

Effect of modified Ohm's and Fourier's laws on magneto thermo-viscoelastic waves with Green-Naghdi theory in a homogeneous isotropic hollow cylinder

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ABSTRACT

This paper deals with the modified Ohm's law, including the temperature gradient and charge density effects, and the generalized Fourier's law, including the present current density impact, the problem of conveyance of thermal stresses and temperature in a generalized Magneto-Thermo-Viscoelastic Solid Cylinder of radius L . The formulation is applied to the generalized thermoelasticity dependent on the Green-Naghdi (G-N II) hypothesis. The Laplace change system is utilized to solve the problem. At last, the outcomes got are introduced graphically to show the impact of Magnetic Field and time and on the field variables.

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1. Introduction

A viscoelastic material is, as the name suggests, one that shows a combination of Newtonian viscous fluid and Hookean elastic solid effects. The viscous term leads to energy dissipation and the elastic term to energy storage. Amorphous polymers such as plastics and synthetic rubber, fibrous materials (e.g., silk and cellulose), glasses, ceramics, biomaterials (e.g., skin and cellulose), glasses, ceramics, biomaterials (e.g. skin and mussel), and nonmetals, in general, may frequently be considered to behave in a linear viscoelasticity manner. For a long time mechanics of deformable bodies has been based upon Hooke's law-that is, upon the assumption of linear elasticity. It was well known that most engineering materials like metals, concrete, soil are not linearly elastic or, are so within limits too narrow to cover the range of practical interest. Nevertheless, almost all routine stress analysis is still based on Hooke's law because of its simplicity.

Interactions between strain and electromagnetic fields are being endeavored on account of its various applications in various pieces of science and advancement. The advancement of magneto elasticity likewise instigates us to consider different issues of geophysics, seismology, and related themes. Without delving into the subtleties of the exploration work distributed so far in the fields of magneto elasticity, magneto-thermo-elasticity, magneto-thermo viscoelasticity we notice some ongoing papers. Acharya and Roy (1978) examined the magneto-thermoelastic surface waves in initially stressed conducting media. Chaudhary et al. (2004) researched the reflection/Transmission of plane SH wave through a self-reinforced elastic layer between two half-spaces. Plane SH-wave response from elastic slab interposed between two different self-reinforced elastic solids studied by Chaudhary et al. (2006). Examined the surface waves in magnetoelastic initially stressed conducting media been illustrated by De and Sengupta (1971). Othman and Song (2006) examined the effect of rotation on the reflection of magneto-thermoelastic waves under thermoelasticity without energy dissipation. Magneto-thermoelastic wave propagation at the interface between two micropolar viscoelastic media examined by Song et al. (2006). Tianhu et al. (2004) contemplated a two-dimensional generalized thermal shock problem for a half-space in

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electromagneto-thermoelasticity. Verma et al. (1988) examined the magneto-elastic transverse surface waves in self-reinforced elastic solids. Effect of the rotation on an infinite generalized magneto-thermoelastic diffusion body with a spherical cavity concentrated by Abd-Alla et al. (2011). Bayones (2012) contemplated the influence of diffusion on the generalized magneto-thermo-viscoelastic problem of homogenous isotropic material. Viscoelastic materials are those for which the connection among anxiety relies upon a time. All materials display some viscoelastic reaction. In like manner metals, for example, steel, aluminum, copper and so on. Abd-Alla and Abo-Dahab (2009) explored time-harmonic sources in a generalized magneto-thermo-viscoelastic continuum with and without energy dissipation. Roychoudhuri and Banerjee (1998) researched the magneto-thermoelastic interactions in an infinite viscoelastic cylinder of temperature rate-dependent material subjected to a periodic loading. Spherically symmetric thermo-viscoelastic waves in a viscoelastic medium with a spherical cavity are talked about by Banerjee and Roychoudhuri (1995). The problem of magneto-thermo-viscoelastic interactions in an unbounded body with a spherical cavity subjected to a periodic loading is examined by Abd-Alla et al. (2004).

Linear viscoelasticity has been a significant zone of research since the period of Maxwell, Boltzmann, Voigt, and Kelvin. Important data with respect to the linear viscoelasticity hypothesis might be gotten in the books of Gross (1968) and Staverman et al. (1956). Many research like Biot (1955), Gurtin and Sternberg (1962), Liioushin and Pobedria (1970), and Tanner (1988) have contributed prominently to Thermo viscoelasticity.

The Kelvin-Voigt model is one of the naturally visible mechanical models regularly used to portray the viscoelastic conduct of material. This model speaks to the deferred elastic reaction exposed to stress when the deformation is time subordinate however recoverable. The dynamic interaction of thermal and mechanical fields in solids has incredible handy applications in present-day aviation atomic reactors and high vitality molecule quickening agents, for instance. Biot (1956) defined the coupled thermoelasticity theory to wipe out the conundrum inalienable in the old-style uncoupled hypothesis that elastic deformation has no impact on the temperature. The field equations for both the hypotheses are of a mixed parabolic-hyperbolic type, which foresees limitless paces for thermoelastic signs, in spite of physical perceptions. Hetnarski and Ignaczak (1999) analyzed five generalizations to the coupled theory of thermoelasticity.

