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Equity home bias and consumption-real exchange rate puzzles: A joint solution

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ABSTRACT

In a standard dynamic stochastic general equilibrium model with a complete asset market, home agents should hold a foreign equity biased portfolio to hedge the non-traded labor income risk, which contradicts home equity biased portfolios observed worldwide. As the labor income share increases, the degree of home bias should decrease because there is more incentive to hold foreign equity. In the data, there is not any evidence that the labor income share and the degree of home bias are negatively correlated. The standard model also predicts that the consumption differential-real exchange rate correlation is positive, while it is negative in the data. I show that a combination of market incompleteness, non-tradable goods, and labor supply can explain the three features above. My model can generate a large equity home bias, despite the strong positive correlation of non-traded human capital return with domestic equity return. The home bias is not sensitive to the labor income share. The consumption differential-real exchange rate unconditional correlation generated by my model simulation is zero.

1. Introduction

To diversify risks, investors in country n, who consume a fraction of μ^n of the world's output, should buy the same fraction of μ^n of global financial assets, (Obstfeld and Rogoff, 1998; Obstfeld et al., 1996). However, investors all over the world mostly hold home equities in their portfolios. Table 1 illustrates the degrees of home equity bias for selected countries. Following Ahearne et al. (2004) and Coeurdacier and Rey (2013), home equity bias is defined as:

$$EHB_i = 1 - \frac{Share\ of\ Foreign\ Equities\ in\ Country\ i\ Equity\ Holdings}{Share\ of\ Foreign\ Equities\ in\ the\ World\ Market\ Portfolio}$$

Baxter and Jermann (1997) pointed out that since non-traded human capital return can be highly correlated with domestic equity return, the optimal portfolio should be foreign biased, which makes the puzzle "worse than you think." A standard dynamic stochastic general equilibrium (DSGE) model predicts that home investors should hold mostly foreign equity. In addition, the fraction of domestic equity in home portfolios should depend on the fraction of labor income share in home GDP. The larger the labor income share is, the less domestic equity home investors should hold, or the smaller the degree of home equity bias. This is not the case in the data. Fig. 1 plots the degree of home equity bias against labor income shares across OECD countries in 2005. Figs. 2 and 3 graph labor income shares and the degree of home equity bias over time for selected countries. The data suggests that there is not a negative relationship between labor income shares and the degree of home equity bias. Also, Fig. 4 shows the real exchange rate and relative consumption.

The early literature on international portfolios that tried to explain the observed level of home bias is based on endowment economy models without labor income. Such models are in Tesar (1993), Baxter et al. (1998), Pesenti and Wincoop (2002), Collard et al. (2007), and others. With an endowment economy, one can avoid the tendency of labor income to generate a foreign-biased portfolio. Although endowment economy models help build our initial foundation for the understanding of optimal international portfolios, they ignore half of the puzzle.

The worldwide increase of asset trade in the last two decades, together with its importance in the global transmission of shocks has generated renewed interest in understanding international

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portfolios in a DSGE context. Tille and Wincoop (2010), Devereux and Sutherland (2011) and Evans and Hnatkovska (2012) developed methods to solve for optimal international portfolios in DSGE models. Matsumoto (2007), Heathcote and Perri (2013), Engel and Matsumoto (2009), Coeurdacier et al. (2007; 2010) are among those who applied these methods in a complete market framework. Matsumoto (2007) built an international portfolio model with tradable and non-tradable sectors. He assumes complete markets, and the stocks in both sectors are traded internationally. He finds that the optimal portfolio depends on parameters' values.

