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Measuring stock performance using stochastic frontier analysis model with dependent error approach

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This paper focuses on analyzing the technical efficiency of Malaysian stock performance over the period of 2013 to 2018. By utilizing the stochastic frontier analysis (SFA) production function Cobb-Douglas, the inefficiency effect of time-invariant is allowed and predicted to estimate the technical efficiency score as well as provide a ranking efficiency based on the model estimation performance. In SFA, the two main errors, random error and inefficiency error are assumed to be independent, and this assumption is not practical in a real-life situation. The assumption for random error is normally distributed and the inefficiency error is half-normal distributed. Therefore, in this paper, when the assumption of SFA is dependent on both errors, the copula is applied to capture the joint distribution of these two error components. These main findings revealed that stock efficiency estimates using copula SFA (CSFA) are appropriate because it uses more practical assumptions and among the seven models, through the AIC method, the Cot copula was selected as the best model. This paper provides new evidence on comparison ranking of technical efficiency based on the three models, yielded by copulas with SFA (CSFA-Cot copula), SFA, and DEA-CCR models. Spearman's rank order was implemented and revealed that there was a high degree of correlation found among the rank efficiency estimates derived from the models of CSFA and SFA applied. However, the scores produced by both models are different. Accurate scores are necessary in order to make correct decisions and predictions. Therefore, the dependence error between random error and inefficiency error cannot be ignored, and the Cot copula in SFA models can be considered as an alternative suitable tool for measuring efficiency performance.

1. Introduction

Performance evaluation is one of the key issues highlighted by various stakeholders in a company, including business analysts, creditors, investors, and financial managers. Delen et al. (2013) described that measuring a company's performance is important to identify its success, current performance, the source of the problem, and the actions that need to be taken to address the issue taking place in the company. Low-performing companies are usually uncompetitive and may experience financial problems, while high-performing companies are able to create jobs and increase wealth (Riedl and Srinivasan, 2010). Therefore, the selection of appropriate analytical methods is important so that the performance obtained is more accurate and helps with strategic planning and better decision-making. The most used approach for measuring efficiency-based performance is Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA). Both measurements can be measured from a variety of criteria using inputs and outputs. DEA and SFA have been widely employed in a variety of disciplines as an efficiency or performance measurement tool for

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comparing a set of entities such as firms, banks, hospitals, nations, and organizations which are generally termed as Decision Making Units (DMUs). These models use frontier, and the results divide the stocks into two sets, the efficient and inefficient stocks. When the stocks operate on the output frontier, it is considered technically effective, and it is not technically efficient if it operates below the frontier. The mathematical models proposed to determine such a frontier can be broadly classified into:

1. Parametric models (e.g., SFA)
2. Non-parametric models (e.g., DEA)

In a parametric model, a functional form of the production function needs to be specified, in contrast to a non-parametric model, where specific assumptions about the form of the production function are not necessary.

When DEA which is initially introduced by Charnes et al. (1978); is a non-parametric method to evaluate the efficiency of DMUs which has no assumption about functional form for the frontier and it evaluates the performance considering various inputs and outputs simultaneously. It also does not require priori assumptions of the relationship between inputs and outputs. There are different versions of the DEA model based on its features. Based on the structure of returns to scale, there are two versions of the DEA model, namely Constant Returns to Scale (CRS) or CCR (Charnes et al., 1978) and Variable Returns to Scale (VRS) or BCC (Banker et al., 1984). In DEA, no functional relationship between production outputs and inputs is presumed, nor any unique statistical distribution of the term of error, and its ability to manage multi-input and output development cycle makes it an attractive alternative and outweighs its statistical shortcomings (Murillo-Zamorano, 2004; Mokhtar et al., 2020). Measuring the performance of companies is most important for investors and financial managers. Based on a study conducted by Gardijan and Kojić (2012), the DEA model is applied for investment purposes with the construction of a stock portfolio in the Croatian stock market. The efficiency of the DMUs, which are in this case the selected stocks from the Zagreb Stock Exchange, is obtained from the output-oriented CCR and BCC models. Besides that, Ismail et al. (2012) examined the effectiveness of the DEA model on portfolio selection for investors over a long horizon in the Malaysian stock market. They employed the technical efficiency of the DEA model to evaluate the firm's efficiency. Then, efficient firms were selected for portfolio formation.

Further, Zohdi et al. (2012) used the DEA approach for the evaluation of the performance of Iranian investment companies via financial statement analysis for the ranking of twelve companies using the CCR and BCC models.

