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Image representation based on fractional order Legendre and Laguerre orthogonal moments

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ABSTRACT

In this paper, we have introduced new sets of fractional order orthogonal basis moments based on Fractional order Legendre orthogonal Functions (FLeFs) and Fractional order Laguerre orthogonal Functions (FLaFs) for image representation. We have generated a novel set of Fractional order Legendre orthogonal Moments (FLeMs) from fractional order Legendre orthogonal functions and a new set of Fractional order Laguerre orthogonal Moments (FLaMs) from the fractional order Laguerre orthogonal functions. The new presented sets of (FLeMs) and (FLaMs) are tested with the recently introduced Fractional order Chebyshev orthogonal Moments (FCMs). This edge detection filter can be used successfully in the gray level image and color images. The new sets of fractional moments are used to reconstruct the gray level image. The numerical results show FLeMs and FLaMs are promised techniques for image representation. The computational time of the proposed techniques is compared with the computational time of Chebyshev orthogonal Moments techniques and gives better results. Also, the fractional parameters give the flexibility of studying global features of the image at different positions of moments.

1. Introduction

Image representation and edge detection play the core of image processing, computer vision, and pattern recognition fields. In recent view months, the image representation based on fractional order moments attracting many researchers. For example, a set of fractional order orthogonal Chebyshev moments is used to represent a gray-scale image (Kazem et al., 2013). Discrete fractional order orthogonal Chebyshev moments for Image encryption and watermarking based on FCMs are investigated in Xiao et al. (2020; 2017), Fernández et al. (2010), and Xu (2005). Shifted Chebyshev polynomials are developed to the new family of basis functions, namely generalized shifted Chebyshev polynomials (Fernández et al., 2011). The bivariate orthogonal polynomials are used to define continuous and discrete orthogonal moments are discussed in Hassani et al. (2020). Only a few papers have used bivariate or multivariate orthogonal polynomials for image analysis and pattern recognition (Xu, 2004; Fernández, 2007).

The orthogonal moments of gray-scale images were firstly studied in Teague (1980), where these orthogonal moments were able to represent digital images with no redundancy or overlap of information. Moreover, orthogonal moments are robust against well-known kind of noise and have an efficient capability of features reconstruction (Papakostas, 2014). The orthogonal moments enable researchers to reconstruct the image from a finite set of moments, using the inverse moment transform (Flusser et al., 2016). Many studies are introduced about the representation of images from orthogonal moments (Hosny et al., 2020a; 2020b; Sweilam et al., 2016).

In this paper, we have introduced new sets of the orthogonal basis of fractional order Legendre moments and fractional order Laguerre moments to represent an image. The rest of this article is organized as follows. In section 2, we have introduced the fractional order Legendre functions and moments. The fractional order Laguerre functions and moments are mentioned in section 3. Finally, we have demonstrated the numerical computations of the proposed FLeMs, FLaMs, FCMs, the effect of fractional parameters of the polynomials in reconstructing images, and CPU elapsed times of different proposed algorithms.

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2. Fractional order Legendre functions

The well-known fractional-order Legendre functions $F_n^\alpha(x), \alpha > 0$ on the interval [0, 1] is defined in Bhrawy (2014) as following (Fig. 1 and Table 1):

$$F_{n+1}^\alpha(x) = \frac{(2n+1)(2x^\alpha - 1)}{n+1} F_n^\alpha(x) - \frac{n}{n+1} F_{n-1}^\alpha(x) \tag{1}$$

where, n=1,2,.., $F_0^\alpha(x) = 1$ and $F_1^\alpha(x) = 2x^\alpha — 1$. The analytic form of $F_n^\alpha(x)$ of degree $n\alpha$ is given by:

$$F_n^\alpha(x) = \sum_{k=0}^{n} \frac{(-1)^{k+n}(n+k)!}{(n-k)!(k!)^2} x^{\alpha k} \tag{2}$$

The fractional order Legendre functions satisfy the orthogonal condition with respect to the weight function $w_\alpha(x) = x^{\alpha-1}$ on the interval [0, 1].