Table 1: Home equity bias for selected countries in 2011

Country	Domestic Market in % of World Market Capitalization	Share of Portfolio in Domestic Equity in %	Degree of Equity Home Bias = EHB_i
United States	33.0	74.6	0.62
Canada	4.0	71.7	0.70
Germany	2.5	47.5	0.46
United Kingdom	6.6	62.8	0.60
Australia	2.6	76.8	0.76
Japan	7.0	79.5	0.78

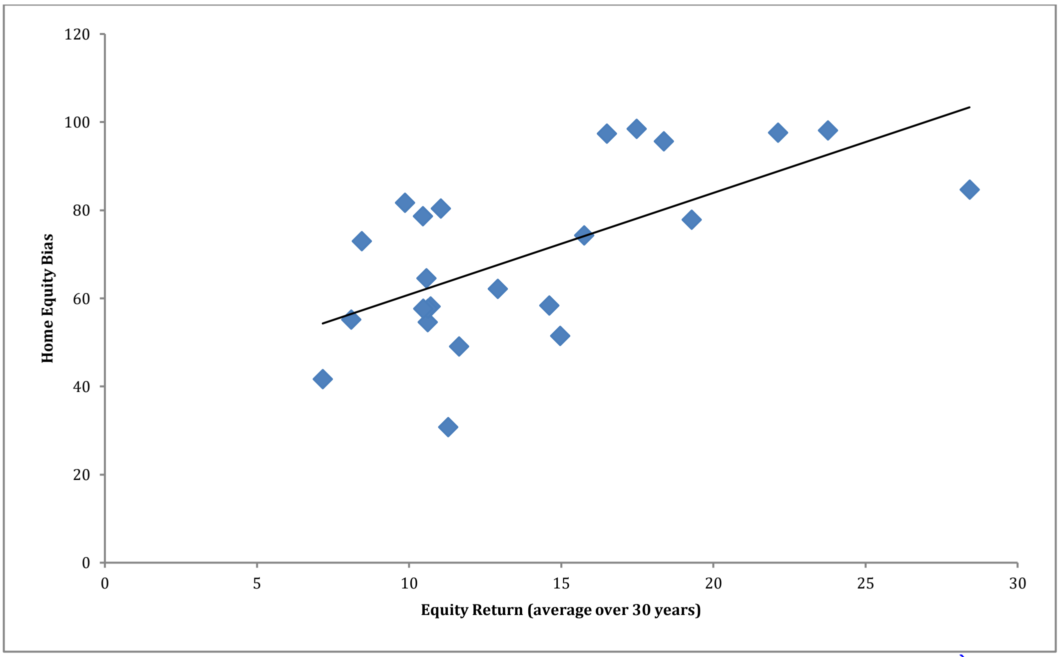

Fig. 1: Labor income share and equity home bias 2007; Data from OECD and Sercu and VanpÈe (2007)