Besides, to overcome the issue of random errors in DEA, SFA was introduced. SFA was proposed independently by Aigner et al. (1977) and Meeusen and van Den Broeck (1977). Compared to DEA, SFA separates inefficiency deviation from actual performance to frontier into two terms: One accounts for actual inefficiency, and the other accounts for random errors such as weather, politics, disease outbreaks, economics, and measurement. This also includes random errors, which means that SFA does not treat all deviations from the frontier as inefficiency. DEA simply treats any deviation from the frontier as inefficiency. However, the inefficiency may be due to random errors rather than technical inefficiency. In this regard, the distinction between actual inefficiency and random errors in SFA does have advantages in explaining possible sources of the inefficiency.

In SFA, there are two main types of errors, namely inefficiency error and random error, which need to be determined by the type of distribution before an estimate is made. Usually, the distribution for random error is normally distributed, while for inefficiency error, there have been many distribution assumptions based on previous studies as there is no specific guideline for the selection of this distribution. The common inefficiency errors are half-normal (Yang, 2010; Iliyasu et al., 2016), truncated normal (Hamidi, 2016; Hasan et al., 2012a), gamma (Greene, 1990; Ritter and Simar, 1997), and exponential distributions (Jondrow et al., 1982). The two main errors in this SFA are also assumed to be independent of each other. The SFA method is used to estimate efficiencies in different fields. Past studies such as Hasan et al. (2012a), Hasan et al. (2012b), Md et al. (2011), and Hasan and Kamil (2014) have used SFA to calculate the efficiency performance of companies' stock in Bangladesh. The inputs used in their study were market return, market capitalization, and book-to-market ratio, while the output was individual returns. Meanwhile, studies conducted by Baten et al. (2014), Wan Ahmad et al. (2010), and Janang et al. (2018); employed SFA to examine the technical efficiency of company stocks in the Malaysian stock market. The assumptions that they used for the two main errors in the SFA were independent. Recently, there have been various situations that are beyond the company's control have happened in Malaysia including the outbreak of diseases especially COVID-19, political uncertainties, and government policies which have subsequently implied the possibility of influencing the inefficiency of companies in managing their inputs well. In real life, the error has a relationship with each other's and independent assumptions are less appropriate to use. This dependency assumption will affect the estimated value of efficiency (Leurcharusmee et al., 2016), thus, giving implications to costs and inaccurate decision-making.

Therefore, the copula model was introduced by Smith (2008) to overcome the problem of error dependence in SFA. Copula modeling in SFA has been used in different fields. The copula SFA (CSFA) model assumes that two main errors, inefficiency errors and random errors are interrelated with each other. This assumption is more realistic and practical in the real world. For example, a study by Najjari et al. (2016) used this copula approach in measuring the efficiency of baseball teams in Japan, while Wiboonpongse et al. (2015) applied the CSFA model to study the efficiency of coffee production in Thailand. The CSFA model is also employed by other studies to measure the efficiency of a company's stock performance. Among them are Tibprasorn et al. (2017) and Tibprasorn et al. (2015) who measured the efficiency of a company's stock market in Thailand in terms of its efficiency to achieve an optimal current stock price based on past price and number of stocks as input variables. The type of copulas they used was Gaussian, Gumbel, t, Frank, and Clayton. Meanwhile, Tibprasorn et al. (2016) measured the efficiency of company stocks in terms of providing weekly stock returns. The copulas used were Gaussian, Frank, Clayton, and Independence.

Therefore, the CSFA model proposed in the present study aims to model the dependency between random error and inefficiency error, as well as to find the efficiency value for each selected company's stocks in Malaysia and to value stocks based on the level of efficiency which gives profit to equity holders. Furthermore, the efficiency study of a company's stock performance using the copula approach is very limited, especially in Malaysia. Past studies have also found that the two main error components (inefficiency and random effect) are dependent on each other, and the dependence of these two components can be proven mathematically and statistically using the SFA model (El Mehdi and Hafner, 2014; Najjari et al., 2016). The copula is used to connect the marginal distribution of the two errors. In addition, research on the performance of stocks in the Malaysian stock market is still lacking and this has subsequently drawn researchers' attention to carrying out a study on this area, especially on the comparison of stock rankings based on CSFA with SFA and DEA. The remainder of the paper is organized as follows. The following section provides some discussion on CSFA, SFA, and DEA. Section 3 describes the materials and methods, data sources, and variable selection. This is followed by the fourth section, which covers the result and discussion of this study, and finally, the conclusion and suggestions for future studies.