$$\int_0^1 w(x) F_n^\alpha(x) F_m^\alpha(x) = h_n \delta_{nm} \tag{3}$$

where $h_n = \frac{1}{(2n+1)\alpha}$ and δ_{nm} is the Kronecker function. The normalized fractional order Legendre functions can be obtained from the formula:

$$F_n^\alpha(x) = \sqrt{\frac{w(x)}{h_n}} \; F_n^\alpha(x) \tag{4}$$

and can easily prove that the normalized fractional order functions are orthogonal on the interval [0, 1].

2.1. Fractional order Legendre orthogonal Moments (FLeMs)

For any two dimensions image $f(x, y) \in L^2([0,1] \times [0, 1])$ the continuously Legendre moment of order (n+m) can be defined as in the following formula:

$$LM_{nm} = \frac{1}{h_n h_m} \int_0^1 \int_0^1 f(x, y)\, F_n^{\alpha_x}(x) F_m^{\alpha_y}\, w(x) w(y) dx\, dy \tag{5}$$

So, the Legendre moment of an image of resolution $N \times M$ Eq. 5 can be rewritten in the form:

$$LM_{nm} = \sum_{i=1}^{N} \sum_{j=1}^{M} f(i,\ j) F_n^{\alpha_x}(x_i)\ F_m^{\alpha_y}(y_j), i = 1,2, \ldots, N; j = 1,2, \ldots M \tag{6}$$

where,

$$x_i = \frac{2i+1}{2N};\ y_j = \frac{2j+1}{2M}, i = 1,2, \ldots, N; j = 1,2,, M \tag{7}$$

Also, an approximation of the original image $f(x, y)$ can be reconstructed from the equation:

$$\hat{f} = \sum_{n=0}^{K} \sum_{m=0}^{L} LM_{nm}\, F_n^{\alpha_x}(x_i) F_m^{\alpha_y}(y_j) \tag{8}$$

3. Fractional order Laguerre functions (FLFs)

Let $L_n^\beta(x), \beta > -1$ be the Fractional order Laguerre Functions (FLFs) of order n. The recurrence relation of FLFs can be defined as (Parand and Delkhosh, 2017; Parand et al., 2017):

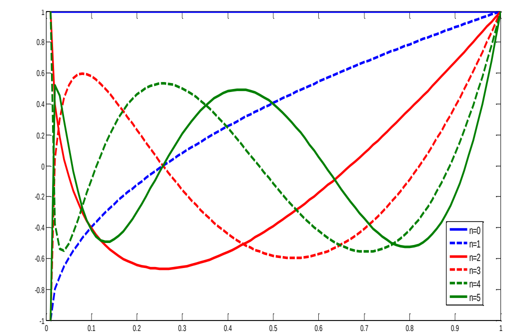
$$L_{n+1}^\beta(x) = (2n - x^\beta + 1) L_n^\beta(x) - n^2\, L_{n-1}^\beta(x) \tag{9}$$

with,

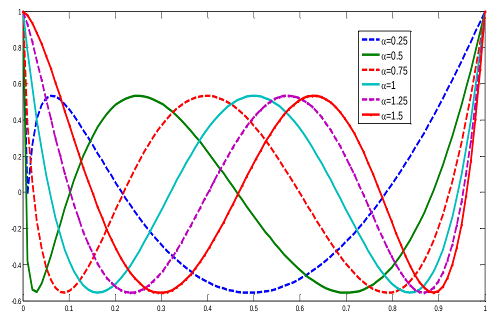
$$n = 0,1, \ldots \text{and } L_0^\beta(x) = 1, \qquad L_1^\beta(x) = 1 - x^\beta$$

and the analytic form of $L_n^\beta(x)$is obtained as:

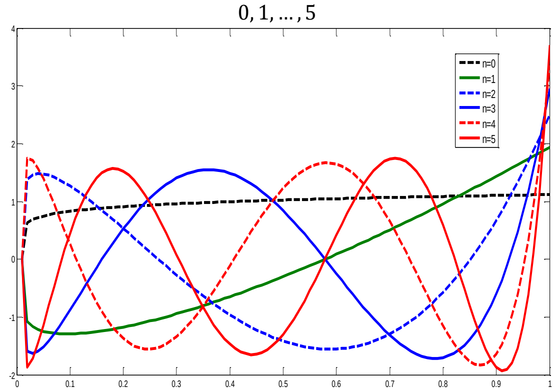
$$L_n^\beta(x) = \sum_{k=0}^{n} (-1)^k \frac{n!}{(n-k)!(k!)^2} x^{k\beta} \tag{10}$$



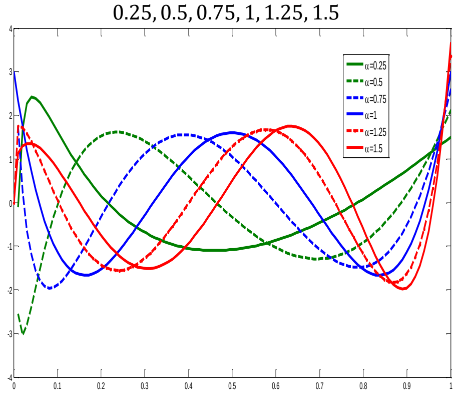
a: shows the fractional Legendre functions with $\alpha = 1.25, n = 0, 1, \ldots, 5$



b: shows the fractional Legendre functions with $\alpha = 0.25, 0.5, 0.75, 1, 1.25, 1.5$



c: shows the normalized fractional Legendre functions with $\alpha = 1.25; n = 0, 1, \ldots, 5$



d: shows the normalized fractional Legendre functions with $\alpha = 0.25, 0.5, 0.75, 1, 1.25, 1.5$

Fig. 1: Fractional Legendre functions and normalized fractional Legendre functions with α

Table 1: Algorithm 1

Fractional Legendre orthogonal Moments (FLeMs)	
Input	$f(x,y), \alpha_x, \alpha_y, n, m$
Step 1	*Generate vectors* x_i, y_j *from Eq. 7*
Step 2	*Compute* $F_n^{\alpha}(x)$ *,n,* x_i, α_x *and* $F_m^{\alpha}(y)$ m, y_j, α_y *Eq. 4*
Step2	*Calculate* LM_{nm} *from Eq. 5*
Output	*Image moment at n, m.*

The fractional Laguerre functions are orthogonal with respect to the weight function $w_\beta(x) = x^{\beta-1}e^{-x^\beta}$ on the interval [0, ∞), and satisfy the orthogonally condition:

$$\int_0^\infty w_\beta(x) L_n^\beta(x) L_m^\beta(x) dx = \frac{1}{\beta}\, \delta_{nm} \quad (11)$$

where δ_{nm} is the Kronecker function. Due to the fractional order, Laguerre functions $L_n^\beta(x)$ expand rapidly with higher orders. In numerical computation, we have used the normalized fractional order Laguerre functions $\hat{L}_n^\beta$ (x) defined by the formula:

$$\hat{L}_n^\beta(x) = \sqrt{\beta\, w_\beta}\, L_n^\beta(x) \quad (12)$$

3.1. Fractional Laguerre orthogonal moments (FLMs)

For any arbitrary function $f(x,y) \in (0,\infty) \times [0,\infty)$, the fractional order Laguerre moments of order (n+m) can be obtained from the continuous integral by the following formula:

$$L_{nm} = \int_0^\infty \int_0^\infty f(x,y) L_n^{\alpha_x}(x) L_m^{\alpha_y}\, dx\, dy \quad (13)$$