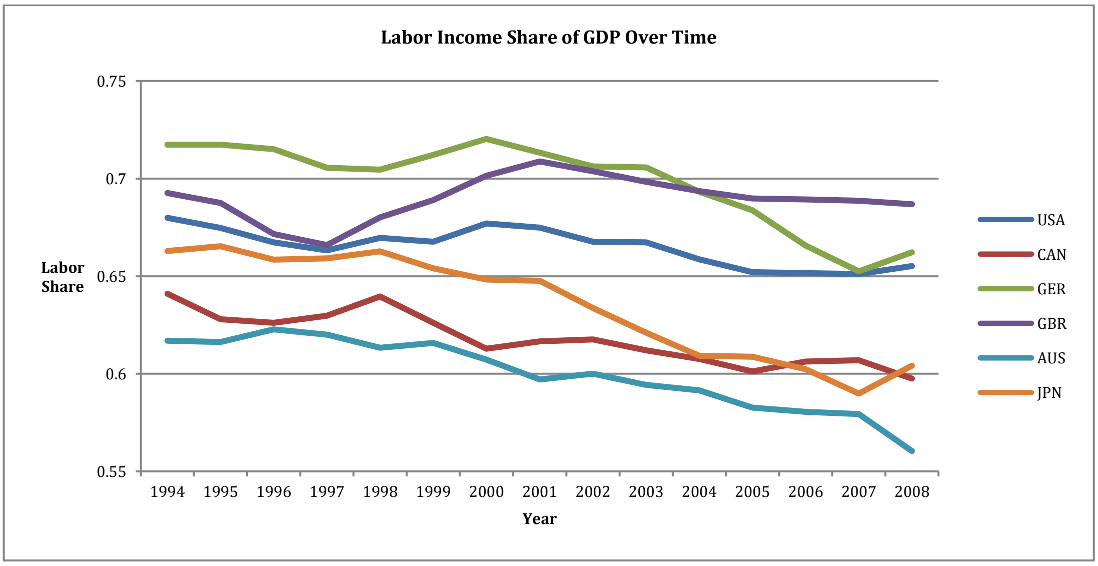

Fig. 2: Labor income share over time; Data from OECD

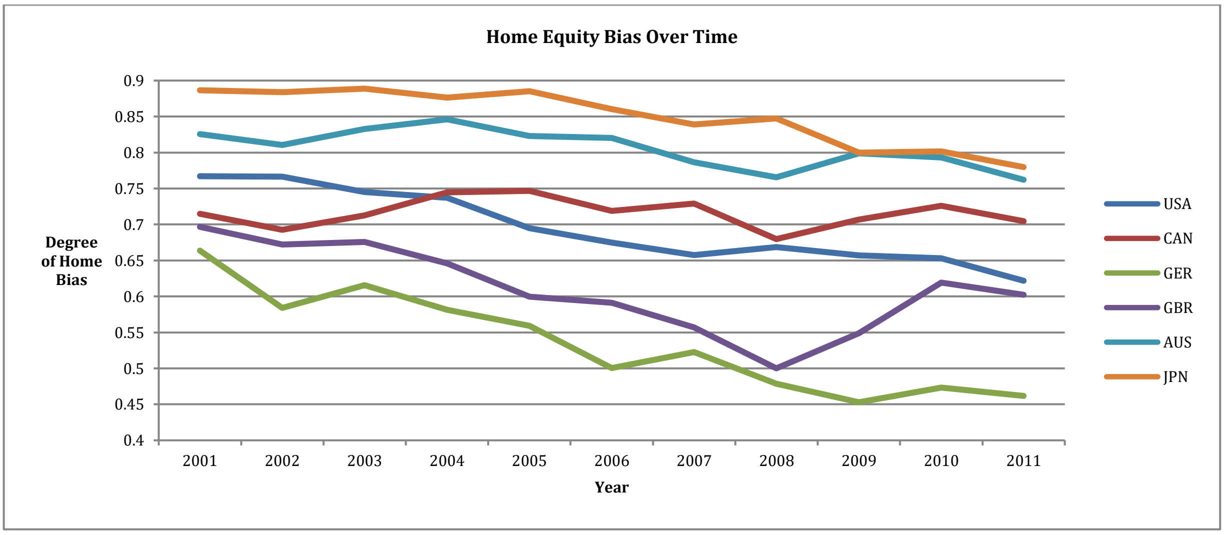

Fig. 3: Home equity bias over time; Home bias calculated from data from CPIS and World Federation of Exchange