The first generalization is because of Lord and Shulman who planned the generalized thermoelasticity theory including one thermal relaxation time. This hypothesis is alluded to as the L-S hypothesis or broadened thermoelasticity in which the Maxwell Cattaneo law replaces the Fourier law of heat conduction by presenting a solitary parameter that goes about as a relaxation time.

The second generalization to the coupled thermoelasticity hypothesis is because of Green and Lindsay (1972), called the G-L hypothesis or the temperature-rate dependent theory, which includes two relaxation times.

The third generalization to the coupled thermoelasticity is known as low-temperature thermoelasticity presented by Hetnarski and Ignaczak (1996), called H-I theory. This model is portrayed by a system of non-linear field equations. Low-temperature non-linear models of heat conduction foresee wave-like thermal signals and which are assumed and concentrated in certain works by Kosiński (1989).

The fourth generalization to the coupled theory is worried about the thermoelasticity theory without energy dissipation presented by Green and Naghdi (1991), alluded to as the G-N hypothesis of type II in which the old-style Fourier law is supplanted by heat flux rate-temperature gradient relation. The heat transport equation doesn't include a temperature-rate term and in that capacity, this model concedes undamped thermoelastic wave in thermoelastic material. With regards to linearized adaptation of this hypothesis, a hypothesis on the uniqueness of arrangement has been built up by Chandrasekharaiah (1986). The fourth generalization of the thermoelasticity hypothesis created by Green and Naghdi additionally includes a heat conduction law, which the conventional law and one that involves the thermal displacement gradient among the constitutive variables. This model is alluded to as the GN model III, which includes the dispersal of vitality as a rule and concedes damped thermoelastic waves.

The fifth generalization to the thermoelasticity theory is known as the dual-phase lag thermoelasticity created by Tzou (1995) and Chandrasekharaiah and Krivoshapko (1998). Tzou (1995) considered micro-structural effects into the delayed response in time in the macroscopic formulation by taking into account that the increase of the lattice temperature is delayed to phonon-electron interactions on the macroscopic level. A macroscopic lagging (or delayed) response between the temperature gradient and the heat flux vector seems to be a possible outcome due to such progressive interactions. Tzou (1995) introduced two-phase lags to both the heat flux vector and the temperature gradient and considered a constitutive equation to describe the lagging behavior in the heat conduction in solids. Here the classical Fourier law is replaced by an approximation to a modification of the law with two different translations for the heat flux vector and the temperature gradient.

Many Applications of the state space approach developed for a different type of problems in thermoelasticity (Ezzat and El-Bary 2009; Youssef et al., 2014; 2017; Ezzat et al., 2015; 2017; El-Bary and Atef, 2016a; 2016b; Amin et al., 2018a; 2018b; Belbachir et al., 2020).

2. Basic governing equations

The governing equations in the context of the theory of generalized thermo viscoelasticity with magnetic field for the isotropic and homogeneous elastic medium are considered as:

- The equation of motion:

$$\sigma_{ij,i} + F_i = \rho \ddot{u}_i \tag{1}$$

where, $F = \mu_o J \times H$

- Heat conduction equation:

$$K\theta_{,ii} = \rho C_E \dot{\theta} + \gamma_e T_o \left(1 + \gamma_o \frac{\partial}{\partial t}\right) \ddot{\theta} + \pi_o \text{div } J \tag{2}$$

- Constitutive relations:

$$\sigma_{ij} = 2 \left(\mu_e + \mu_v \frac{\partial}{\partial t}\right) e_{ij} + \left(\lambda_e + \mu_v \frac{\partial}{\partial t}\right) e \delta_{ij} - \gamma \theta \delta_{ij} \tag{3}$$

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}). \tag{4}$$

We take the linearized Maxwell's equations governing the electromagnetic field for a perfectly conducting medium as,

$$\text{curl } h = J + \epsilon_o \frac{\partial E}{\partial t}, \tag{5}$$

$$\text{curl } E = -\mu_o \frac{\partial h}{\partial t}, \tag{6}$$

$$E = -\mu_o \left(\frac{\partial u}{\partial t} \times H\right) + k_o \text{grad } \theta, \tag{7}$$

$$\text{div } h = 0. \tag{8}$$

We derive the analytical formulation of the problem in cylindrical coordinates (r, ϕ, z) with the z-axis coinciding with the axis of the cylinder. We consider the strain axis symmetrical about the z-axis. We have only the radial displacement $u_r = u(r, t)$, the circumferential displacement $u_\theta = 0$ and the longitudinal displacement $u_z = 0$.

we consider a perfectly conducting isotropic homogeneous generalized thermo viscoelastic cylinder subjected to a constant magnetic field $H(0, H_o, 0)$ which produce an induced magnetic field $h(0, h_o, 0)$ and induced electric field $E(0, 0, E)$. We assume one-dimensional motion for which all the field quantities are functions of r and t .