2. Copula stochastic frontier analysis

SFA decomposes the error terms into two components. One part represents random events outside of the decision-making unit's control and another part is a non-negative term that captures inefficiency. The SFA model is a parametric technique, which requires assumptions about the functional form of the production function and the distribution of the error terms. The frontier model without random components can be written as:

$$y_i = f(x_i; \beta).TE_i \quad i = 1,2,\ldots\ldots,I. \tag{1}$$

where, y_i is the observed scalar output of the producer i and x_i is a vector of N inputs used by the producer i. $f(x_i; \beta)$ is the production frontier and β is a vector of technology parameters to be estimated.

TE_i denotes the technical efficiency defined by the ratio of observed output to maximum feasible output. $TE_i = 1$ shows that the i^{th} firm obtains the maximum feasible output, while $TE_i < 1$ provides a measure of the shortfall of the observed output from the maximum feasible output. Let TE_i is a stochastic variable, so we can write it as $TE_i = exp(-u_i)$ where $u_i \geq 0$. By adding a component of random shocks (which may come from weather changes, economic adversities, or plain luck) and it is assumed to be as $exp(v_i)$ to Eq. 2, and presented as:

$$y_i = f(x_i; \beta).exp(-u_i).exp(v_i). \tag{2}$$

Now, if we also assume that $f(x_i; \beta)$ takes the log-linear Cobb-Douglas form, Eq. 2 can be written as the following:

$$lny_i = \beta_0 + \sum_n \beta_n lnx_{ni} + v_i - u_i. \tag{3}$$

where, $\varepsilon_i = v_i - u_i$ and $i = 1,2,\ldots\ldots,I$ denotes firms. v_i is the random error component, which almost always it is considered as a two-sided normally distributed variable, and u_i is the non-negative technical inefficiency error component. Together they represent a compound error term, with a specific distribution to be determined, hence the name of composed error models is often referred to. Common choices for u include the exponential, the half-normal, the truncated normal, and the gamma distributions, and for v it is typically the normal distribution. Assume that there is potential dependence between u and v, also v_i (and u_i) are independent over i (where $i = 1,2\ldots\ldots,I$).

Let G_1 and G_2 denote the distribution functions of u and v respectively, and H be the joint distribution function of u and v. Then, by the Sklar theorem, there is a copula, C_θ which satisfies in relation, $H(u,v) = C_\theta(G_1(u), G_2(v))$, and so its joint density function is as follows:

$$h(u,v) = g_1(u)g_2(v)C_\theta(G_1(u), G_2(v)) \tag{4}$$

As $\varepsilon = v - u$, by the marginal distribution of h, we get:

$$h(\varepsilon) = \int_0^{+\infty} g_1(u)\, g_2(u+\varepsilon)C_\theta(G_1(u), G_2(u+\varepsilon))du. \tag{5}$$

Replacing $\varepsilon = lny - f(x;\beta)$ in Eq. 5 gives the density of y. Using the maximum likelihood estimator (MLE) is a way to obtain a more efficient estimator of stochastic frontier models. Clearly, copulas allow us to model marginal distributions separately from their dependence structure, so we have a flexible joint distribution function, whose marginals are specified by the researcher. After estimating stochastic frontier models, we desire to calculate the technical efficiency of DMUs. This technical efficiency is defined as follows:

$$TE = E(exp\{-u\}|\varepsilon). \quad (6)$$

By using Eq. 5 and Eq. 6, the technical efficiency is presented as follows:

$$TE = \frac{1}{h(\varepsilon)} \int_{\mathbb{R}^+} exp\{-u\} h(u, \varepsilon) du. \quad (7)$$

3. Data envelopment analysis

The DEA model is used to describe how efficient the decision-making units are in transforming the inputs into outputs or outcomes. In the DEA model, the efficiency of the unit is expressed as the ratio of the sum-weighted outputs to sum-weighted inputs. The DEA model is also able to handle multiple outputs and inputs simultaneously. In the DEA approach, efficiency is the objective function value of a multi-criteria linear programming model (Murillo-Zamorano, 2004). The objective of the DEA is to determine relative performance indicators amongst productive units, considering specific groups of inputs and outputs. It is a multi-factor productivity analysis model for measuring the relative efficiencies of a homogenous set of DMUs. The efficiency score in the presence of multiple input and output factors is defined as:

$$\text{Efficiency} = \frac{\text{Weighted sum outputs}}{\text{Weighted sum inputs}} \quad (8)$$