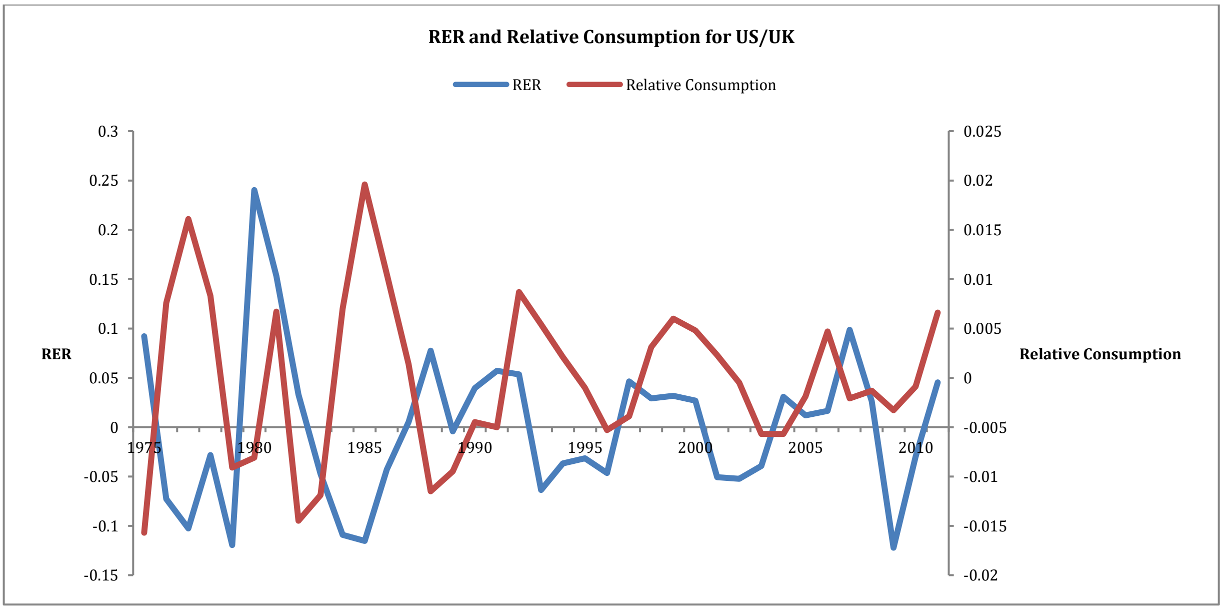

Fig. 4: Real exchange rate and relative consumption for USA and GBR; Annual data from World Bank. Both series are logged and HP-filtered using the smoothing parameter $\lambda = 6.25$

A very foreign biased portfolio of stocks in the non-tradable sector is needed to generate equity home bias in the portfolio of stocks in the tradable sector. In the data, however, the degree of equity home bias in the non-tradable sector is much higher relative to that of the tradable sector (Kang and Stulz, 1997; Denis and Huizinga, 2004; Hnatkovska, 2010). Heathcote and Perri (2013) generated home bias with capital accumulation. This results in pro-cyclical investment expenditure and counter-cyclical dividends. Thus, home equity is perfectly negatively correlated with home labor income, which makes it useful to hedge labor income risk. However, labor income and dividend payment are positively correlated in the data for G7 countries, casting doubt on Heathcote and Perri's key mechanism for generating equity home bias (Coeurdacier et al., 2010). Engel and Matsumoto (2009) showed that a forward position in the foreign exchange market can ensure perfect risk sharing with nominal rigidity. Thus, a complete market equilibrium can be achieved even with an equity home biased portfolio. However, the implied long position of home investors on domestic bond contradicts the fact that the U.S appears short on the dollar and long on foreign exchange (Obstfeld, 2007). Tille and Wincoop (2010) used cost in asset trade to generate equity home bias. Fitzgerald's (2012) empirical tests found that the null hypothesis of frictionless asset markets within developed countries cannot be rejected. Coeurdacier et al. (2007) used redistributive shocks and "iPod" shocks. Such shocks are of debatable origin and need more micro-foundation. Coeurdacier and Rey (2013) explained home bias with investment efficiency shocks.

International portfolio models that assume market incompleteness include Pesenti and Wincoop (2002), Hnatkovska (2010), and Feng (2013). Pesenti and Wincoop (2002) built a portfolio balance, endowment economy model where stocks of the non-tradable endowment are not traded. They obtain moments of stock returns and tradable and non-tradable consumptions, and they conclude that the optimal portfolio should be slightly home-biased.

