of nonlinear systems. A continuous fuzzy model with parameter uncertainties can be described by,

R_i: If θ_1 is F_i^1 and If θ_2 is $F_i^2 \cdots$ If θ_s is F_i^s, Then

$$\begin{cases} \dot{x}(t) = A_i(t)x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^q$ are the state vector, the input vector, the output vector, respectively. A_i, B_i, and C_i are constant real matrices with appropriate dimensions; r is the number of model rules; $\theta(t) = [\theta_{1,t}, \theta_{2,t}, \dots, \theta_{p,t}]$ is the premise variable vector and $h_i(\theta(t))$ denotes the normalized membership function which satisfies $h_i(\theta(t)) \geqslant 0, i \in \mathbb{S} \triangleq \{1,2,\dots,r\}$ and $\sum_i^r h(\theta(t)) = 1$ for all t.

Assume that $A_i(t) = A_i + \Delta A_i$, is a time-varying system matrix. A_i, B_i, and C_i are constant matrices with appropriate dimensions. Parameter uncertainty ΔA_i is assumed to be of the form,

$$\Delta A_i = M_i \Delta N_i \quad (2)$$

where M_i, N_i and N_{ui} are known real constant matrices and Δ is unknown time-varying matrix function satisfying,

$$\Delta^T \Delta \leqslant I \quad (3)$$

The overall fuzzy model is inferred as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r h_i(\theta(t))\{A_i(t)x(t) + B_{2i}u(t)\} \\ y(t) = \sum_{i=1}^r h_i(\theta(t))C_{2i}x(t) \end{cases} \quad (4)$$

where $h_i(\theta)$ are the normalized weight functions defined by,

$$h_i(\theta) = \frac{\prod_{j=1}^s F_j^i(\theta_j)}{\sum_{i=1}^r \prod_{j=1}^s F_j^i(\theta_j)}, \; i = 1,2,\cdots,r$$

and $F_j^i(\theta_j)$ represents the membership degrees of θ_j in the fuzzy set F_j^i . Note that the normalized weights $h_i(\theta)$ satisfy,

$$h_i(\theta) \geqslant 0, \; i = 1,2,\dots,r \; \sum_{i=1}^r h_i h_i(\theta) = 1. \quad (5)$$

Consider the following reference model:

$$\dot{x}_r(t) = A_r x_r(t) + B_r r(t) \quad (6)$$

where $x_r(t)$ represents the reference state, A_r is an asymptotically stable matrix, $r(t)$ is a bounded reference input.

The main objective of this paper is to design a PDC fuzzy controller able to stabilize the system under consideration and guarantee the following H_∞ tracking performance according to the tracking error:

$$\int_0^\infty e_r^T(t)Qe_r(t)dt \leq \gamma^2 \int_0^\infty r^T(t)r(t)dt \quad (7)$$

where,

$$er(t) = x(t) - xr(t) \quad (8)$$

Lemma 2.1: (Petersen, 1987) Given matrices M, N, and P of appropriate dimensions and with P symmetrical, then,

$$P + M\Delta N + N^T F^T(t) M^T < 0 \quad (9)$$

for any Δ satisfying $\Delta^T\Delta \leq I$, if and only if there exists a scalar $\epsilon > 0$ such that,

$$P + \epsilon MM^T + \epsilon^{-1} N^T N < 0 \quad (10)$$

Lemma 2.2: For given matrices P $>$ 0, S, and R of appropriate dimensions the following two inequalities are equivalent,

$$-P + R^T S^{-1} R < 0 \Leftrightarrow \begin{bmatrix} -P & S^T \\ * & -R \end{bmatrix} < 0 \quad (11)$$

Lemma 2.3: The following inequality holds (Lo and Lin, 2004):

$$\sum_{i=1}^r \sum_{j=1}^r h_i h_j \Upsilon_{ij} < 0 \quad (12)$$

if

$$\Upsilon_{ii} < 0, i = 1,2,\cdots,r \quad (13)$$

$$\frac{2}{r-1}\Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} < 0, \; j > i \quad (14)$$

For the purpose of control design, we consider different feedback schemes including fuzzy state feedback and fuzzy observer-based state feedback.

3. State-feedback tracking controller design

In this section, we develop a procedure to design a state-feedback controller to achieve the objective control requirement when all state variables are available for measurement.

$$u(t) = \sum_{i=1}^r h_i(t)(K_{1i}x(t) + K_{2i}x_r(t)) \quad (15)$$

where K_{1i} and K_{2i} are controller gains to be designed.

