

Rayleigh-Benard convection in a binary fluid-saturated anisotropic porous layer with variable viscosity effect



Nurul Hafizah Zainal Abidin^{1,*}, Norfadzillah Mohd Mokhtar², Izzati Khalidah Khalid¹, Norazam Arbin¹, Roslah Arsad¹

¹Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, Shah Alam, Malaysia

²Laboratory of Computational Sciences and Mathematical Physics, Institute for Mathematical Research (INSPEM), Universiti Putra Malaysia, Seri Kembangan, Malaysia

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ABSTRACT

Rayleigh-Benard convection due to buoyancy that occurred in a horizontal binary fluid layer saturated anisotropic porous media is investigated numerically. The system is heated from below and cooled from above. The temperature-dependent viscosity effect was applied to the double-diffusive binary fluid and the critical Rayleigh number for free-free, rigid-free, and rigid-rigid representing the lower-upper boundary were obtained by using the single-term Galerkin expansion procedure. Both boundaries are conducted to temperature. The effect of temperature-dependent viscosity, mechanical anisotropy, thermal anisotropy, Soret, and Dufour parameters on the onset of stationary convection are discussed and shown graphically.

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1. Introduction

The linear stability analysis of a binary fluid is subjected when there exist temperature and concentration gradients in a fluid. The existence of both gradients that diffuse at a different rate which is known as double-diffusive has attracted interest in convection problems. In a binary mixture, Dufour analyses the thermo-diffusion, and Soret analyses the diffusion-thermo effects on the flow. The research on this type of mixture was first reported by Nield and Kuznetsov (2011) where both stationary and oscillatory mode for a thermosolutal convective binary fluid layer induced by thermal and solutal gradients is investigated. Bergeon et al. (1998) highlighted the Soret effect and a year later Slavtchev et al. (1999) extended the research by considering the nonlinear Soret effect. Saravanan and Sivakumar (2009) examined the effects of Soret with through flow in a binary fluid layer system.

Double diffusive convection in a porous medium has been studied due to the importance in geophysics where groundwater usually contains salts in solution and hence both thermal expansion and solute concentration variations can produce

variations in density. This phenomenon was explained by Prats (1966). Early research on the onset of convection in a horizontal layer of a saturated porous heated from below was done experimentally by Morrison et al. (1949) and theoretically by Horton and Rogers (1945) and Lapwood (1948). Hirata et al. (2012) studied the small scalar process occurring in the global climate system where a natural double-diffusive was considered under-ice melt. Nield (1968) has studied the onset of double-diffusive convection in a horizontal layer of a saturated porous medium. The linear perturbation analysis was performed in both steady and oscillatory instability. The previous studies concerned homogeneous isotropic porous (Horton and Rogers, 1945; Lapwood, 1948; Nield and Bejan, 2006) and later in an anisotropy porous (Tyvand and Storesletten, 1991; Storesletten, 1993; Shivakumara et al., 2011). The properties of an isotropy material are uniform in all directions but in anisotropy, the properties of the material have a dependent direction. Hence, anisotropic porous is more complex compared to the isotropic porous.

In earlier research, fluid may be assumed to have a constant viscosity whereby we knew that viscosity might vary to other factors. Fluid may possess a temperature-dependent viscosity where the viscosity decreases exponentially with temperature (Griffiths, 1986). Temperature-dependent viscosity influences the heat transport and the spatial structure of a fluid. Few researchers studied the temperature-dependent viscosity in various types of

* Corresponding Author.

Email Address: nurul354@uitm.edu.my (N. H. Z. Abidin)

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Corresponding author's ORCID profile:

<https://orcid.org/0000-0003-4965-7444>

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problems. Torrance and Turcotte (1971) and Stengel et al. (1982) studied the temperature-dependent viscosity effect in Benard instability and Kozhoukharova and Rozé (1999) and Lam and Bayazitoglu (1987) in Marangoni instability. Arifin and Abidin (2009) and Abidin et al. (2017) have studied the effect of temperature-dependent viscosity together with others effects such as the Coriolis effect, feedback control, and Soret effect in a fluid layer. Ramirez and Saez (1990) stated that temperature-dependent viscosity should be taken into account for every case studied including in a porous medium since the effect gave a significant impact on the instability of convection. Viscosity variation in a double-diffusive nanofluid layer was studied by Yadav et al. (2013; 2017) where the results showed that the viscosity variation delayed the onset of convection.