The displacement components take the form,

$$u_r = u(r, t), u_\theta = u_z = 0. \tag{9}$$

The strain components become,

$$e = \frac{\partial u}{\partial r} + \frac{u}{r}. \tag{10}$$

The components of magnetic field vectors are,

$$H_r = 0, H_\theta = H_o, H_z = 0. \tag{11}$$

The electric intensity vector E is parallel to the current density vector J . Hence components of E and J are given by,

$$E_r = E_z = 0, E_\theta = E \tag{12}$$

$$J_r = J_\theta = 0, J_\phi = J. \tag{13}$$

Now, the Maxwell's Eqs. 6-9 provide the following results:

$$E = \mu_o H_o \frac{\partial u}{\partial t} + \frac{k_o \partial \theta}{\mu_o \partial r}, \tag{14}$$

$$h = -H_o \left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) - \frac{k_o \partial \theta}{\mu_o \partial r}, \tag{15}$$

$$J = -\frac{\partial h}{\partial r} + \frac{k_o \partial \theta}{\mu_o \partial r}. \tag{16}$$

Using Eq. 11 and Eqs. 14-16 into the relation,

$$F = \mu_o J \times H$$

we obtain,

$$F_r = \mu_o H_o^2 \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2}\right) + k_o H_o \frac{\partial \theta}{\partial r}. \tag{17}$$

From Eq. 3 we obtain the components of the stress tensor as,

$$\sigma_{rr} = \left(\lambda_e + 2\mu_e\right) + \left(\lambda_v + 2\mu_v\right) \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) + \left(\lambda_e + \lambda_v \frac{\partial}{\partial t}\right) \frac{u}{r} - \gamma \theta, \tag{18}$$

$$\sigma_{\phi\phi} = \left(\lambda_e + \lambda_v \frac{\partial}{\partial t}\right) \left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) + 2 \left(\mu_e + \mu_v \frac{\partial}{\partial t}\right) \frac{u}{r} - \gamma \theta, \tag{19}$$

$$\sigma_{zz} = \left(\lambda_e + \lambda_v \frac{\partial}{\partial t}\right) \left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) - \gamma \theta, \tag{20}$$

$$\sigma_{r\phi} = \sigma_{r\psi} = \sigma_{\psi\phi} = 0. \tag{21}$$

Also, we arrived at:

$$\rho \frac{\partial^2 u}{\partial t^2} = \left(\lambda_e + 2\mu_e\right) + \left(\lambda_v + 2\mu_v\right) \frac{\partial}{\partial t} + \mu_o H_o^2 \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2}\right) - \gamma \frac{\partial \theta}{\partial r} + k_o H_o \frac{\partial \theta}{\partial r}. \tag{22}$$

Now, we shall use the following non-dimensional variables:

$$\hat{r} = C_1 \eta r, \hat{u} = C_1 \eta u, \hat{t} = C_1^2 \eta t, \sigma_{ij} = \frac{\sigma_{ij}}{\mu_e}, \hat{\theta} = \frac{\theta}{T_o}, \hat{h} = \frac{h}{H_o}, \hat{E} = \frac{E}{\mu_o H_o C_1}, \hat{J} = \frac{J}{\eta H_o C_1}.$$

Eqs. 14-16 and Eqs. 18-22 take the following form (dropping the primes for convenience).

$$J = \frac{\partial e}{\partial r} + A \frac{\partial \theta}{\partial r} \tag{23}$$

$$h = -\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) - A \frac{\partial \theta}{\partial r} \tag{24}$$

$$E = \frac{\partial u}{\partial t} + A \frac{\partial \theta}{\partial r} \tag{25}$$

$$\sigma_{rr} = \left(1 + a \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial r} + \left(b_1 + b_2 \frac{\partial}{\partial t}\right) \frac{u}{r} - \theta, \tag{26}$$

$$\sigma_{\phi\phi} = \left(1 + a \frac{\partial}{\partial t}\right) \frac{u}{r} + \left(b_1 + b_2 \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial r} - \theta, \tag{27}$$

$$\sigma_{zz} = \left(b_1 + b_2 \frac{\partial}{\partial t}\right) \left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) - \theta \tag{28}$$

$$\sigma_{r\phi} = \sigma_{r\psi} = \sigma_{\psi\phi}, \tag{29}$$

$$\frac{\partial^2 u}{\partial t^2} = \left(1 + A + R_H + a \frac{\partial}{\partial t}\right) \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2}\right) - \frac{\partial e}{\partial r} \tag{30}$$

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} = \left(1 + \tau_o \frac{\partial}{\partial t}\right) \frac{\partial \theta}{\partial t} + \varepsilon \left(1 + \tau_o \frac{\partial}{\partial t}\right) \left(\frac{\partial u}{\partial r} + \frac{u}{r}\right). \tag{31}$$

where,

$$C_1^2 = \frac{\lambda_e + 2\mu_e}{\rho}, C_2^2 = \frac{\lambda_v + 2\mu_v}{\rho}, a = \frac{C_2^2}{K}, b_1 = \frac{\lambda_e}{\rho C_1^2}, b_2 = \frac{\lambda_v}{\rho K}, \varepsilon = \frac{\gamma^2 \theta_o}{\rho^2 C_E C_1^2}, R_H = \frac{\mu_o H_o^2}{\lambda_e + 2\mu_e}, A = \frac{k_o}{\mu_o H_o}.$$