In this study, the organization (firm) unit is identified as an efficient unit if they obtain an efficiency score of 100%. Companies that fail to achieve 100% efficiency, will be classified as an inefficient unit. The DEA-CCR model is formulated as follows: Eq. 9 is an objective function that maximizes the efficiency for k-decision-making unit (DMU). The DEA primal model in linear form can be written as follows:

$$Maximize\ h_k = \sum_{r=1}^{s} u_r y_{rk}$$
$$\text{for } DMU_k. \quad (9)$$

Subject to:

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0$$
$$j = 1,2, \dots \dots \dots, n \quad (10)$$

$$\sum_{i=1}^{m} v_i x_{ik} = 1 \quad (11)$$

$$u_r \ge 0 \quad r = 1,2, \dots \dots \dots, s \quad (12)$$

$$v_i \ge 0 \quad i = 1,2, \dots \dots \dots, m \quad (13)$$

where,
h_k: the relative efficiency (objective function) of DMU_k,
s: the number of outputs,
u_r: the weight to be determined for output r,
y_{rj}: observed magnitude of r type output for $DMU\ j$,
m: the no of inputs,
u_i: the weight to be determined for input i,
x_{ij}: observed magnitude of i-type output for $DMU\ j$,
n: no of DMU.

This primal model of DEA can solve based on input-oriented or output oriented. The model of input-oriented is used for minimizing the input at a particular level of output, however, the output-oriented is used for maximizing output at a certain input. In this study, the focus is only on the output-oriented by determining how DMU maximizes their outputs with certain inputs. Therefore, a dual model for output-oriented based on the DEA model under the constant return to scale is employed to evaluate the DMU which is defined as follows:

$$Maximize\ \phi_k \quad (14)$$

subject to

$$-x_{ki} + \textstyle\sum_{j=1}^{n} \lambda_j x_{ij} = 0 \qquad i = 1,2, \dots \dots, m \quad (15)$$
$$\phi_k y_{rk} + \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} = 0 \qquad r = 1,2, \dots \dots, s \quad (16)$$
$$\lambda_j \ge 0 \quad j = 1,2, \dots \dots, n \quad (17)$$
$$\phi_k \text{ unconstrained,}$$

where, ϕ_k is a maximum possible proportional output amount that DMU_k can produce. The technical efficiency score, θ_k for DMU_k can be defined as:

$$\theta_k = \frac{1}{\phi_k} \quad (18)$$

where, the score produced is $0 \le \theta_k \le 1$.

4. Material and methods

4.1. Data sources

This study estimated the technical efficiency of companies listed under Bursa Malaysia. The datasets included balance panel data for the years 2013-2018 (6 years). This study used secondary data which were mainly taken from the annual reports of the companies extracted from the DataStream of Thomson Reuters. The efficiency score was calculated using R-programming software, employing Benchmarking package for DEA and MATLAB software for CSFA. The 30 selected listed companies as shown in Table 1.

4.2. Variable constructions

The sample dataset was collected from the database which includes asset turnover (AT), market capitalization (MC), debt-to-equity ratio (DE), and returns on equity (ROE). There are three inputs and one output for computing the efficiency score, which is listed as follows:

- Input 1: The value of the sales and revenue of a company in relation to the value of its assets was measured by AT. It is used as a measure of the efficiency with which the assets are used to generate revenue.

• Input 2: MC shows the size of a company which is known as the fundamental determinant of different characteristics, including risk which investors are interested in.
• Input 3: DE ratio is used in this study to evaluate a company's leverage.
• Output: ROE indicates how much profit a company gained compared to the total shareholders' equity listed on balance sheets.

Three inputs and one output of this study were selected by referring to the studies and shown in Table 2.