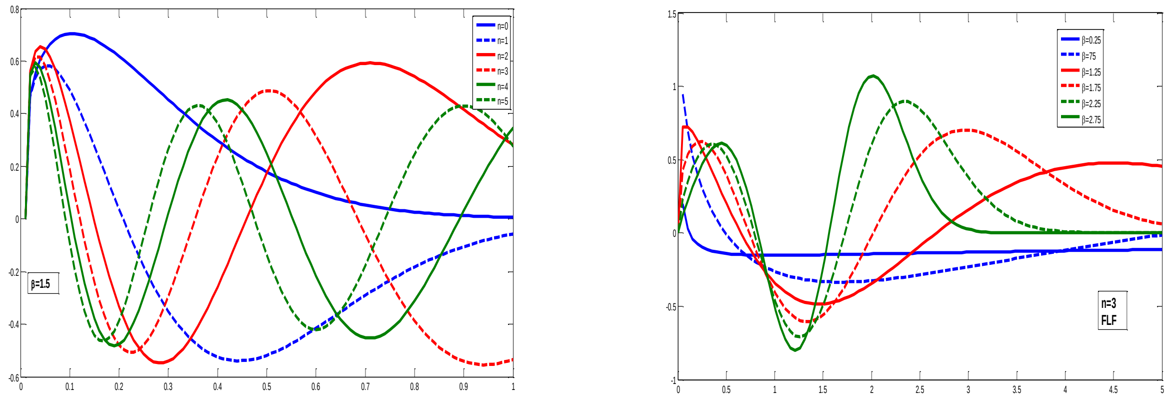
For a digital image $f(i,j)$ of the resolution $N \times M$, the fractional order Laguerre moments can be written the formula:

$$L_{nm} = \frac{1}{NM} \sum_{i=0}^{N} \sum_{j=0}^{M} f(i,j)\, L_n^{\alpha_x}(x) L_m^{\alpha_y}(y) \quad (14)$$

The reconstructed image can be obtained from the formula:

$$\hat{f} = \sum_{n=0}^{K} \sum_{m=0}^{L} L_{nm}\, L_n^{\beta_x}(x) L_m^{\beta_y}(y) \quad (15)$$

where K and L are the maximum number of orders, in our computation, we but K=L (Fig. 2 and Table 2).



a: normalized fractional Laguerre functions with different orders $\beta = 1.5; n = 0,1,\ldots,5$

b: normalized fractional Laguerre functions with different values of $\beta = 0.25, 0.75, 1.25, 1.75, 2.25, 2.75; n = 3$

Fig. 2: Fractional Legendre functions and normalized fractional Legendre functions with β

Table 2: Algorithm 2

Fractional Laguerre orthogonal Moments (FLFs)	
Input	$f(x,y), \beta_x, \beta_y, n, m$
Step 1	*Compute* $\hat{L}_n^{\alpha}(x)$ *,n,* x, β_x *and* $\hat{L}_m^{\alpha}(y)$ m, y, α_y *Eq. (12)*
Step2	*Calculate* L_{nm}*from Eq. (14)*
Output	*Image moment at n, m.*

4. Discussion and numerical results

To demonstrate the performance of the newly introduced algorithms, FLeMs, FLaMs, and FLLMs, we have done a set of numerical experiments on dataset images that are displayed in Fig. 3. All the algorithms and the numerical experiments are implemented and executed in MATLAB8.2 under Microsoft Windows environment using a PC with IntelCore i5 CPU 2.4 GHz and 4 GB RAM.

4.1. Image representation

To check the stability of the proposed new algorithms to reconstruct an image, we have used Mean Square Error (MSE) to measure the performance of the proposed FLeMs constructed from fractional Legendre orthogonal functions, FLaMs from fractional order orthogonal Laguerre functions, and compare our results with FCMs recently introduced in Benouini et al. (2019) and Hassani et al. (2019). The MSE between the original image $f(x,y)$ and reconstructed image $\hat{f}(x,y)$ is computed from the following formula:

$$MSE = \frac{1}{NM} \sum_x^N \sum_y^M \left[f(x,y) - \hat{f}(x,y)\right]^2 \quad (16)$$

The reconstructed image MSE of the proposed FLeMs and FLaMs, Lena gray level image is compared with FCMs and displayed in Fig. 4. In our computation for FLeMs, we have used by $\alpha = 0.8, 0.9, 1.2$ Eq. 4, Eq. 6, and Eq. 7. In the case of FLaMs, we have substituted by $\beta = 0.8, 1.2, 0.2$ in Eq. 11, Eq. 13, and Eq. 14. For comparison purposes, we have plotted the MSE errors of the FLeMs from fractional order Legendre, FLaMs from fractional order Laguerre, and against FCMs.