In this research, we study the effect of temperature-dependent viscosity in a double-diffusive binary fluid layer saturated in a porous layer. To the author's best knowledge, this problem has not been reported in the literature. Three types of bounding surfaces (lower boundary-upper boundary) are considered in this investigation: free-free, rigid-free and rigid-rigid. We assume that the upper surface to be non-deformable and employ the stability analysis theory. The resulting eigenvalue problem is solved numerically using the Galerkin method.

2. Mathematical formulation

A Boussinesq binary fluid which is saturated in a horizontal porous layer with a depth d is considered in this system. The plane is infinitely extended horizontally in the x and y -direction with the vertically downward gravity force \mathbf{g} acting in it. Velocity, $\mathbf{v} = (u, v, w)$ and the density, ρ of the binary fluid are assumed to be linearly dependent upon the temperature gradient, T and the solute concentration, S .

For a Boussinesq approximation, we assumed the physical properties of the fluid are constant except the kinematic viscosity, μ , and density, ρ . These two parameters are assumed to vary upon the temperature, T , and the solute concentration, S where the equations are given by:

$$\mu = \mu_0 \exp[-\mu_t(T - T_0) + \mu_s(S - S_0)], \tag{1}$$

$$\rho = \rho_0 [1 - \alpha_t(T - T_0) + \alpha_s(S - S_0)]. \tag{2}$$

Here, μ_0 and ρ_0 are the reference values at the reference temperature, T_0 , and the reference concentration, S_0 . μ_t and α_t are the rate of change of kinematic viscosity and density with temperature. μ_s and α_s are the rate of change of kinematic viscosity and rate of change of density with concentration. The derivation will start from four governing Eqs. 3-6 used for the Rayleigh-Benard convection following the analysis by Nield and Kuznetsov (2011), Nanjundappa et al. (2013), and Malashetty and Swamy (2010).

$$\nabla \cdot \mathbf{v} = 0, \tag{3}$$

$$\frac{\rho}{\xi} \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla p + \mu \mathbf{K} \cdot \mathbf{v} + \rho \mathbf{g}, \tag{4}$$

$$\rho c \left[\eta \frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla) T \right] = \rho c D_{TC} \nabla^2 C + \nabla \cdot (D \cdot \nabla T), \tag{5}$$

$$\xi \frac{\partial C}{\partial t} + (\mathbf{v} \cdot \nabla) C = D_S \nabla^2 C + D_{CT} \nabla^2 T, \tag{6}$$

where ξ is the porosity, t is the dimensionless time, p is the pressure, μ is the kinematic viscosity, $\mathbf{K} = K_x^{-1}(\mathbf{ii} + \mathbf{jj}) + K_z^{-1}(\mathbf{kk})$ is the inverse of the anisotropic permeability tensor, \mathbf{g} is the gravity, c is the specific heat, η is the specific heat ratio, D_{TC} is the Dufour diffusivity, $D = D_x(\mathbf{ii} + \mathbf{jj}) + D_z(\mathbf{kk})$ is the anisotropic heat diffusion tensor, D_S is the solutal diffusivity and D_{CT} is the Soret diffusivity.