Table 1: List of selected Malaysian companies

DMU	Companies	DMU	Companies	DMU	Companies
1	Hap Seng Consolidated Bhd	11	Air Asia Group	21	UZMA Bhd
2	Westports Holdings Bhd	12	Luxchem Corporation Bhd	22	Harbour-Link Group Bhd
3	AWC Berhad	13	My E.G Services Bhd	23	KPJ Healthcare Bhd
4	Bintulu Port Holdings Bhd	14	Pharmaniaga Bhd	24	AYS Ventures Bhd
5	Cypark Resources Bhd	15	UOA Development Bhd	25	Kumpulan FIMA Bhd
6	Deleum Bhd	16	Yinson Holdings Bhd	26	OCK Group Bhd
7	Eita Resources Bhd	17	RGB International Bhd	27	Pansar Bhd
8	Freight Management Holdings Bhd	18	Suiwah Corporation Bhd	28	Pantech Group Holdings Bhd
9	GD Express Carrier Berhad	19	Suria Capital Holdings Bhd	29	Mega First Corporation Bhd
10	George Kent (Malaysia) Bhd	20	Tiong Nam Logistics Holdings Bhd	30	CJ Century Logistics Holdings Bhd

Table 2: Variables for input and output

Variables		Value symbol	Weight symbol
Return on equity	output	y_1	u_1
Asset turnover	input	x_1	v_1
Market capitalization	input	x_2	v_2
Debt to equity ratio	input	x_3	v_3

Asset turnover has been identified in this analysis as one of the inputs and its measurement is determined by the income and profit value of a business compared to its assets. It is used as a measure of how well the resources were used to generate revenue. The reason for selecting asset turnover as one of the input variables is that the changes in the asset turnover ratio provide information on future profitability (Fairfield and Yohn, 2001). Market capitalization is very essential to estimate stock return and risk. Dias (2013) studied the roles of market capitalization by estimating the value at risk (VaR) in order to diversify the portfolio with different market capitalizations. The study by Reinganum (1983) investigated the relationship between stock return and market capitalization and found that market capitalization was an excellent indicator of a long-run rate of return. Besides that, the average portfolio return was found systematically related to market capitalization. In a study by Bhandari (1988), the expected returns on common stocks are found positively linked to the beta (risk) and firm size controlling debt per equity ratio. In another report, Mokhtar et al. (2014) identified the debt-equity ratio as one of the most significant financial ratios used by the Fuzzy Delphi Method (FDM) to measure stock performance.

4.3. Production model specification

A production function defines the relationship between the level of inputs and the resulting level of outputs. The form production function used to measure stock's efficiency is the Cobb-Douglas production model, which is expressed as:

$$ln(y)_{it} = \beta_0 + \beta_1 ln(x_1)_{it} + \beta_2 ln(x_2)_{it} + \beta_3 ln(x_3)_{it} + v_{it} - u_{it}$$

$$ln(ROE)_{it} = \beta_0 + \beta_1 ln(AT)_{it} + \beta_2 ln(MC)_{it} + \beta_3 ln(DE)_{it} + v_{it} - u_{it}. \quad (19)$$

where, subscripts i and t represent the i^{th} company (decision-making unit) for $i = 1,2, \dots \dots \dots, 30$, and t^{th} year of observation for $t = 1,2, \dots \dots, 6$. ln representing the natural logarithm, β is a vector of unknown parameters to be estimated. The v_{it} term corresponds to random error, statistical noise, measurement error, and other random events that are beyond the company's control and it is assumed to be independently and identically distributed (i.i.d) normal random variables with zero means and variances; $v_{it} \sim N(0, \sigma_v^2)$ and the u_{it} term is a non-negative random variable associated with technical inefficiency in production and is assumed to be independently and identically distributed (i.i.d). It is further assumed that v_{it} and u_{it} has independently distributed each other. Variable x_1 is denoted as assets turnover (AT), x_2 is market capitalization (MC) and x_3 is the debt-to-equity ratio (DE). For this study, half-normal distribution was chosen for the Cobb-Douglas form. When u_{it} and v_{it} are dependent, copula will use to join the distribution for both errors, and the model is called copula SFA (CSFA). For the assumption of technical inefficiency error, assuming that the technical inefficiency error has a time-invariant effect, $u_{it} = u_i$.

The technical efficiency of individual i is defined as $TE = exp(-u_i)$. The copula used is from the Archimedean group. Several copula functions were selected for this study and three copulas have a trigonometric and hyperbolic generator and they are more flexible in modeling dependence structures. The Cot copula function has a trigonometric generator and was proposed by Pirmoradian and Hamzah (2011). Also, Csch-copula and Coth-copula families have hyperbolic generators and were proposed by Hasan and Najjari (2013) and Najjari et al. (2014), respectively. Their details are provided in Table 3. Once the technical efficiency using the copula SFA model is obtained, the criterion information used in this study is Akaike Information Criterion (AIC), which can be calculated as follows:

$$AIC = (-2LL + 2m)/N \quad (20)$$

where, LL is the likelihood log for the model, m is the estimated number of parameters, and N is the sample size for the data. AIC is used to determine the best model. The model that has a small AIC value will be considered the best model.