Fig. 5 shows the reconstructed Lena image at the different orders from the different proposed FLeMs, FLaMs, and FCMs.

Fig. 3: Original dataset

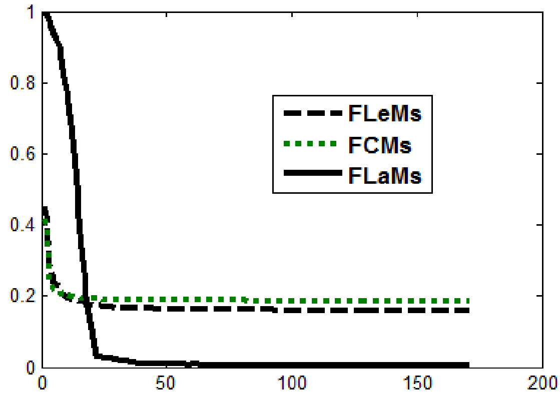
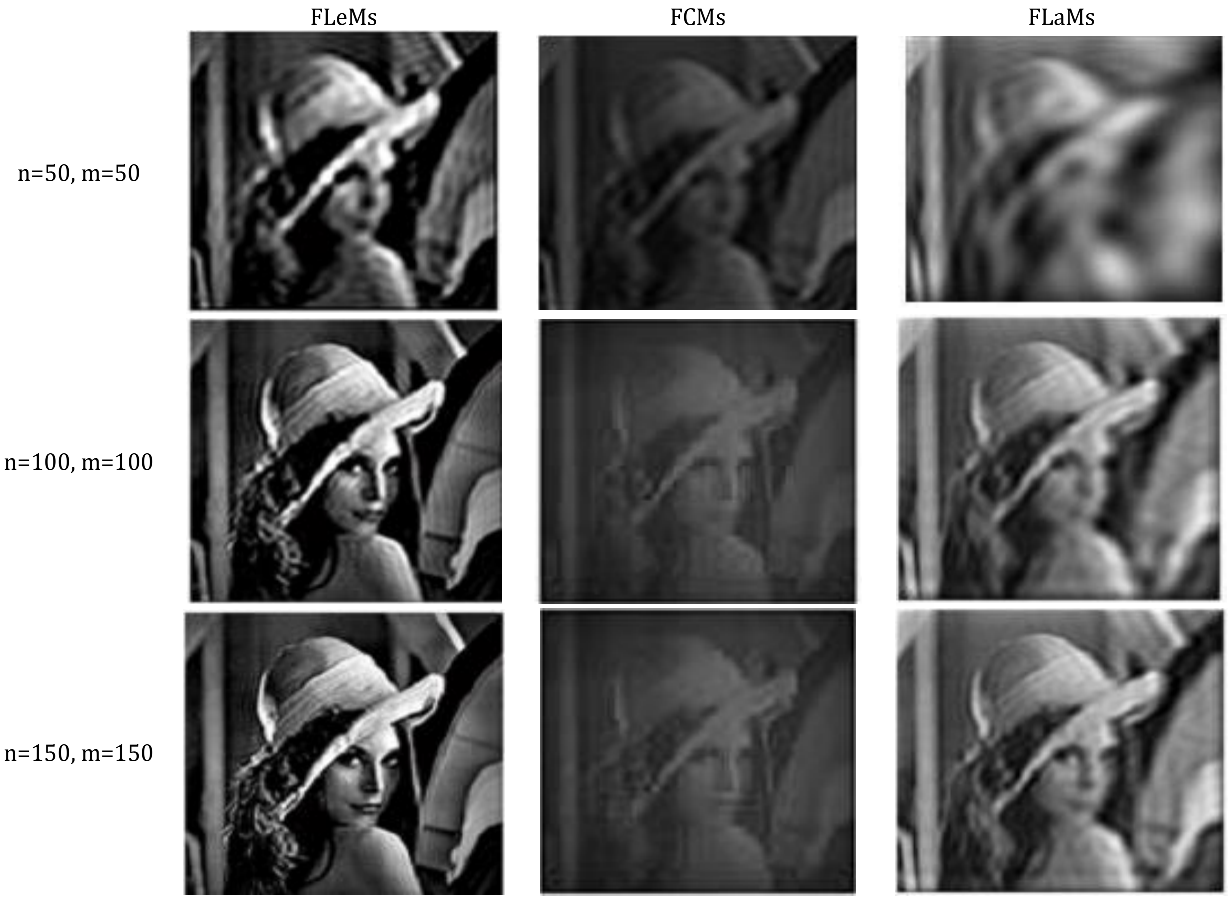