3. Laplace transform domain

Taking the Laplace transform of Eqs. 23-31 by using homogeneous initial conditions, defined and denoted as,

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t) dt \quad , s > 0$$

we obtain,

$$\bar{j} = \frac{\partial \bar{e}}{\partial r} + A \frac{d\bar{\theta}}{dr} \tag{32}$$

$$\bar{h} = -\left(\frac{\partial \bar{u}}{\partial r} + \frac{\bar{u}}{r}\right) - A \frac{d\bar{\theta}}{dr} \tag{33}$$

$$\bar{E} = s\bar{u} + A \frac{d\bar{\theta}}{dr} \tag{34}$$

$$\frac{d^3 \theta}{dr^3} + \frac{1}{r} \frac{d^2 \theta}{dr^2} - \frac{1}{r^2} \frac{d\bar{\theta}}{dr} = C_{21}\bar{u} + C_{22} \frac{d\bar{\theta}}{dr} \tag{35}$$

$$\frac{d^2 \bar{u}}{dr^2} + \frac{1}{r} \frac{d\bar{u}}{dr} - \frac{\bar{u}}{r^2} = C_{11}\bar{u} + C_{12} \frac{d\bar{\theta}}{dr} \tag{36}$$

$$\sigma_{rr} = (1 + as) \frac{d\bar{u}}{dr} + (b_1 + b_2s) \frac{\bar{u}}{r} - \theta \tag{37}$$

$$\sigma_{\phi\phi} = (1 + as) \frac{\bar{u}}{r} + (b_1 + b_2s) \frac{d\bar{u}}{dr} - \theta \tag{38}$$

$$\sigma_{zz} = (b_1 + b_2s) \left(\frac{d\bar{u}}{dr} + \frac{\bar{u}}{r}\right) - \theta \tag{39}$$

where,

$$C_{11} = \frac{s^2}{1+A+as+R_H}, C_{12} = \frac{C_{11}}{s^2}, C_{21} = \varepsilon s(1+A + \tau_o s)C_{11}, C_{22} = s(1+A + \tau_o s)(1 + \varepsilon C_{12}).$$

4. Solution of the problem

Now to solve Eqs. 35 and 36 put $\phi = \frac{d\bar{\theta}}{dr}$ and $\bar{u} = \frac{dv}{dr}$ we get,

$$\nabla^4 v - (C_{11} + C_{22})\nabla^2 v + (C_{11}C_{22} - C_{21}C_{12})v = 0 \tag{40}$$

Eq. 40 can be written as,

$$(\nabla^2 - K_1^2)(\nabla^2 - K_2^2)v = 0 \tag{41}$$

where K_1^2 and K_2^2 are the roots of the characteristic equation,

$$K^4 - (C_{11} + C_{22})K^2 + (C_{11}C_{22} - C_{21}C_{12}) = 0 \tag{42}$$

The solution of Eq. 41 takes the form,

$$v = v_1 + v_2$$

where, v_1 satisfy the equation,

$$(\nabla^2 - K_1^2)v_1 = 0 \tag{43}$$

The solution of Eq. 43 takes the form:

$$v_1 = A_1 I_0(K_1 r) + B_1 I_1(K_1 r) \tag{44}$$

where, $I_0(K_1 r)$ is the Modified Bessel function of the first kind of order zero; $I_1(K_1 r)$ is the Modified Bessel function of the second kind of order zero; A_1, B_1 are parameters depending on s only to be determined from the boundary conditions.

For boundedness, we take $B_1 = 0$ then,

$$v_1 = A_1 I_0(K_1 r) \tag{45}$$

by the same way we get:

$$v_2 = A_2 I_0(K_2 r) \tag{46}$$

then,

$$v = A_1 I_0(K_1 r) + A_2 I_0(K_2 r) \tag{47}$$

finally, we get,

$$\bar{u} = A_1 K_1 I_1(K_1 r) + A_2 K_2 I_1(K_2 r) \tag{48}$$

in a similar manner, we get,

$$\bar{\theta} = A_1 I_0(K_0 r) + A_2 I_0(K_2 r) \tag{49}$$

5. Numerical inversion of Laplace transforms

In order to invert the Laplace transforms in the above equations we shall use a numerical technique based on Fourier expansions of functions. Let $\bar{g}(s)$ be the Laplace transform of a given function $g(t)$. The inversion formula of Laplace transforms states that,

$$g(t) = \frac{1}{2\pi i} \int_{d-i\infty}^{d+i\infty} e^{st} \bar{g}(s) ds$$

where d is an arbitrary positive constant greater than all the real parts of the singularities of $\bar{g}(s)$. Taking $s = d + iy$, we get,

$$g(t) = \frac{e^{dt}}{2\pi} \int_{-\infty}^{\infty} e^{iky} \bar{g}(d + iy) dy.$$

This integral can be approximated by,

$$g(t) = \frac{e^{dt}}{2\pi} \sum_{k=-\infty}^{\infty} e^{ikt\Delta y} \bar{g}(d + ik\Delta y) \Delta y.$$