5. Findings and discussion

This research was carried out on 30 companies in Malaysia for the years 2013-2018. The MLE method was used to estimate the parameters. The estimated parameters for SFA, copula-based SFA models, and Kendall's τ are given in Table 4. Kendall's τ is the probability of concordance minus the probability of discordance and is thus standardized to the interval $[-1,1]$ (Smith, 2008), and it has a direct relationship with the copula parameter, θ. Therefore, we could interpret the degree of association between technical inefficiency, u and random error, v based on Kendall's τ parameter value. For the Clayton copula, the relationship between the copula parameter, θ and Kendall's τ is given by $\tau = \theta/\theta + 2$. Hence, the estimated value Kendall's τ is $\tau = 0.740$, which corresponds to the parameter $\theta = 5.703$. These results indicate that there is a strong relationship between u and v. Similar to Clayton copula, Gumbel copula has a parameter $\theta = 0.5660$ with Kendall's tau value is 0.823, which also indicates a strong relationship between u and v. The strongest association is shown by the A12 family with $\tau = 0.889$.

Table 3: Specifics of the selected copula families

Copula	Generator $\ell_\theta(t)$	Kendall's τ	Interval θ
Clayton	$\frac{1}{\theta}\left(\frac{1}{t^\theta} - 1\right)$	$\frac{\theta}{\theta + 2}$	$(0, \infty)$
A12	$\left(\frac{1}{t} - 1\right)^\theta$	$1 - \frac{2}{3\theta}$	$[1, \infty)$
Gumbel	$(-lnt)^\theta$	$\frac{\theta - 1}{\theta}$	$[1, \infty)$
Csch copula	$csch(t^\theta) - csch(1)$	$\frac{\theta}{\theta + 2}$	$(0, \infty)$
Coth copula	$coth(\theta t) - coth(\theta)$	$1 + \frac{2}{\theta^2} - \frac{2}{\theta} coth(\theta)$	$[1, \infty)$
Cot copula	$cot^\theta\left(\frac{\pi t}{2}\right)$	$1 - \frac{8}{\pi^2\theta}$	$[1, \infty)$

Referring to the last column of Table 4, overall, most of Kendall's τ estimates for the model's copula, reveal the presence strong relationship between u and v. The coefficient of the parameter gamma, γ which can be estimated by $\gamma = \sigma_u^2/\sigma_u^2 + \sigma_v^2 = \lambda^2/\lambda^2 + 1$, provides information regarding the presence of efficiency in production. The parameter γ lies between o and 1, where $\gamma = 0$ specifies that all frontier deviations are due to random error (noise), while $\gamma = 1$ means all deviations are triggered by technical inefficiency (Aigner et al., 1977). For the SFA model, the value γ is 0.718. Since this value is close to one, the deviation comes mainly from technical inefficiency. The approach for determining the best model copula is by checking the information criterion (AIC or BIC) (Wiboonpongse et al., 2015). Therefore, this study used the AIC approach for determining the best model to estimate technical efficiencies for DMUs, so that the prediction of performance is more accurate and not biased. The negligence of the error dependence assumption in the SFA model will result in inaccurate parameters estimations and efficiency scores.

Table 4: The average parameters for CSFA, SFA models, and Kendall's τ for 2013-2018

Copula	γ	β_0	β_1	β_2	β_3	θ	τ
Clayton	0.518	-4.903	0.180	0.251	-0.031	5.703	0.740
A12	0.515	-5.443	0.229	0.291	-0.035	6.021	0.889
Gumbel	0.508	-5.102	0.167	0.265	-0.028	5.660	0.823
Csch	0.477	-5.066	0.169	0.262	-0.030	5.510	0.734
Coth	0.454	-5.062	0.170	0.262	-0.030	5.447	0.700
Cot	0.512	-4.984	0.175	0.256	-0.034	3.212	**0.748**
Product	0.506	-5.088	0.167	0.264	-0.028	-	-
SFA	0.718	-5.893	0.409	0.308	-0.050	-	-

Thus, based on Table 5, the calculation of the information value of the Akaike criterion (AIC) using Eq. (20) is obtained. The Cot copula was selected as the best copula model because it has the smallest AIC value and describes the relationship between u and v. So, the researchers calculated the value of Kendall τ to get the level of strength between the error u and v. So, through $1 - 8/\pi^2\, \theta$, and the estimated value of $\theta = 3.212$, making the calculated Kendall value of τ is 0.748. This explains the strong relationship between the variables u and v, and the strength between those variables is 74.8%.