Fig. 4: MSE for the different proposed FLeMs, FLaMs, and compared with FCMs for the Lena gray level image reconstruction error

The results of the proposed algorithms for different orders $^{n+m}$ are reconstructed for different image datasets displayed in Fig. 3. In Fig. 5, we have displayed the reconstructed the first image at order n=m=50, n=m=100, n=m=150, n=m=200 and n=m=300. We observe in the numerical results that; the higher orders give better results for all algorithms. Also, we have noted the accuracy of the reconstruction depends on α and β as well as the FLaMs is highly depend on β.



n=200, m=200

n=300, m=300

Fig. 5: The reconstructed Lena image at the different orders from the different proposed FLeMs, FLaMs and FCMs

4.2. Computational time

In order to examine the priority of the proposed novel FLeMs, FLaMs, and FCMs, we have computed the computational performance of the proposed fractional-order moments. Fig. 6 shows the elapsed CPU times in seconds for the moment's computation of the first test image, with size 256×256 pixels. In Fig. 6, we have plotted the natural logarithm of the CPU elapsed times against the number of moments. According to the results presented in Fig. 6, one can observe that the computation time taken by FLeMs, FLaMs is less than the CPU elapsed times were taken by FCMs. In numerical computation, we have observed that the CPU elapsed times increase with increasing order of moments and also depend highly on the scale parameter in FLeMs and FLaMs α and β, respectively. The shift parameters α are not highly affected by the CPU elapsed times. All curves in Fig. 6 are computed with shifts parameters α-0.8 and β-12.

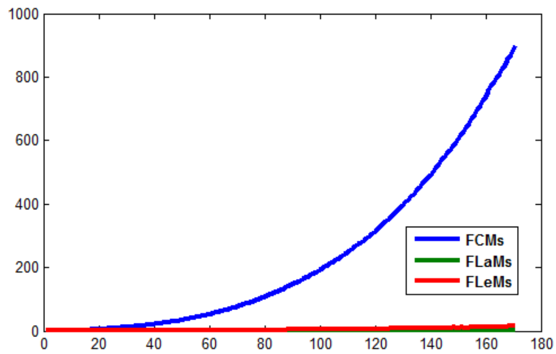


Fig. 6: Comparative study of FLeMs, FLaMs, and FCMs

5. Conclusion

In this paper, we have presented a novel set of fractional order Legendre orthogonal moments (FLeMs) and fractional order Laguerre orthogonal moments. These moments have been used to reconstruct an image. Also, we have compared our results with fractional order Chebyshev orthogonal moments, which is recently introduced by Hassani et al. (2020). This set of moments mathematically is able to represent any two-dimension image, and in practice, we noticed the numerical result perfectly represents an image as shown in Fig. 5. Finally, we have computed the computational time, and our algorithms are less than the consumed time by FCMs, and it can be used in image processing, pattern recognition to reconstruct and analyze its contents.

Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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