The closed-loop system of 4 and 15 is given by,

$$\dot{\bar{x}}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j (\mathcal{A}_{ij}(t)\bar{x}(t) + \mathcal{B}r(t)) \quad (16)$$

where $\bar{x} = [x(t)x_r(t)]^T$,

$$\begin{aligned} \mathcal{A}_{ij}(t) &= \mathcal{A}_{ij} + \Delta\mathcal{A}_{ij}(t), \Delta\mathcal{A}_{ij}(t) = \bar{M}_i \Delta \bar{N}_i \\ \mathcal{A}_{ij} &= \begin{bmatrix} A_i + B_{2i}K_{1j} & B_{2i}K_{2j} \\ 0 & A_r \end{bmatrix} \bar{M}_i = \begin{bmatrix} M_i \\ 0 \end{bmatrix} \\ \bar{N}_i &= [N_i 0] \mathcal{B} = \begin{bmatrix} 0 \\ B_r \end{bmatrix} \end{aligned} \tag{17}$$

Moreover, the H_∞ tracking performance can be written as,

$$\int_0^\infty \bar{x}^T(t)\bar{Q}\bar{x}(t)dt \leq \gamma^2 \int_0^\infty r^T(t)r(t)dt, \bar{Q} = \begin{bmatrix} Q & -Q \\ -Q & Q \end{bmatrix} \tag{18}$$

Theorem 1: Closed-loop fuzzy system (16) is robustly stable if there exists a matrix $P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} > 0$ such that,

$$\Upsilon_{ii} < 0, i \in \mathbb{S} \tag{19}$$
$$\frac{2}{r-1}\Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} < 0,\ j > i \tag{20}$$

where,

$$\Upsilon_{ij} = \begin{pmatrix} P\mathcal{A}_{ij} + \mathcal{A}_{ij}^T P & \mathcal{B} & P\bar{M}_i & \bar{N}_i^T \\ * & -\gamma^2 I & 0 & 0 \\ * & * & -\epsilon^{-1} I & 0 \\ * & * & * & -\epsilon I \end{pmatrix} \tag{21}$$

Proof: Under the conditions of the theorem, we first establish the stability of the system in 16. In the sequel, we choose a Lyapunov function candidate for system 16 as follows:

$$V(\bar{x}(t)) = \bar{x}^T(t)P\bar{x}(t) \tag{22}$$

Then, the derivative of Lyapunov function (22) gives:

$$\dot{V}(\bar{x}(t)) = 2\bar{x}^T(t)P\dot{\bar{x}}(t) \tag{23}$$

Considering (16), we get

$$\dot{V}(\bar{x}(t)) = 2\bar{x}^T(t)P \sum_{i=1}^r \sum_{j=1}^r h_i h_j \big(\mathcal{A}_{ij}(t)\bar{x}(t) + \mathcal{B}r(t)\big) \tag{24}$$

Moreover, we have,

$$\dot{V}(\bar{x}(t)) + \bar{x}^T(t)\bar{Q}\bar{x}(t) - \gamma^2 r^T(t)r(t) \tag{25}$$
$$= 2\bar{x}^T(t)P \sum_{i=1}^r \sum_{j=1}^r h_i h_j \big(\mathcal{A}_{ij}(t)\bar{x}(t) + \mathcal{B}r(t)\big) + \bar{x}^T(t)\bar{Q}\bar{x}(t) - \gamma^2 r^T(t)r(t) \tag{26}$$

which can be written as,

$$\dot{V}(\bar{x}(t)) + \bar{x}^T(t)\bar{Q}\bar{x}(t) - \gamma^2 r^T(t)r(t)$$
$$\sum_{i=1}^r \sum_{j=1}^r h_i h_j \begin{bmatrix} P\mathcal{A}_{ij}(t) + \mathcal{A}_{ij}(t)^T P + \bar{Q} & \mathcal{B} \\ * & -\gamma^2 I \end{bmatrix} \tag{27}$$