In this study, the researchers also applied one of the models in the DEA which is the CCR model and SFA model to compute the efficiency score of the companies. As a reference, for the DEA-CCR, SFA, and

CSFA models, when the efficiency score is at 1, it indicates an efficient company, and a score of less than 1 indicates an inefficient company. For ease of reference, each company is referred to as DMU1, DMU30.

Table 5: AIC estimates for CSFA models for 2013-2018

	Clayton	A12	Gumbel	Csch	Coth	Cot	Product
Log L	40.041	40.148	40.026	40.013	40.036	50.064	40.042
AIC	-66.082	-66.295	-66.052	-66.026	-66.072	**-86.127**	-66.085

Therefore, based on Table 6, through the Cot copula model, the findings have identified that the five most efficient companies are DMU13 (My E.G Services Berhad), DMU5 (Cypark Resources Berhad), DMU7 (Eita Resources Berhad), DMU12 (Luxchem Corporation Berhad), and DMU10 (George Kent (Malaysia) Berhad). Meanwhile, the five lowest-performing companies are DMU23 (KPJ Healthcare Berhad), DMU18 (Suiwah Corporation Berhad), DMU14 (Pharmaniaga Berhad), DMU25 (FIMA Group Berhad), and DMU19 (Suria Capital Holdings Berhad). My E.G Services Berhad ranks the highest in efficiency performance, while KPJ Healthcare Berhad is the lowest-performing company. Compared to the other two models, namely DEA-CCR and SFA, the company with the highest performance is DMU 5 (Cypark Resources Berhad). The average efficiency for 30 companies using Cot copula for the period 2013-2018 was 56.8%. This score value is lower than the efficiency score value obtained from the SFA model (70.8%) and the DEA-CCR model (68.2%). Interestingly, through this Cot copula model, improvement information can be obtained. These companies should increase their efficiency by 43.2% (Full-average efficiency=100%-56.8%=43.2%). These obtained score values also need to be accurate because the score values can be used as input or output variables to predict the company's performance in the future.

Table 6: Efficiency score of DMUs from the year 2013 until 2018 (Output-oriented)

DMU	DEA-CCR		SFA (CD-SN)		CSFA (Cot copula)	
	Score	Ranking	Score	Ranking	Score	Ranking
1	0.560	27	0.620	21	0.448	23
2	0.884	8	0.855	8	0.685	7
3	0.885	7	0.769	11	0.586	11
4	0.605	17	0.765	12	0.520	18
5	**1**	**1**	**0.933**	**1**	0.835	2
6	0.693	13	0.811	9	0.657	8
7	0.929	4	0.869	6	0.823	3
8	0.619	16	0.747	14	0.619	10
9	0.568	25	0.602	22	0.569	16
10	0.944	2	0.889	4	0.765	5
11	0.569	23	0.563	25	0.475	20
12	0.592	19	0.802	10	0.777	4
13	0.922	5	0.916	2	**0.867**	**1**
14	0.327	29	0.522	27	0.405	28
15	0.919	6	0.659	19	0.441	24
16	0.755	11	0.861	7	0.627	9
17	0.937	3	0.898	3	0.742	6
18	0.498	28	0.481	29	0.355	29
19	0.696	12	0.683	17	0.422	26
20	0.799	9	0.871	5	0.585	12
21	0.500	24	0.584	23	0.557	17
22	0.681	14	0.757	13	0.575	15
23	0.225	30	0.400	30	0.271	30
24	0.570	22	0.575	24	0.456	21
25	0.764	10	0.558	26	0.410	27
26	0.578	21	0.680	18	0.585	13
27	0.564	26	0.494	28	0.453	22
28	0.579	20	0.723	15	0.580	14
29	0.634	15	0.704	16	0.496	19
30	0.601	18	0.648	20	0.441	25
Average score	**0.682**		**0.708**		**0.568**	

Table 7 shows the correlation rank matrix for the methods used to measure performance in this study. The pairs for the CSFA models were found to have very high correlations with each other with correlation values exceeding 0.9. Specifically, the pairs of Gumbel and Product copulas showed the same rank results with an equivalent correlation value of one. These findings are in line with the study of Najjari et al. (2016). The relationship between the CSFA and DEA-CCR model pairs was moderate with a range between 0.6 and 0.7. For the pair of the SFA model and the DEA-CCR, a strong relationship was found.