Taking $\Delta y = \frac{\pi}{t_1}$ we obtain:

$$g(t) = \frac{e^{dt}}{t_1} \left(\frac{1}{2} \bar{g}(d) + \text{Re} \left(\sum_{k=1}^{\infty} e^{ikt\pi/t_1} \bar{g}(d + ik\pi/t_1) \right) \right).$$

For numerical purposes, this is approximated by the function,

$$g_N(t) = \frac{e^{dt}}{t_1} \left(\frac{1}{2} \bar{g}(d) + \text{Re} \left(\sum_{k=1}^N e^{ikt\pi/t_1} \bar{g}(d + ik\pi/t_1) \right) \right) \tag{50}$$

where N is a sufficiently large integer chosen such that,

$$\frac{e^{dt}}{t_1} \text{Re} \left(e^{iN\pi t/t_1} \bar{g}(d + iN\pi/t_1) \right) < \eta$$

where η is a reselected small positive number that corresponds to the degree of accuracy to be achieved Formula 49 is the numerical inversion formula valid for $0 \leq t \leq t_1$. In particular, we choose $t = t_1$, getting,

$$g_N(t) = \frac{e^{dt}}{t_1} \left(\frac{1}{2} \bar{g}(d) + \text{Re} \left(\sum_{k=1}^N (-1)^k \bar{g}(d + ik\pi/t_1) \right) \right)$$

6. Numerical results and discussions

The copper material was chosen for purposes of numerical evaluations and constants of the problem were taken as follows:

$$\begin{aligned}
 K &= 386 \text{ N/K S}, \alpha_T = 17.8(10)^{-5} K^{-1}, \\
 C_E &= 383.1 \text{ m}^2/K, T_o = 293K, \\
 \rho &= 8954 \text{ kg/m}^2, \mu_e = 3.86(10)^{10} \text{ N/m}^2 \\
 \lambda_e &= 7.76(10)^{10} \text{ N/m}^2, \tau_o = 0.001, \\
 \alpha_o &= 6.8831(10)^{-13}, \alpha_1 = 6.8831(10)^{-13}, R = 1, \theta_1 = 1
 \end{aligned}$$

In order to study the effect of time t and study the comparison between two theories on temperature, radial stress, displacement, and strain, we now present our results in the form of graphs (Figs. 1-8).

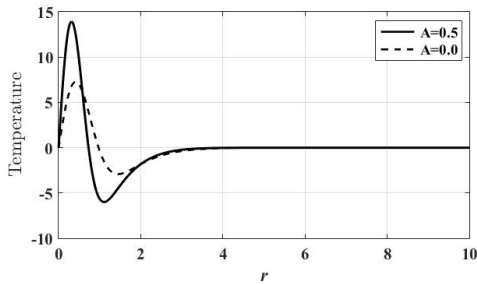


Fig. 1: Variation of temperature θ with distance r for the coefficient of Ohm and Fourier laws

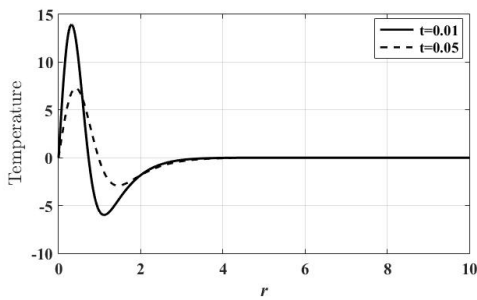


Fig. 2: Variation of temperature θ with distance r for different value of time t

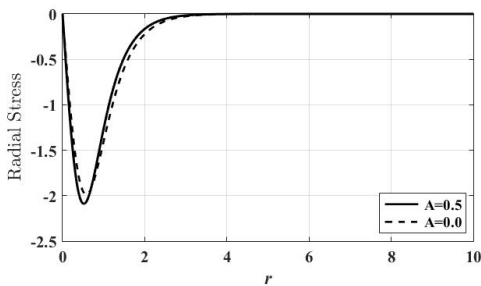


Fig. 3: Variation of radial Stress σ_{rr} with distance r for the coefficient of Ohm and Fourier laws

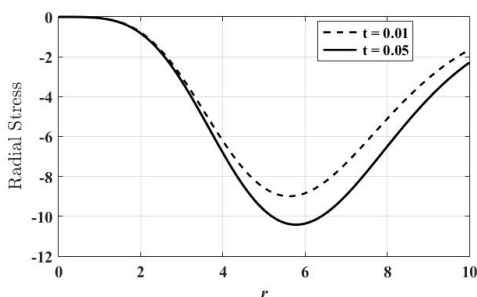


Fig. 4: Variation of radial stress σ_{rr} with distance r for different value of time t

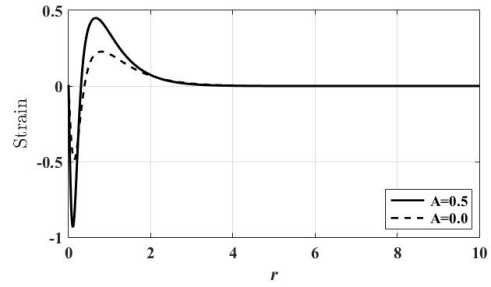


Fig. 5: Variation of strain e with distance r for the coefficient of Ohm and Fourier laws