Hnatkovska (2010) built a DSGE model with similar assumptions. In her model, bias in the consumption of tradable goods generates home bias. When home non-tradable consumption increases above foreign consumption, home demand for tradable goods increases. Since home consumption of tradable goods consists of large home goods, home agents should hold home equity in the tradable sector to hedge non-tradable sector technology shocks. The findings of these papers suggest that market incompleteness could be an answer to the international portfolio puzzle. However, it is uncertain whether their results still hold when the labor income is present. In addition, it is complicated, if not currently impossible, for one to extend their models to include labor income in a standard DSGE framework. The numerical method used to solve for the optimal portfolio in Hnatkovska (2010) relies critically on the closed-form solution for dynamic portfolio holdings given conditional means and variance of returns, which was developed by Campbell et al. (2003). With labor income, this method does not yield a closed-form solution for portfolio holdings (Viceira, 2001; Campbell and Viceira, 2002). Feng (2013) built a model that can generate home equity bias with the incomplete market, endogenous labor supply, and taste shock. She solves for the optimal portfolio which depends on the covariance of labor income and tastes shocked with foreign equity excess return. With the correlation measured from data, a home equity bias portfolio is implied. However, it is unclear whether the model generates a high positive correlation between home equity return and home labor income. To see why this is the case, log linearizes the consumption, leisure first order conditions to get: $\hat{T}_t + \widehat{w}_t - p\hat{C}_t = k\hat{L}_t$ where T_t, w_t, C_t *and* L_t are taste shock, wage, consumption, and labor supply. k is the inverse of the Frisch elasticity of labor supply. For simplicity, assume further that labor is inelastically supplied and therefore $k = 0$, the equation becomes: $\hat{T}_t + \widehat{w}_t - p\hat{C}_t = 0$. Thus, when a positive taste shock hits, wage tends to be negatively correlated with consumption, and consequently, negatively correlated with domestic equity return. Since it is unclear whether the model generates a strong positive correlation between domestic equity return and human capital return in a DSGE setting, it is unclear whether the model has solved the puzzle identified by Baxter and Jermann (1997).

In this paper, I extend the work by Pesenti and van Wincoop (2002) and Hnatkovska (2010) and include a production economy. The percentage of home equity held in the home portfolio of stocks in the tradable sector generated by my model is 94%, despite a 64% labor income share in the GDP and the strongly positive unconditional correlation of human capital return and equity return. The optimal portfolio is insensitive to the change in the labor income share. In addition, the unconditional correlation of consumption differential and the real exchange rate is zero.

In my model, market incompleteness and non-tradable goods tilt the home portfolio toward home equity. When the market is incomplete, home agents cannot fully ensure against non-tradable sector relative technology shocks. When favorable non-tradable sector relative technology shocks hit, home non-tradable consumption is high, and therefore, the home marginal utility of tradable is high, due to the complementary relationship between the two goods. At the same time, labor mobility across sectors increases home tradable output, making home tradable sector equity a good asset to hedge non-tradable sector relative technology shocks. When labor income is a negligible part of GDP, it is intuitive that home agents will hold a home-biased portfolio of tradable sector equity, as seen in Pesenti and van Wincoop (2002) and Hnatkovska (2010). As labor income share increases, on the one hand, home agents would like to hold more foreign equity to hedge the positive correlation of home equity return and labor income. On the other hand, as capital share decreases, home agents need more home equity to hedge non-tradable sector relative technology shocks. The change in the degree of home bias when the labor income share increases are small, which is what we observe in the data.

This paper is also related to the literature on the consumption differential-real exchange rate correlation puzzle. In a standard complete market framework, consumption differential and real exchange rates between two countries should be perfectly positively correlated (Backus and Smith, 1993). This is not the case in the data since the correlation is low and often negative. Figs. 5 and 6 graphs the consumption differential and real exchange rate for the last 37 years between the U.S. and U.K., and the U.S. and Japan. Table 2 reports the correlation between the two series (Data is from the World Bank. The HP filter parameter is 6.25 as in Ravn and Uhlig (2002) for annual data).

Benigno and Thoenissen (2008) showed that with an incomplete market, tradable sector technology shocks generate a strong negative consumption differential-real exchange rate correlation while non-tradable sector technology shocks generate a strong positive correlation. The model is convincing if one believes that tradable sector technology stocks are the prevailing source of fluctuation. The model can generate a low consumption-real exchange rate correlation when the tradable sector productivity shocks are seven times more volatile and three times more persistent than the non-tradable sector. Corsetti et al. (2008) generated a negative consumption differential-real exchange rate in line with data using highly persistent shocks perfectly correlated across sectors.