Then, according to Lemma 2.3, the conditions in Theorem 1 are equivalent to,

$$\sum_{i=1}^r \sum_{j=1}^r h_i h_j \Upsilon_{ij} \tag{28}$$

$$= \sum_{i=1}^r \sum_{j=1}^r h_i h_j \begin{pmatrix} P\mathcal{A}_{ij} + \mathcal{A}_{ij}^T P & \mathcal{B} & P\bar{M}_i & \bar{N}_i \\ * & -\gamma^2 I & 0 & 0 \\ * & * & -\epsilon^{-1} I & 0 \\ * & * & * & -\epsilon I \end{pmatrix} < 0 \tag{29}$$

Equivalently, the following inequality holds according to Lemma 2.1,

$$\sum_{i=1}^r \sum_{j=1}^r h_i h_j \begin{pmatrix} P\mathcal{A}_{ij}(t) + \mathcal{A}_{ij}(t)^T P + \bar{Q} & \mathcal{B} \\ * & -\gamma^2 I \end{pmatrix} < 0 \tag{30}$$

Thus, it is easy to verify that,

$$\dot{V}(\bar{x}(t)) + \bar{x}^T(t)\bar{Q}\bar{x}(t) - \gamma^2 r^T(t)r(t) < 0 \tag{31}$$

Integrating 33 from $t = 0$ to $t = \inf$ gives,

$$\int_0^\infty \dot{V}\big(\bar{x}(t)\big) + \bar{x}^T(t)\bar{Q}\bar{x}(t) - \gamma^2 r^T(t)r(t) = V(\infty) - V(0) + \int_0^\infty \bar{x}^T(t)\bar{Q}\bar{x}(t) - \gamma^2 r^T(t)r(t) < 0 \tag{32}$$

Hence, under zero initial condition, we get,

$$\int_0^\infty \bar{x}^T(t)\bar{Q}\bar{x}(t)dt < -\gamma^2 \int_0^\infty r^T(t)r(t)dt \tag{33}$$

and the H_∞ criterion is verified.

Theorem 2: Consider the fuzzy system 4 and the PDC fuzzy controller 15. If there exist matrices $P > 0$, Y_{1i} and Y_{2i} such that the following LMI is verified,

$$\bar{\Upsilon}_{ii} < 0, i \in \mathbb{S} (34)$$
$$\frac{2}{r-1}\bar{\Upsilon}_{ii} + \bar{\Upsilon}_{ij} + \bar{\Upsilon}_{ji} < 0,\ j > i \tag{35}$$

where,

$$\bar{\Upsilon}_{ij} = \begin{pmatrix} \bar{\Upsilon}_{11ij} + \bar{\Upsilon}_{11ij}^T & \bar{\Upsilon}_{12ij} & \epsilon\bar{M}_i & \bar{\Upsilon}_{14i}^T \\ * & -\gamma^2 I & 0 & 0 \\ * & * & -\epsilon I & 0 \\ * & * & * & -\epsilon I \end{pmatrix} \tag{36}$$

where,

$$\begin{aligned} \bar{\Upsilon}_{11ij} &= \begin{bmatrix} A_i X_1 + B_{2i}Y_{1j} & B_{2i}Y_{2j} \\ 0 & A_r X_2 \end{bmatrix} \\ \bar{\Upsilon}_{12ij} &= \begin{bmatrix} 0 \\ X_2 B_r \end{bmatrix} \\ \bar{\Upsilon}_{14i} &= [X_1 N_i 0] \end{aligned} \tag{37}$$

then, the closed-loop system (4) is robustly stable, and the feedback gains are given by $K_{1i} = Y_{1i}X_1^{-1}$, and $K_{2i} = Y_{2i}X_2^{-1}$

Proof: Let $X = P^{-1}$. By performing the congruence transformation to 19, and 20 by diag (X, I, I, I), inequalities 37 and 38 hold by setting $Y_{1i} = K_{1i}X_1$, and $Y_{2i} = K_{2i}X_1$

4. Observer-based controller

The establishment of a PDC control law requires the measurement of the state vector. As this condition is rarely verified in practice, the use of a fuzzy observer is necessary for this case. The observer shares the same fuzzy sets as the model taken into account. The fuzzy observer is given by the following model,

$$\begin{cases} \dot{x}_c(t) & = \sum_{i=1}^{r} h_i(\theta(t))\{A_i(t)x_c(t) + B_{2i}u(t) \\ & +L_i(y(t) - y_c(t))\} \\ y_c(t) & = \sum_{i=1}^{r} h_i(\theta(t))C_{2i}x_c(t) \end{cases} \quad (38)$$

where $x_c(t)$ is the state estimation of $x(t), y_c$ is the observer output, $L_i \in \mathbb{R}^{n\times q}$ and are the observer gain matrices. Suppose the following control law is used:

Let us denote the estimation errors as

$$e(t) = x(t) - x_c(t). \quad (39)$$

By differentiating 43, we obtain,

$$\dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j\{(A_i - L_iC_{2i})e(t) + \Delta A_i x(t)\}. \quad (40)$$

Suppose the following control law is used:

$$u(t) = \sum_{i=1}^{r} h_i(K_{1i}x_c(t) + K_{2i}x_r(t)). \quad (41)$$

The closed-loop system of 4 and 46 is shown as fol. lows:

$$\{\dot{\tilde{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j\{\tilde{\mathcal{A}}_{ij}(t)\tilde{x}(t)\}. \quad (42)$$

where,

$$\tilde{x}(t) = [x(t)e(t)x_r(t)]^T$$
$$\tilde{\mathcal{A}}_{ij}(t) = \tilde{\mathcal{A}}_{ij} + \tilde{\mathcal{M}}_{ij}\Delta\tilde{\mathcal{N}}_{ij}$$
$$\tilde{\mathcal{A}}_{ij} = \begin{bmatrix} A_i + B_{2i}K_{1i} & -B_{2i}K_j & B_{2i}K_{2i} \\ 0 & A_i - L_iC_{2i} & 0 \\ 0 & 0 & A_r \end{bmatrix}$$
$$\tilde{\mathcal{M}}_i = \begin{bmatrix} M_i \\ 0 \\ 0 \end{bmatrix}, \tilde{\mathcal{N}}_i = [N_i \quad 0 \quad 0]$$

Theorem 3: Closed-loop fuzzy system 47 is robustly stable, if there exist matrices $P > 0$ such that:

$$\tilde{\Upsilon}_{ii} < 0, i \in \mathbb{S} \quad (43)$$
$$\frac{2}{r-1}\tilde{\Upsilon}_{ii} + \tilde{\Upsilon}_{ij} + \tilde{\Upsilon}_{ji} < 0,\ j > i \quad (44)$$

where,

$$\tilde{\Upsilon}_{ij} = \begin{pmatrix} \mathbf{A}_{ij} + \mathbf{A}_{ij}^T & \mathcal{B} & \mathbf{M}_i & \tilde{\mathcal{N}}_i^T \\ * & -\gamma^2 I & 0 & 0 \\ * & * & -\epsilon I & 0 \\ * & * & * & -\epsilon I \end{pmatrix} \quad (45)$$

where,

$$\mathbf{A}_{ij} = \begin{pmatrix} Z_{ij}^1 & -P_2(B_{2i}K_{1j}) & P_3(B_{2i}K_{21j}) \\ * & Z_{ij}^2 & 0 \\ * & * & A_rP_3 \end{pmatrix}$$
$$Z_{ij}^1 = P_1(A_i + B_{2i}K_j)$$
$$Z_{ij}^2 = P_2A_i - F_iC_{2j} \quad (46)$$
$$\mathbf{M}_i = \begin{pmatrix} P_1M_i \\ 0 \\ 0 \end{pmatrix}$$

Proof: By following the same lines to prove Theorem 1, it is easy to verify that,

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j\big(\tilde{P}\tilde{\mathcal{A}}_{ij} + \tilde{\mathcal{A}}_{ij}^T\tilde{P} + \epsilon(\tilde{P}\tilde{\mathcal{M}}_i)(\tilde{P}\tilde{\mathcal{M}}_i)^T + \epsilon^{-1}\tilde{\mathcal{N}}_i^T\tilde{\mathcal{N}}_i\big) < 0 \quad (47)$$

By setting $\tilde{P} = \begin{bmatrix} P_1 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & P_3 \end{bmatrix}$, inequalities 48-49 hold using Lemma 2.3.

Note that, there is no effective algorithm for solving the parameters P_1, P_2, P_3, K_i, and L_i in Theorem 3. However, we can use the two-step procedure to solve them.

4.1. Design procedure

1. From Theorem 2, solve the state feedback controller Ki.
2. Substitute Ki into 48 and then solve the obtained LMIs to get $P1, P2, P3$, and Li.