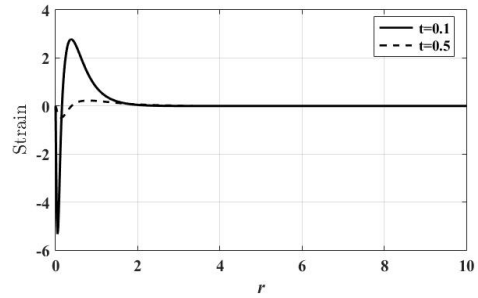


Fig. 6: Variation of Strain e with distance r for different value of time t

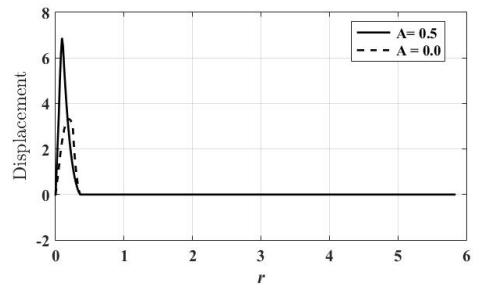


Fig. 7: Variation of displacement u with distance r for the coefficient of Ohm and Fourier laws

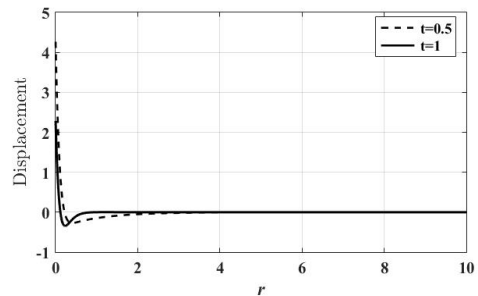


Fig. 8: Variation of displacement u with distance r for different value of time t

Fig. 1 shows the variation of temperature θ for the coefficient of Ohm and Fourier laws. It is noticed that the modified Fourier and Ohm laws influence is significant, the temperature in the modified model records values higher than those in the old model.

Fig. 2 shows the variation of temperature θ against r for wide range of $r, 1 \leq r \leq 3$ for different values of time ($t = 0.01, t = 0.05$) and we have noticed that the time t has significant effects on temperature. The increasing of the value of t causes increasing of the value of temperature, and temperature θ vanishes more rapidly.

Fig. 3 shows the variation of radial stress for the coefficient of Ohm and Fourier laws. It is noticed that

the modified Fourier and Ohm model affects the radial stress by increasing their values.

Fig. 4 shows the variation of the radial stress σ_{rr} against r for wide range of $r, (1 \leq r \leq 8)$, for different values of time ($t = 0.01, t = 0.05$) and we have noticed that the time t has significant effects on stress, for a wide range of $(r, 1 \leq r \leq 2.7)$ the increasing of the value of t causes decreasing of the value of the radial stresses σ_{rr} and for wide range of $r, 2.7 \leq r \leq 8$ the increasing of the value of t causes increasing of the value of the radial stresses and radial stresses σ_{rr} vanishes more rapidly.

Fig. 5 shows the variation of strain e for the coefficient of Ohm and Fourier laws. It is noticed that the modified Fourier and Ohm laws influence is significant, the strain in the modified model records values higher than those in the old model.

Fig. 6 shows the variation of strain e against r for wide range of $r, (1 \leq r \leq 3)$, at fractional parameter $\alpha = 0.5$ and for different values of time ($t = 0.1, t = 0.5$) and we have noticed that the time t has significant effects on the strain, for the wide range ($1 \leq r \leq 1.4$) the increasing of the value of t causes decreasing of the value of strain e , for the wide range ($1.4 \leq r \leq 3$) the increasing of the value of t causes increasing of the value of strain and strain e vanishes more rapidly.

Fig. 7 is plotted to shows variation of displacement u for the coefficient of Ohm and Fourier laws. It is noticed that the modified Fourier and Ohm laws influence is significant, the displacement in the modified model records values higher than those in the old model.

Fig. 8 shows the variation of displacement u against r for wide range of $r, 1 \leq r \leq 3$, for different values of time ($t = 0.5, t = 1$) and we have noticed that the time t has significant effects on displacement u . The increasing of the value of t causes increasing of the value of displacement u , and displacement u vanishes more rapidly.

7. Conclusion

As per this work, numerous analyses in the field of generalized thermoelasticity have applied Green-Naghdi for the elastic problem and not many of them can effectively be applied for magneto thermo Viscoelastic problem. From the above discussion, one can conclude that the new model of generalized magneto-thermo-viscoelasticity predicts new qualities for the temperature, displacement, stresses, and strain. The impact of the temperature gradient at adequately low temperatures may cause new sorts of the magneto-thermo-viscoelastic wave with explicit strength properties. The expansion in the estimations of temperature might be clarified as the lost heat-producing from the development of electric flow; this heat might be the primary motivation behind why the deformation of the medium will, in general, be typical and the magnetic field records esteem more prominent than the qualities in the old model.

It very well may be noticed that the speed of spread of every single physical amount is limited and correspond to the physical conduct of Viscoelastic material.