Each boundary wall is assumed to be thermally conducted to the temperature where the temperature is $T_0 + \Delta T$ and ΔT at the lower and upper boundary respectively. Meanwhile, the solute concentration is taken to be $S_0 + \Delta S$ and ΔS . Since the temperature and concentration change in the fluid is small, and for simplicity, the variable is marked with asterisks.

$$\nabla^* \cdot \mathbf{v}^* = 0, \tag{7}$$

$$\frac{\rho}{\xi} \left[\frac{\partial \mathbf{v}^*}{\partial t^*} + (\mathbf{v}^* \cdot \nabla^*) \mathbf{v}^* \right] = -\nabla^* p^* + \mu \mathbf{K}^* \cdot \mathbf{v}^* + \rho \mathbf{g}, \tag{8}$$

$$\rho c \left[\eta \frac{\partial T^*}{\partial t^*} + (\mathbf{v}^* \cdot \nabla^*) T^* \right] = \rho c D_{TC} \nabla^{*2} C^* + \nabla^* \cdot (D \cdot \nabla^* T^*), \tag{9}$$

$$\xi \frac{\partial C^*}{\partial t^*} + (\mathbf{v}^* \cdot \nabla^*) C^* = D_S \nabla^{*2} C^* + D_{CT} \nabla^{*2} T^*. \tag{10}$$

Infinitesimal disturbances are introduced by setting,

$$(x, y, z) = \frac{(x^*, y^*, z^*)}{d}, t = \frac{t^* \kappa}{d^2}, (u, v, w) = \frac{d(u^*, v^*, w^*)}{\alpha_f}, p = \frac{p^* d^2}{\mu \alpha_f}, T = \frac{T^* - T_0^*}{\Delta T^*}, C = \frac{C^* - C_0^*}{\Delta C^*},$$

where α_f is the thermal diffusivity of the fluid. Hence, the governing Eqs. 7-10 take the form:

$$\nabla \cdot \mathbf{v} = 0, \tag{11}$$

$$\frac{1}{\xi Pr} \left[\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right] = -\nabla p + \nabla v^2 + Ra T \hat{e}_z + Rs C \hat{e}_z, \tag{12}$$

$$\eta \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla^2 T + Df \nabla^2 C, \tag{13}$$

$$\xi_n \frac{\partial C}{\partial t} + \mathbf{v} \cdot \nabla C = \frac{1}{Le} \nabla^2 C + Sr \nabla^2 T, \tag{14}$$

where $\eta = \frac{D_x}{D_z}$ is the thermal anisotropy parameter,

$Pr = \frac{\xi v d^2}{\rho \alpha_f}$ is the Prandtl number, $\xi = \frac{K_x}{K_z}$ is the

mechanical anisotropy parameter, $Ra = \frac{\alpha g \Delta T K_z}{\mu \alpha_f}$ is the

Rayleigh number, $Sr = \frac{\alpha c g d \Delta C K_z}{\mu D_S}$ is the Solutal

Rayleigh number, $Df = \frac{D_{TC} \Delta C}{\alpha_f \Delta T}$ is the Dufour

parameter, $Le = \frac{\alpha_f}{D_S}$ is the Lewis number and $Sr =$

$\frac{D_{CT} \Delta T}{\alpha_f \Delta C}$ is the Soret parameter. Here, $\xi_n = \frac{\xi}{\eta}$ is the

normalized porosity where in this problem we set $\xi = \eta = 1$ to restrict the parameter space to the minimum (Malashetty and Swamy, 2010).

The quiescent basic state is in the form given by:

$$(u, v, w) = (0,0,0), T = T_b(z), p = p_b(z), \rho = \rho_b(z), C = C_b(z). \tag{15}$$

Eqs. 11-14 are reduced by using Eq. 15:

$$\frac{\partial p_b}{\partial z} = RaT_b + RsC_b \tag{16}$$

$$\frac{\partial^2 T_b}{\partial z^2} = -Df \frac{\partial^2 C_b}{\partial z^2} \tag{17}$$

$$\frac{1}{Le} \frac{\partial^2 C_b}{\partial z^2} = -Sr \frac{\partial^2 T_b}{\partial z^2} \tag{18}$$