Based on Table 7, too, the findings showed that the SFA model has a very strong relationship with the CSFA models. This study used the basic DEA model, which is the output-oriented CCR model. Through this method, the evaluation of the company's efficiency performance is based on whether the company can maximize production output to provide a high-profit return by maintaining the level of available inputs (asset turnover, market capitalization, and debt to equity). This method is preferred by many researchers because it does not require any assumptions about inputs and outputs.

Table 7: Correlation matrix for company rank according to the 2013-2018 method based on average efficiency

No	Method	1	2	3	4	5	6	7	8	9
1	Clayton	1.000	0.982	0.991	0.993	0.948	0.952	0.991	0.911	0.664
2	A12		1.000	0.987	0.989	0.939	0.944	0.987	0.923	0.693
3	Gumbel			1.000	0.999	0.954	0.956	1.000	0.899	0.665
4	Csch				1.000	0.956	0.958	0.999	0.903	0.667
5	Coth					1.000	0.997	0.954	0.881	0.656
6	Cot						1.000	0.956	0.893	0.667
7	Product							1.000	0.901	0.665
8	SFA								1.000	0.835
9	CCR									1.000

Note: Numbers 1-7 are CSFA models using the Cobb-Douglas production function and the type of technical inefficiency distribution is half normal. 1: Clayton copula; 2: A12 copula; 3: Gumbel copula; 4: Csch copula; 5: Coth copula; 6: Cot copula; 7: Product copula; 8: SFA; 9: DEA-CCR

However, the DEA does not consider random factors that also contribute indirectly to company inefficiencies. This study found that the neglect of these random errors in company efficiency calculations resulted in estimates being made less accurately, either overestimated or underestimated when compared to the SFA model in this study. In the world of investment reality, a company's performance is prone to random errors or things beyond the company's control such as financial policy, financial crisis, political instability, disease outbreaks, and weather problems that may contribute to company inefficiency. The maximum output target of giving a profit return to the company's investors is also affected by the presence of these problems. When these factors are not considered in determining inefficiencies, it will result in the measured estimates being less accurate. These components, called random errors, need to be identified and isolated in the calculation of a company's technical inefficiencies so that accurate calculations can be obtained. This factor called random error needs to be considered in estimating company inefficiencies because the estimates are made using more realistic assumptions.

Therefore, SFA is the proposed method because this model considers random factors in the technical inefficiency of the company. However, the SFA uses the assumption that these two major errors are independent of each other making it an unrealistic assumption. This study proposes a more accurate dependency assumption to assess a company's stock efficiency. These accurate estimates can help investors plan properly and make the right decision. Based on the previous discussion the dependence of the parameters shows that the two errors, technical inefficiency, and random errors are related to each other. The value of these parameters further reinforces that the CSFA model is more suitable to be used for analyzing efficiency performance.

6. Conclusion

This study measured the technical efficiency score of the companies' stock in Malaysia in the years 2013 to 2018. Estimation of the efficiency score of the companies was based on the CSFA approach which consists of single output, return to equity and three inputs, assets turnover, market capitalization, and debt to equity ratio. This study found that the relationship between SFA and CSFA is strong. This is evidenced when these two models still maintain relatively consistent rankings with each other. Yet the efficiency scores produced by these two models were different. The importance of efficiency scores should be emphasized because, through these scores, various information and predictions can be obtained. This information is important because it can help companies make plans to formulate strategies in order to improve their respective efficiency performance. For investors and researchers, the efficiency score can be used as a variable to estimate performance. However, if the score information is inaccurate, then the estimates made will result in biased, and inaccurate results and can result in errors in strategizing and have financial implications.

This study also showed that copula could describe the dependence or relationship between two main types of errors, inefficiency errors and random errors. However, there are some constraints that are inherent in this study, the first is that the use of the SFA model is limited to one output only and the use of other financial ratios is not covered within the scope of the study. The copula model used is limited to seven types of copulas only. Therefore, further studies can use other suitable copula types to estimate efficiency. Although the efficiency calculation method using the CSFA model is complicated, the advantage of this model is a more accurate estimate with more realistic assumptions.

Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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