List of symbols

λ_e, μ_e	Lame elastic constants
ρ	Density
C_E	Specific heat at constant strain
K	Thermal conductivity
α_t	Coefficient of linear thermal expansion
γ	$= (3\lambda_e + 2\mu_e)\alpha_t$
λ_v, μ_v	Viscoelastic relaxation time
t	Time
q_i	Components of heat flux vector
σ_{ij}	Components of stress tensor
e_{ij}	Components of strain tensor
u_i	Components of displacement vector
T_o	Reference temperature
θ	Temperature increment
δ_{ij}	Kronicker delta
e	Cubical dilatation
L	Radius of the cylinder
μ_o	Magnetic permittivity
E	Electric displacement vector
J	Current density vector
H	Total magnetic intensity vector
h	Induced magnetic field vector
H_o	Initial uniform magnetic field
F_i	Components of Lorentz body force
π_o	Coefficient connecting the current density with the heat flow density
k_o	Coefficient connecting the temperature gradient and electric current density

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Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

References

- Abd-Alla AM and Abo-Dahab SM (2009). Time-harmonic sources in a generalized magneto-thermo-viscoelastic continuum with and without energy dissipation. Applied Mathematical Modelling, 33(5): 2388-2402. <https://doi.org/10.1016/j.apm.2008.07.008>
- Abd-Alla AM, Abo-Dahab SM, and Bayones FS (2011). Effect of the rotation on an infinite generalized magneto-thermoelastic diffusion body with a spherical cavity. International Review of Physics, 5(4): 171-181.
- Abd-Alla AM, Hammad HAH, and Abo-Dahab SM (2004). Magneto-thermo-viscoelastic interactions in an unbounded body with a spherical cavity subjected to a periodic loading. Applied Mathematics and Computation, 155(1): 235-248. [https://doi.org/10.1016/S0096-3003\(03\)00773-2](https://doi.org/10.1016/S0096-3003(03)00773-2)