In this state, we superpose perturbations on the basic solution where

$$(u, v, w, T, p, \rho, C) = \begin{bmatrix} 0 + u', 0 + v', 0 + w' \\ T_b(z) + T', p_b(z) + p' \\ \rho_b(z) + \rho', C_b(z) + C' \end{bmatrix}, \tag{19}$$

is substituted in Eqs. 11-14 and linearized by neglecting products of primed quantities. The following equations were obtained:

$$\nabla \cdot \mathbf{v}' = 0, \tag{20}$$

$$\left[\frac{1}{\partial \eta \Pr} + \left(\hat{e}_z + \frac{1}{\xi} \nabla^2 \right) \right] \mathbf{v}' = -\nabla p + \nabla \mathbf{v}' + RaT' \hat{e}_z + RsC' \hat{e}_z, \tag{21}$$

$$\eta \frac{\partial T'}{\partial t} - w' = \nabla^2 T' + Df \nabla^2 C', \tag{22}$$

$$\xi_n \frac{\partial C'}{\partial t} - w' = \frac{1}{Le} \nabla^2 C' + Sr \nabla^2 T'. \tag{23}$$

Operating Eq. 21 by eliminating the pressure term by using curl identity together with Eq. 20, Eq. 21 can be written as:

$$\frac{1}{Pr} \frac{\partial}{\partial t} \nabla^2 w' - \nabla^4 w' = RaT' \nabla_a^2 + \frac{Rs}{Le} C' \nabla_a^2. \tag{24}$$

A normal mode representation is introduced in the form:

$$(w', T', C') = [W(z), \Theta(z), \Phi(z)] e^{st+i(\alpha_x x + \alpha_y y)} \tag{25}$$

and substitute into the differential Eqs. 22-24 to obtain:

$$\bar{f} \left[\left(\frac{D^2}{\xi} - \alpha^2 \right) + \frac{s}{Pr} (D^2 - \alpha^2) \right] W + \alpha Ra \Theta - \alpha Rs \Phi = 0 \tag{26}$$

$$(D^2 - \eta \alpha^2 - s) \Theta + W + Df (D^2 - \alpha^2) \Phi = 0, \tag{27}$$

$$\frac{1}{Le} (D^2 - \alpha^2 - s) \Phi + W + Sr (D^2 - \alpha^2) \Theta = 0, \tag{28}$$

where $\alpha = (\alpha_x^2 + \alpha_y^2)^{\frac{1}{2}}$, $D = \frac{d}{dz}$ and $\bar{f}(z) = e^{B(z-\frac{1}{2})}$. α represented here is the dimensionless horizontal wavenumber and since in this research paper, we only considered the stationary mode, we now set the growth parameter, $s=0$. B is the dimensionless viscosity parameter. The average viscosity and average temperature between the upper and lower boundary are taken as the reference parameters.

To obtain the neutral stability of the convection, both boundaries are conducted to the temperature where the boundaries are as follow:

- Lower and upper boundaries are free-slip,

$$W = D^2 W = \Theta = 0. \tag{29}$$

- Lower and upper boundaries are rigid-slip,

$$W = DW = \Theta = 0. \tag{30}$$

- Lower boundary is rigid-slip and the upper-boundary is free-slip,

$$W = DW = \Theta = 0 \text{ at } z = 0, \tag{31}$$

$$W = D^2 W = \Theta = 0 \text{ at } z = 1. \tag{32}$$

The method used to find an approximate solution to the system is by Galerkin-type weighted residuals method where the three variables are written in a series of basis functions:

$$W = \sum_{n=1}^N A_n W_n, \Theta = \sum_{n=1}^N M_n \Theta_n, \Phi = \sum_{n=1}^N E_n \Phi_n \tag{33}$$

where A_n, M_n and E_n are unknown coefficients. To approximate the solutions, W_n, Θ_n , and Φ_n are chosen generally based on the lower-upper boundaries conditions, namely free-free, rigid-free and rigid-rigid.