Table 2: Consumption differential and real exchange rate correlation

	$Cor(C_t^D, RER_t)$	$Cor(\Delta C_t^D, \Delta RER_t)$
US-UK	-0.33	-0.17
US-JPN	0.42	0.39

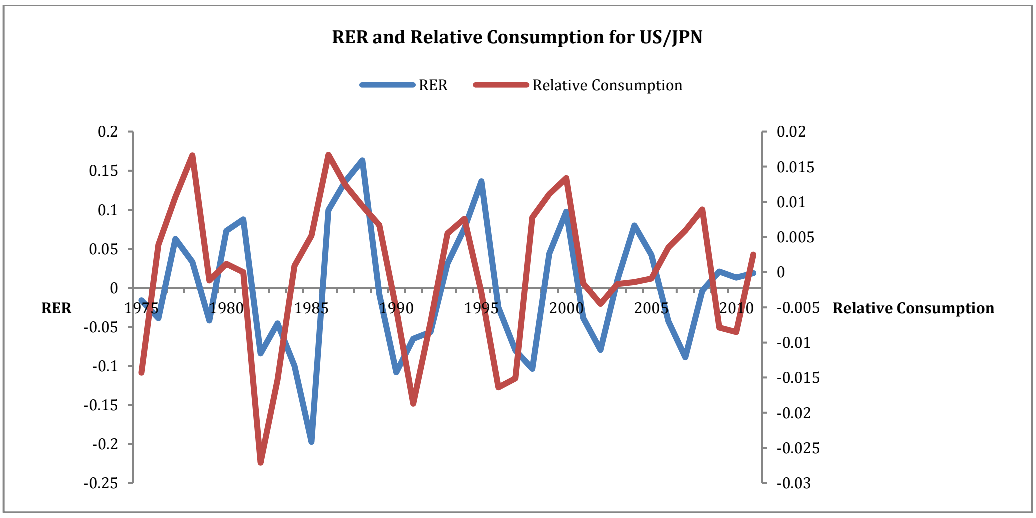

Fig. 5: Real exchange rate and relative consumption (for USA and JPN); Annual data from World Bank. Both series are logged and HP-filtered using the smoothing parameter $\lambda = 6.25$

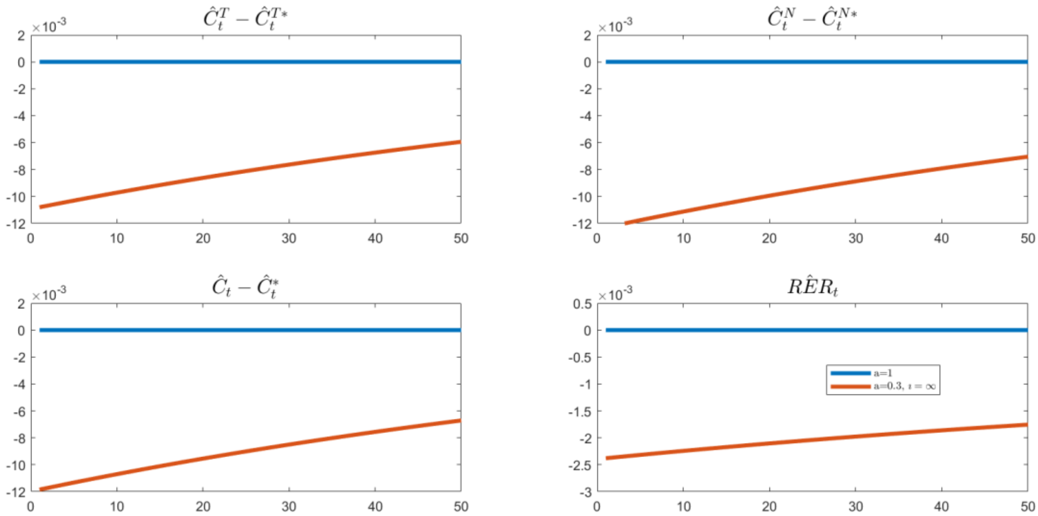

Fig. 6: Complete market impulse responses, tradable sector relative technology shock

In my model, I investigate the consumption differential-real exchange rate puzzle jointly with the home equity bias puzzle. Doing so helps me identify one channel that can generate low correlation that has not been identified before. Following tradable sector relative technology shocks, the correlation between consumption and the real exchange rate is negative, which is similar to the results in the previous literature.