5. A simulation example

We consider the following problem of balancing an inverted pendulum on a cart. The dynamic equations of motion of the pendulum are given as,

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{g\sin(x1) - amlx_2^2\sin(2x_1) - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)} + w \\ z(t) = x_1 + x_2 + 0.001u(t) \\ y(t) = x_1 + 0.01w(t) \end{cases} \quad (48)$$

where x_1 denotes the angle of the pendulum from the vertical axis, and x_2 is the angular velocity, $g = 9.8\ \mathrm{m/s^2}$ is the gravity constant, m is the mass of the pendulum, $2l$ is the length of the pendulum, $a = 1/(m+M), M$ is the mass of the cart, and u is the force applied to the cart. In this simulation, the pendulum parameters are chosen as $= 2kg, M = 8$ kg, and $2l = 1$ m. Let us consider the following fuzzy model to design a nonfragile observer-based fuzzy controller that achieves H_∞ performance.

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t))\{A_i(t)x(t) + B_{2i}u(t)\} \\ y(t) = \sum_{i=1}^{r} h_i(\theta(t))C_{2i}x(t) \end{cases} \quad (49)$$

where,

$$A_1 = \begin{bmatrix} 0 & 1 \\ 17.2941 & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 1 \\ 12.6305 & 0 \end{bmatrix}$$
$$B_{21} = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix}, B_{22} = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix},\ C_{2i} = [10],\ i = 1,2$$

We use the following membership functions,

$$h_1(x_1) = 1 - \frac{1}{1 + \exp\left(-7\left(x_1 - \frac{\pi}{4}\right)\right)}\ h_2(x_1) = 1 - h_1(x_1)$$

Assume that there are additive perturbations in the system, coefficients. We give the known parameters as,

$M_i = \begin{bmatrix} -0.1 \\ 0 \end{bmatrix}$, $N_i = [0.1 \quad 0]$

The first step is achieved by solving the LMIs in Theorem 2, a feasible solution is obtained with the corresponding control gain matrices defined as,

$$\begin{aligned} K_{11} &= [4759.72080.7] \\ K_{12} &= [-4267.3 - 1953.4] \\ K_{21} &= [70388\ 31320] \\ K_{22} &= [-64519 - 29511] \end{aligned} \quad (50)$$

Then, by substituting the gains K_i into 48-49 the obtained LMIs can be solved with the following observer gains:

$$L_1 = \begin{bmatrix} 1.9712 \\ 19.725 \end{bmatrix},\ L_2 = \begin{bmatrix} 1.4987 \\ 11.41 \end{bmatrix} \quad (51)$$

The simulation result gives a potent verification of the effectiveness of the suggested control scheme and shows its robustness in spite of the uncertainties. Fig. 1 shows trajectories of $x_1(t)$, $\hat{x}_{c1}(t)$ and $\hat{x}_{r1}(t)$ and Fig. 2 shows trajectories of $x_2(t), \hat{x}_{c2}(t)$ and $\hat{x}_{r2}(t)$.