$$W = \Theta = \Phi = \sin(z\pi), \tag{34}$$

$$W = z^2 - 2z^3 + z^4, \Theta = z(1-z), \Phi = z^2(1-z), \tag{35}$$

$$W = z^2(1-z)(3-2z), \Theta = z(1-z), \Phi = z(1-z). \tag{36}$$

Using expression in Eqs. 34-36 for W, Θ and Φ in the linearized Eqs. 26-28 as well as multiplying all equations with the base functions respectively and integrating the functions, a system of 3×3 linear algebraic equations in 3 unknowns A_n, M_n and $E_n, n = 1, 2, 3, \dots, N$ where N is the natural number is obtained. Rayleigh number, Ra act as the eigenvalue when the determinant of the coefficient matrix is vanished to obtain a system with a non-trivial solution.

Now, we perform integration by Prats (1966) with respect z between $z \in [0, 1]$. By using the boundary conditions 29-32, we obtain the system of linear homogeneous algebraic equations:

$$A_{ji} W_i + M_{ji} \Theta_i + E_{ji} \Phi_i = 0, \tag{37}$$

$$F_{ji} W_i + G_{ji} \Theta_i + H_{ji} \Phi_i = 0, \tag{38}$$

$$I_{ji} W_i + J_{ji} \Theta_i + K_{ji} \Phi_i = 0. \tag{39}$$

The above set of homogeneous algebraic equations can have a non-trivial solution if and only if the determinant of the matrix is zero.

The obtained eigenvalue which is the Rayleigh number, Ra for the lower-upper free-free boundary conditions is:

$$Ra = \frac{1}{\alpha^2 \lambda_1} \left[4 \left(-\frac{\lambda_2 \lambda_3}{2 \xi Le} - \frac{\lambda_2 \lambda_4 \lambda_5}{\xi} - \alpha^2 Rs \lambda_6 \right) Le \right], \tag{40}$$

$$\lambda_1 = Df Le \pi^2 + Df Le \alpha^2 - \pi^2 - \alpha^2,$$

$$\lambda_2 = B^2 \pi^2 \xi + B^2 \alpha^2 \xi - \alpha^2 \xi - \pi^2,$$

$$\lambda_3 = \left(-\frac{1}{2} \eta^2 \alpha^2 - \frac{1}{2} \pi^2 \right) (\pi^2 + \alpha^2),$$

$$\lambda_4 = -\frac{1}{2} \pi^2 Df - \frac{1}{2} Df \alpha^2, \tag{41}$$

$$\lambda_5 = -\frac{1}{2} \pi^2 Sr - \frac{1}{2} Sr \alpha^2,$$

$$\lambda_6 = -\frac{1}{4} \pi^2 Sr - \frac{1}{4} Sr \alpha^2 + \frac{1}{4} \eta \alpha^2 + \frac{1}{4} \pi^2.$$

3. Results and discussion

The onset of Rayleigh convection was solved in three different boundaries condition which is the rigid-rigid, rigid-free, and free-free surface

representing the lower-upper boundaries. Both boundaries were set to be conducted to temperature. When we set $\xi = \eta = 1$ and $Rs=0$ which represent the isotropic fluid, we obtain the same result as Horton and Rogers (1945) and Lapwood (1948) where the critical Rayleigh number obtained is $Ra_c=39.48$. Similar results were also obtained for an isotropic binary fluid (Nield and Bejan, 2006) and in an anisotropic binary fluid (Malashetty and Swamy, 2010). In a binary fluid-saturated anisotropic porous layer, the expression for stationary Rayleigh number is given by:

$$Ra = \frac{\pi^2 + \eta\alpha^2}{\alpha^2} \left(\frac{\pi^2}{\xi} + \alpha^2 + \frac{\alpha^2 RsLe}{\pi^2 + \alpha^2} \right), \tag{42}$$

which is obtained by Malashetty and Swamy (2010). In this paper, we extended their research by integrating the temperature-dependent viscosity effect, B into a double-diffusive binary fluid-saturated in a porous layer. Table 1 list the comparison values of the critical Rayleigh number for various boundary conditions where $Le=5$, $Rs=10$, $\xi=0.5$, $\eta=0.3$, $Sr=0$ and $Df=0$. The present study shows a good agreement with Malashetty and Swamy (2010) for free-free boundary conditions. When the solutal Rayleigh number, Rs increases, it is found that the critical Rayleigh number increased, therefore the effect of solutal Rayleigh number stabilized the system. It is also shown that rigid-rigid boundary has the highest critical Rayleigh number followed by rigid-free and free-free boundaries for any values of solutal Rayleigh number.