Following non-tradable sector relative technology shocks, home non-tradable consumption, and therefore tradable consumption, rise above foreign consumption. Since the home portfolio does not contain enough home equity to support such an increase in consumption, home agents have to spend a fraction of their permanent wealth. Thus, in subsequent periods when home wealth deteriorates, home consumption decays at a much faster rate than other variables in the model, including the real exchange rate. The results with non-tradable sector productivity shocks are different to those found in previous literature, which usually find that non-tradable technology shocks generate a perfect correlation between consumption differential and real exchange rate. The unconditional consumption differential-real exchange rate in my model is close to zero with more convincing shock processes.

2. Model

The model framework is built upon Ghironi et al. (2009) and Devereux and Sutherland (2010). I use Devereux and Sutherland's (2011) solution method to solve for the optimal portfolio. There are two symmetric countries, each has the size of 1/2, with tradable and non-tradable sectors. Following Pesenti and Wincoop (2002) and Hnatkovska (2010), I assume market incompleteness, and equities of firms in the non-tradable sector are not traded internationally (Kang and Stulz, 1997; Denis and Huizinga, 2004; Hnatkovska, 2010). Prices are flexible and labor is endogenous and mobile across sectors. Shocks are log AR (1), sectoral technology shocks, and uncorrelated across countries and sectors.

The basket of tradable goods consumed at home is given by:

$$C_t^T = \left[\left(\frac{1}{2}\right)^{\frac{1}{\omega}} (C_t^H)^{\frac{w-1}{w}} + \left(\frac{1}{2}\right)^{\frac{1}{\omega}} (C_t^F)^{\frac{w-1}{w}}\right]^{\frac{w}{w-1}} \quad w > 0$$

where, C_t^H and C_t^F denote consumption sub-baskets consumed at the home of both home and foreign tradable goods, given by Dixit-Stiglitz aggregates:

$$C_t^H = \left[2^{\frac{1}{\epsilon}} \int_0^{\frac{1}{2}} c_t^H(z)^{\frac{\epsilon-1}{\epsilon}} dz\right]^{\frac{\epsilon}{\epsilon-1}},$$

$$C_t^F = \left[2^{\frac{1}{\epsilon}} \int_{\frac{1}{2}}^1 c_t^F(z^*)^{\frac{\epsilon-1}{\epsilon}} dz^*\right]^{\frac{\epsilon}{\epsilon-1}} \quad \text{with } \epsilon > 1.$$

The corresponding price indexes are:

$$P_t^T = \left[\frac{1}{2}(P_t^H)^{1-\omega} + \frac{1}{2}(P_t^F)^{1-\omega}\right]^{\frac{1}{1-\omega}},$$

$$P_t^H = \left[2\int_0^{\frac{1}{2}} (P_t^H(z))^{1-\epsilon} dz\right]^{\frac{1}{1-\epsilon}},$$

$$P_t^F = \left[2\int_{\frac{1}{2}}^1 (P_t^F(z^*))^{1-\epsilon} dz^*\right]^{\frac{1}{1-\epsilon}}$$

The non-tradable consumption aggregate and price index are:

$$C_t^N = \left[\int_0^1 c_t^N(z)^{\frac{\epsilon-1}{\epsilon}} dz\right]^{\frac{\epsilon}{\epsilon-1}},$$

$$P_t^N = \left[\int_0^1 P_t^N(z)^{1-\epsilon} dz\right]^{\frac{1}{1-\epsilon}}$$

Home agents' maximization problem is:

$$max\ E_0 \sum \gamma_t \left\{\frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right\}, \sigma > 0, \varphi > 0.$$

I follow Schmitt-Grohe and Uribe (2003) and assume endogenous discount factors that follow the following process:

$$\gamma_{t+1} = \gamma_t \beta (C_{At}^T)^{-n} / (\bar{C}_{At}^T)^{-n}, n > 0$$

where, C_{At}^T and $\bar{C}_{At}^T$ are country aggregate tradable good consumption at time t and its initial symmetric steady state. Agents take γ_t as exogenous and do not internalize the impact of their consumption on the discount factor. Consumption is an aggregate of tradable and non-tradable consumption:

$$C_t = [a^{\frac{1}{\theta}}(C_t^T)^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}}(C_t^N)^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}}, \theta > 0$$