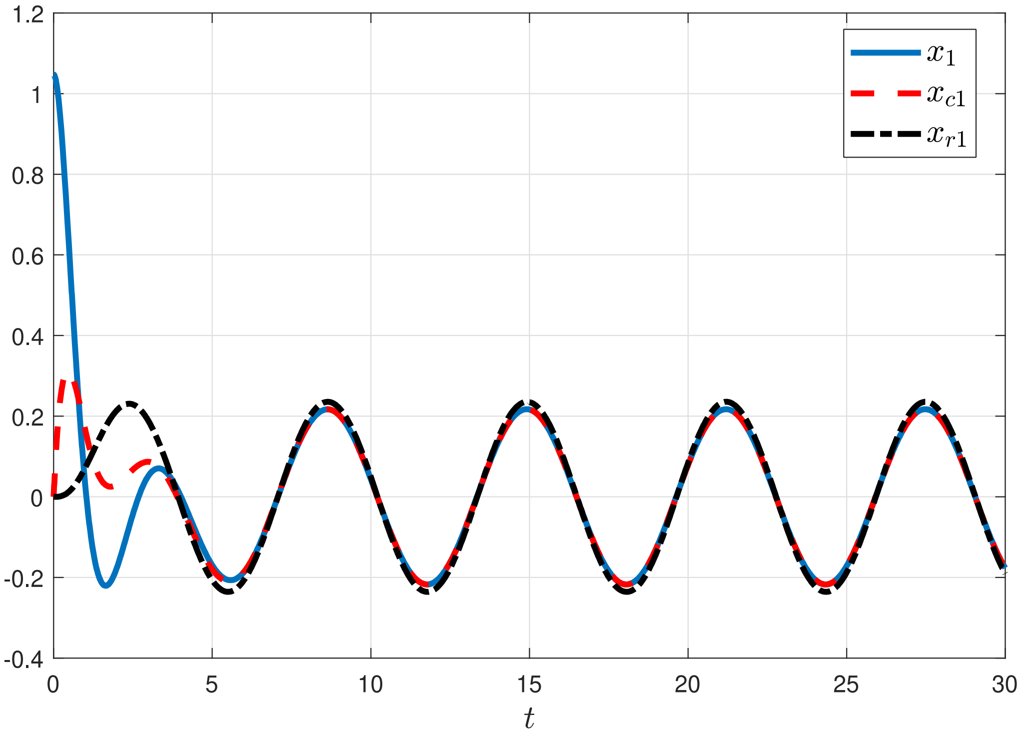

Fig. 1: Trajectories of $x_1(t), \hat{x}_{c1}(t)$ and $\hat{x}_{r1}(t)$

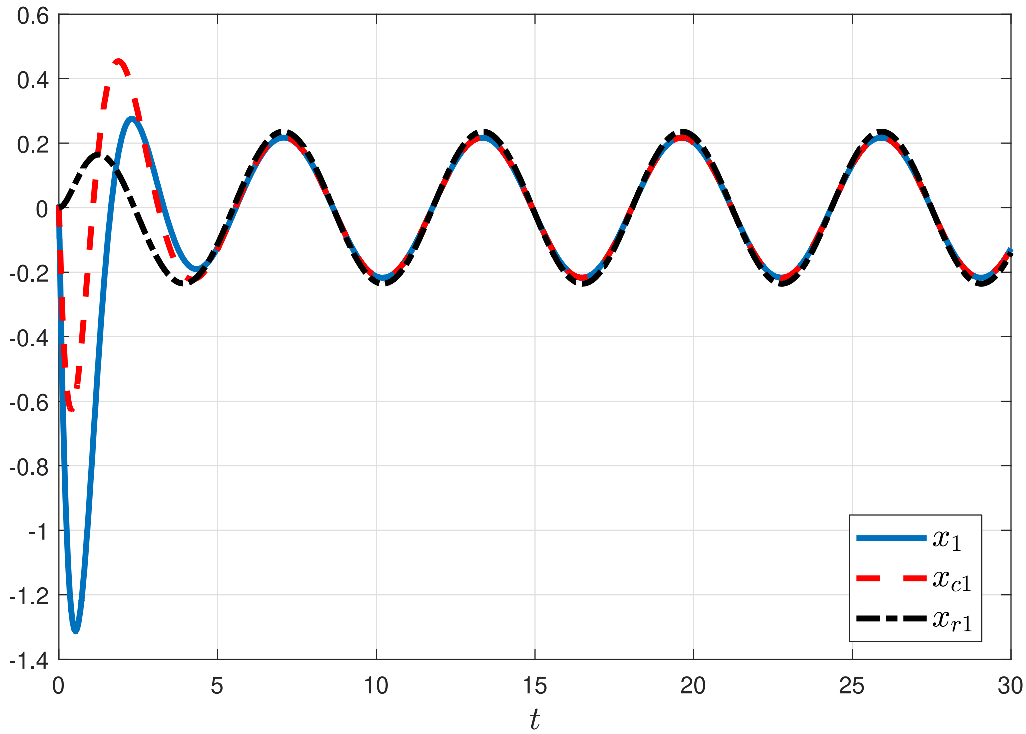

Fig. 2: Trajectories of $x_2(t), \hat{x}_{c2}(t)$ and $\hat{x}_{r2}(t)$

The simulation result gives a potent verification of the effectiveness of the suggested control scheme and shows its robustness in spite of the uncertainties.

6. Conclusion

This paper is concerned with the robust tracking control for non-linear systems described by (TS) fuzzy models. On the basis of Lyapunov theory, the principal aspects of the proposed control scheme lie in the design of an observer to estimate the unmeasured states and the synthesis of a fuzzy controller for the nonlinear system. Moreover, sufficient conditions have been developed in terms of strict LMI, to guarantee the robust stability of the closed-loop system. Finally, the proposed control scheme has been validated through numerical simulations based on the pendulum system.

Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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