Table 1: The comparison of critical values of Rayleigh number Ra_c with Malashetty and Swamy (2010) for a binary fluid-saturated anisotropic porous layer in the absence of temperature-dependent viscosity ($B=0$) for various boundary conditions

Solutal Rayleigh number, Rs	Malashetty and Swamy (2010)		Present study	
	Free-free	Free-Free	Rigid-Free	Rigid-Rigid
$Rs = 10$	54.53	54.54	58.18	342.14
$Rs = 25$	86.6	86.6	90.24	394.73
$Rs = 50$	136.2	136.2	140	477.92
$Rs = 100$	229.18	229.18	233.33	629.81

In the addition of temperature-dependent viscosity within the mathematical formulation, it is shown that the marginal stability curves shift downwards as the values of temperature-dependent viscosity, B increases, thus destabilized for all wavenumbers, α . The exact figure is shown in Table 2 and graphically in Fig. 1 with $Le=5$, $Rs=10$, $\xi=0.5$, $\eta=0.3$, $Sr = 0.005$ and $Df=0.005$. We included and studied both the effect of Soret and Dufour parameters as these parameters exist in a double-diffusive system. Similar to Table 1, it shows that rigid-rigid boundary is the most stable system. However, the critical Rayleigh number for the rigid-free boundary is only higher than the free-free boundary for a temperature-dependent viscosity value less than 0.1 ($B<0.1$).

Fig. 2 shows the neutral stability curves for different values of mechanical anisotropy parameters, where the effect is to hasten the onset of convection. This is due to the mobility of the fluid in the vertical direction being accelerated when the horizontal permeability, K_x increased and the vertical permeability, K_y is fixed.

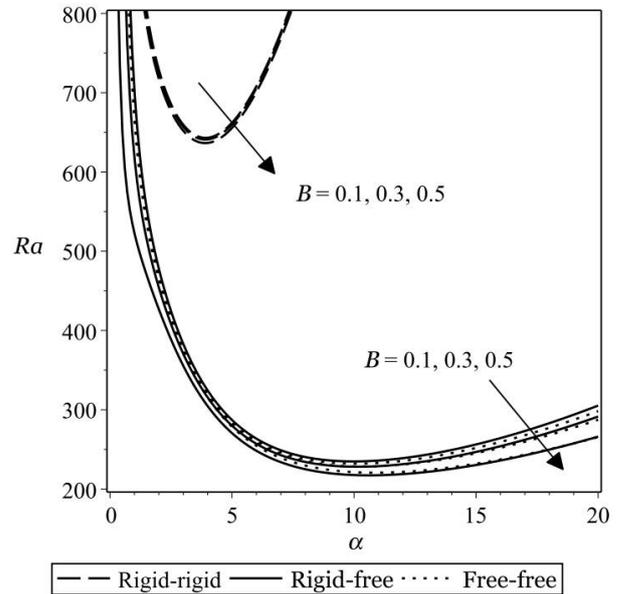


Fig. 1: Marginal stability curves for $Le=5$, $Rs=10$, $\xi=0.5$, $\eta=0.3$, $Sr=Df=0.005$ and various values of temperature dependent viscosity parameter, B

Table 2: The critical values of Rayleigh number Ra_c for a binary fluid-saturated anisotropic porous layer with temperature-dependent viscosity effect for various boundary conditions