- Acharya DP and Roy I (1978). Magneto-thermo-elastic surface waves in initially stressed conducting media. *Acta Geophysica Polonica*, 26: 299-311.
- Amin M, El-Bary A, and Atef H (2018a). Effect of viscous fractional parameter on generalized magneto thermo-viscoelastic thin strip exposed to moving heat source. *Materials Focus*, 7(6): 814-823. <https://doi.org/10.1166/mat.2018.1591>
- Amin MM, El-Bary AA, and Atef HM (2018b). Modification of kelvin-voigt model in fractional order for thermoviscoelastic isotropic material. *Materials Focus*, 7(6): 824-832. <https://doi.org/10.1166/mat.2018.1592>
- Banerjee S and Roychoudhuri SK (1995). Spherically symmetric thermo-visco-elastic waves in a visco-elastic medium with a spherical cavity. *Computers and Mathematics with Applications*, 30(1): 91-98. [https://doi.org/10.1016/0898-1221\(95\)00070-F](https://doi.org/10.1016/0898-1221(95)00070-F)
- Bayones FS (2012). The influence of diffusion on generalized magneto-thermo-viscoelastic problem of a homogenous isotropic material. *Advances in Theoretical and Applied Mechanics*, 5(2): 69-92.
- Belbachir N, Bourada M, Draiche K, Tounsi A, Bourada F, Bousahla AA, and Mahmoud SR (2020). Thermal flexural analysis of anti-symmetric cross-ply laminated plates using a four variable refined theory. *Smart Structures and Systems*, 25(4): 409-422.
- Biot MA (1955). Variational principles in irreversible thermodynamics with application to viscoelasticity. *Physical Review*, 97(6): 1463-1469. <https://doi.org/10.1103/PhysRev.97.1463>
- Biot MA (1956). Thermoelasticity and irreversible thermodynamics. *Journal of Applied Physics*, 27(3): 240-253. <https://doi.org/10.1063/1.1722351>
- Chandrasekharaiah DS (1986). Thermoelasticity with second sound: A review. *Applied Mechanics Reviews*, 39(3): 355-376. <https://doi.org/10.1115/1.3143705>
- Chandrasekharaiah DS and Krivoschapko SN (1998). Hyperbolic thermoelasticity: A review of recent literature. *Applied Mechanics Reviews*, 51(12): 705-729. <https://doi.org/10.1115/1.3098984>
- Chaudhary S, Kausik VP, and Tomar SK (2006). Plane SH-wave response from elastic slab interposed between two different self-reinforced elastic solids. *Applied Mechanics and Engineering*, 11(4): 787-801.
- Chaudhary S, Kaushik VP, and Tomar SK (2004). Reflection/transmission of plane SH wave through a self-reinforced elastic layer between two half-spaces. *Acta Geophysica Polonica*, 52(2): 219-236.
- De SN and Sengupta PR (1971). Surface waves in magneto-elastic initially stressed conducting media. *Pure and Applied Geophysics*, 88(1): 44-52. <https://doi.org/10.1007/BF00877891>
- El-Bary AA and Atef H (2016a). On effect of viscous fractional parameter on infinite thermo viscoelastic medium with a spherical cavity. *Journal of Computational and Theoretical Nanoscience*, 13(1): 1027-1036. <https://doi.org/10.1166/jctn.2016.4332>
- El-Bary AA and Atef M (2016b). Modified approach for stress strain equation in the linear Kelvin-Voigt solid based on fractional order. *Journal of Computational and Theoretical Nanoscience*, 13(1): 579-583. <https://doi.org/10.1166/jctn.2016.4843>
- Ezzat MA and El-Bary AA (2009). On three models of magneto-hydrodynamic free-convection flow. *Canadian Journal of Physics*, 87(12): 1213-1226. <https://doi.org/10.1139/P09-071>
- Ezzat MA, El-Karamany AS, and El-Bary AA (2015). On thermo-viscoelasticity with variable thermal conductivity and fractional-order heat transfer. *International Journal of Thermophysics*, 36(7): 1684-1697. <https://doi.org/10.1007/s10765-015-1873-8>
- Ezzat MA, El-Karamany AS, and El-Bary AA (2017). Thermoelectric viscoelastic materials with memory-dependent derivative. *Smart Structures and Systems*, 19: 539-551. <https://doi.org/10.12989/sss.2017.19.5.539>
- Green AE and Lindsay KA (1972). Thermoelasticity. *Journal of Elasticity*, 2(1): 1-7. <https://doi.org/10.1007/BF00045689>
- Green AE and Naghdi PM (1991). A re-examination of the basic postulates of thermomechanics. *Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences*, 432(1885): 171-194. <https://doi.org/10.1098/rspa.1991.0012>
- Gross B (1968). *Mathematical structure of the theories of viscoelasticity*. Hermann Publisher, Paris, France.
- Gurtin ME and Sternberg E (1962). On the linear theory of viscoelasticity. *Archive for Rational Mechanics and Analysis*, 11: 291-356. <https://doi.org/10.1007/BF00253942>
- Hetnarski RB and Ignaczak J (1996). Soliton-like waves in a low temperature nonlinear thermoelastic solid. *International Journal of Engineering Science*, 34(15): 1767-1787. [https://doi.org/10.1016/S0020-7225\(96\)00046-8](https://doi.org/10.1016/S0020-7225(96)00046-8)
- Hetnarski RB and Ignaczak J (1999). Generalized thermoelasticity. *Journal of Thermal Stresses*, 22(4-5): 451-476. <https://doi.org/10.1080/014957399280832>
- Kosiński W (1989). Elastic waves in the presence of a new temperature scale. *North-Holland Series in Applied Mathematics and Mechanics*, 35: 629-634. <https://doi.org/10.1016/B978-0-444-87272-2.50099-3>
- Liioushin AA and Pobedria BE (1970). *Mathematical theory of thermal viscoelasticity*. Nauka, Moscow, Russia.
- Othman MIA and Song Y (2006). The effect of rotation on the reflection of magneto-thermoelastic waves under thermoelasticity without energy dissipation. *Acta Mechanica*, 184(1-4): 189-204. <https://doi.org/10.1007/s00707-006-0337-4>
- Roychoudhuri SK and Banerjee S (1998). Magneto-thermo-elastic interactions in an infinite viscoelastic cylinder of temperature-rate dependent material subjected to a periodic loading. *International Journal of Engineering Science*, 36(5-6): 635-643. [https://doi.org/10.1016/S0020-7225\(97\)00096-7](https://doi.org/10.1016/S0020-7225(97)00096-7)
- Song YQ, Zhang YC, Xu HY, and Lu BH (2006). Magneto-thermoviscoelastic wave propagation at the interface between two micropolar viscoelastic media. *Applied Mathematics and Computation*, 176(2): 785-802. <https://doi.org/10.1016/j.amc.2005.10.027>
- Staverman AJ, Schwarzl F, and Stuart HA (1956). *Die physic der Hochpolymeren*. Volume 4, Springer, Berlin, Germany.
- Tanner RI (1988). *Engineering rheology*. Oxford University Press, Oxford, UK.
- Tianhu H, Yapeng S, and Xiaogeng T (2004). A two-dimensional generalized thermal shock problem for a half-space in electromagneto-thermoelasticity. *International Journal of Engineering Science*, 42(8-9): 809-823. <https://doi.org/10.1016/j.ijengsci.2003.09.006>
- Tzou D (1995). A unified field approach for heat conduction from macro-to micro-scales. *Journal of Heat Transfer*, 117(1): 8-16. <https://doi.org/10.1115/1.2822329>
- Verma PDS, Rana OH, and Verma MEENU (1988). Magnetoelastic transverse surface waves in self-reinforced elastic bodies. *Indian Journal of Pure and Applied Mathematics*, 19(7): 713-716.
- Youssef HM, El-Bary AA, and Elsbai KA (2014). Vibration of gold nano beam in context of two-temperature generalized thermoelasticity subjected to laser pulse. *Latin American Journal of Solids and Structures*, 11(13): 2460-2482. <https://doi.org/10.1590/S1679-78252014001300008>

Youssef HM, Elsibai KA, and El-Bary AA (2017). Effect of the speed, the rotation and the magnetic field on the Q-factor of

an axially clamped gold micro-beam. *Meccanica*, 52(7): 1685-1694. <https://doi.org/10.1007/s11012-016-0498-8>