Temperature-dependent viscosity, B	Present study		
	Free-Free	Rigid-Free	Rigid-Rigid
$B = 0.1$	55.39	57.09	350.21
$B = 0.3$	53.20	50.59	347.37
$B = 0.5$	48.71	41.68	341.64

Meanwhile, the decreasing wavenumber indicates that the cell width increases when the mechanical anisotropy, ξ increases. Fig. 3 indicates the effect of the thermal anisotropy parameter, η on the onset of convection. It is found from the rigorous investigation, the effect of thermal anisotropy is to stabilize the system since the increase in the values of the thermal anisotropy parameter increased the Rayleigh number, Ra .

The trends of stability for Soret and Dufour parameters that exist in a double-diffusive are investigated. The stability curves for these effects are shown in Figs. 4 and 5. It is clearly seen that, in Fig. 4, the Rayleigh number decreases when the Soret number, Sr increases, thus the onset of convection is advanced by the Soret parameter. Temperature flux increases when the system is heated from below and contributes to the initiation of natural convection in binary fluid mixtures. As for the Dufour parameter, Df , it is examined that from Fig. 5, the effect of the Dufour parameter contrast with the effect of the Soret parameter.

The increase in the values of the Dufour parameter increased the Rayleigh number, Ra monotonically. These findings agree well with those reports by Hurle and Jakeman (1971). The energy flux from lower to higher solute concentration is driven by the mass gradient in the binary system. The results also coincide with the results obtained in Abidin et al. (2017) where the coupled effect of Dufour and Soret were considered.

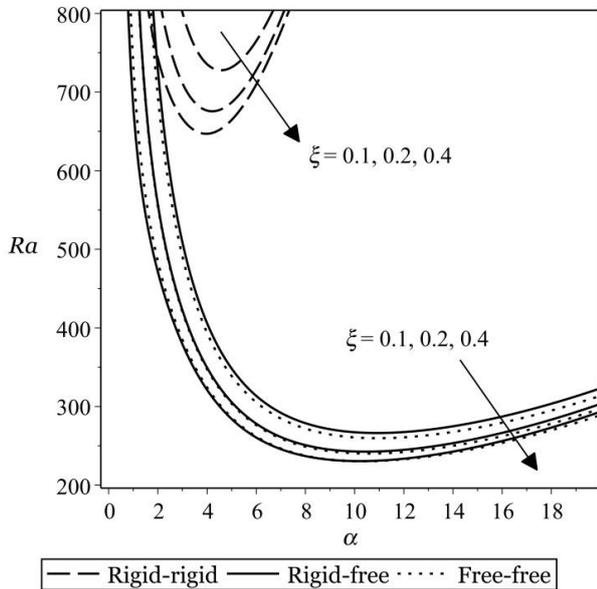


Fig. 2: Marginal stability curves for $Le=5, Rs=10, B=0.3, \eta=0.3, Sr=Df=0.005$ and various values of mechanical anisotropy parameter, ξ

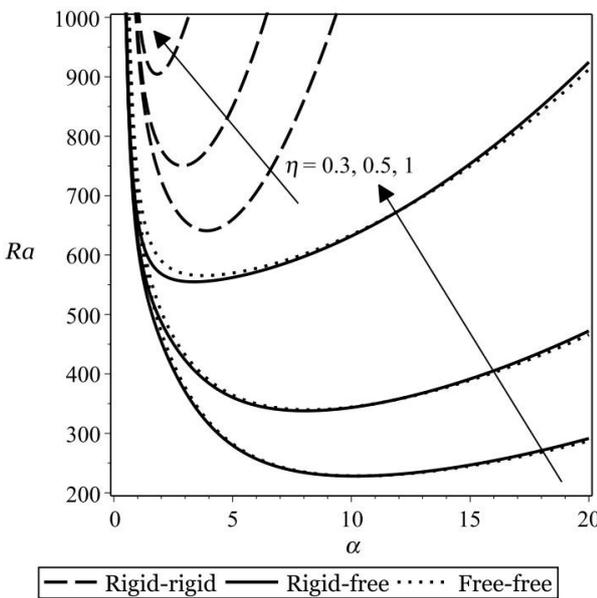


Fig. 3: Marginal stability curves for $Le=5, Rs=10, B=0.3, \xi=0.5, Sr=Df=0.005$ and various values of thermal anisotropy parameter, η

4. Conclusion

The effect of temperature-dependent viscosity in double-diffusive convection in a fluid-saturated anisotropic porous layer is examined. Boussinesq fluid is heated and cooled from below using linear stability theories. The onset double-diffusive

convection is advanced in the presence of temperature-dependent viscosity B , mechanical anisotropy parameter ξ , Soret parameter Sr and Lewis number Le . Meanwhile, in the existence effect of solutal Rayleigh number Rs , thermal anisotropy parameter η and Dufour parameter Df stabilized the onset of double-diffusive convection. Finally, the lower-upper boundary conditions of rigid-rigid are the most stable system compared to rigid-free and free-free respectively.

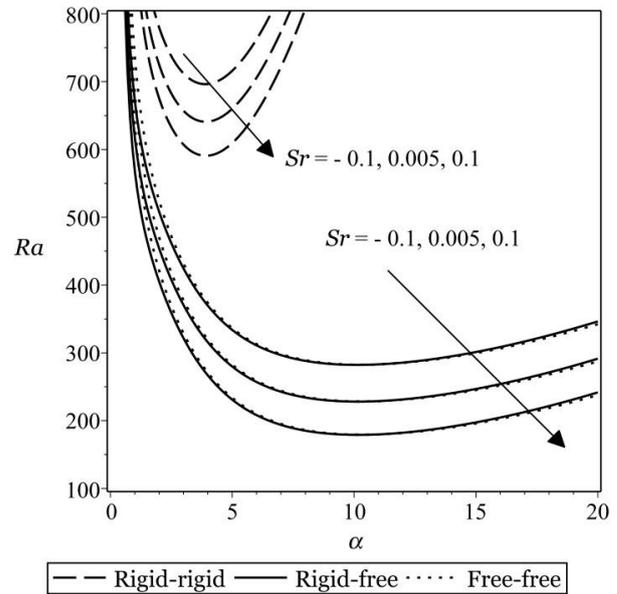


Fig. 4: Marginal stability curves for $Le=5, Rs=10, \eta=B=0.3, \xi=0.5, Df=0.005$ and various values of Soret number, Sr

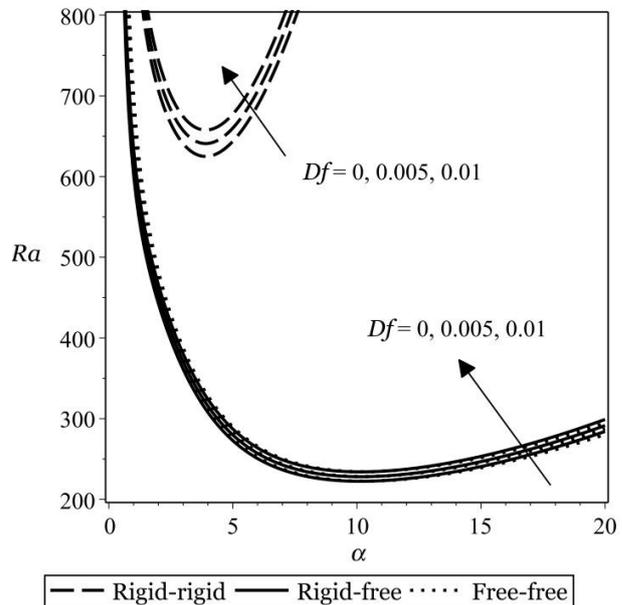


Fig. 5: Marginal stability curves for $Le=5, Rs=10, \eta=B=0.3, \xi=0.5, Sr=0.005$ and various values of Dufour parameter, Df

Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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