The parameter a controls for the relative size of tradable and non-tradable sectors. The budget constraints in units of tradable consumption baskets are given by:

$$C_t^T + p_t^N C_t^N + a_{1t} + a_{2t}$$

$$= r_{1t}a_{1t-1} + r_{2t}a_{2t-1} + d_t^T + d_t^N + w_t L_t$$

where, p_t^N is the price of the basket of non-tradable in terms of tradable (i.e. $p_t^N = P_t^N/P_t^T$). a_{1t} and a_{2t} are home real holdings of domestic and foreign tradable equities. r_{1t} and r_{2t} are returns on home and foreign equities in the tradable sector. d_t^T and d_t^N are dividends of home tradable and non-tradable sectors. w_t is the wage and L_t is the total labor supply. The problem for foreign agents is similar. Foreign variables are denoted with asterisks.

The first-order conditions for home agents are:

$$\frac{C_t^{\frac{1}{\theta}-\frac{1}{\sigma}} a^{\frac{1}{\theta}}}{(C_t^T)^{\frac{1}{\theta}}} = \lambda_t, \tag{1}$$

$$\frac{C_t^{\frac{1}{\theta}-\frac{1}{\sigma}} (1-a)^{\frac{1}{\theta}}}{(C_t^N)^{\frac{1}{\theta}}} = p_t^N \lambda_t, \tag{2}$$

$$\chi L_t^{\frac{1}{\varphi}} = w_t \lambda_t, \tag{3}$$

$$\lambda_t = \frac{\gamma_{t+1}}{\gamma_t} E_t[\beta \lambda_{t+1} r_{1t+1}], \tag{4}$$

$$\lambda_t = \frac{\gamma_{t+1}}{\gamma_t} E_t[\beta \lambda_{t+1} r_{2t+1}], \tag{5}$$

where, λ_t is the Lagrangian multiplier of the budget constraint. The discount factor at $t+1$ is known at time t and appears outside the expectation operator.

Firm z's production is linear in labor and is given by:

$$y_t^j(z) = Z_t^j L_t^j(z), \qquad j = T, N,$$

where, $y_t(z)^T$, $y_t^N(z)$, $L_t^T(z)$ and $L_t^N(z)$ are the outputs and labor demands of individual firms in the tradable and non-tradable sectors. Z_t^T and Z_t^N are technologies in the tradable and non-tradable sectors and their log deviations from steady-state, $\hat{z}_{t+1}^T$ and $\hat{z}_{t+1}^N$, follow AR(1) processes as follows:

$$\begin{bmatrix} \hat{z}_{t+1}^T - \hat{z}_{t+1}^{T^*} \\ \hat{z}_{t+1}^N - \hat{z}_{t+1}^{N^*} \end{bmatrix} = \begin{bmatrix} \rho^T & 0 \\ 0 & \rho^N \end{bmatrix} \begin{bmatrix} \hat{z}_t^T - \hat{z}_t^{T^*} \\ \hat{z}_t^N - \hat{z}_t^{N^*} \end{bmatrix} + \begin{bmatrix} e_{t+1}^T \\ e_{t+1}^N \end{bmatrix}$$

where, e_t^T and e_t^N are jointly normally distributed with mean zero and covariance matrix:

$$E_t \left[\begin{bmatrix} e_{t+1}^T \\ e_{t+1}^N \end{bmatrix} \begin{bmatrix} e_{t+1}^T & e_{t+1}^N \end{bmatrix}\right]$$
$$= \begin{bmatrix} (\sigma^T)^2 & COV(e_{t+1}^T, e_{t+1}^N) \\ COV(e_{t+1}^T, e_{t+1}^N) & (\sigma^N)^2 \end{bmatrix}$$
$$= (\sigma^N)^2 \begin{bmatrix} \iota & \rho^{TN}\sqrt{\iota} \\ \rho^{TN}\sqrt{\iota} & 1 \end{bmatrix}.$$

where, $\sigma^T, \sigma^N, \rho^{TN}$ are standard deviations and correlation of e_t^T and e_t^N. ι is the variance ratio of the two shocks. Firm revenues are distributed as labor income and dividends. Firms' profit maximizing behaviors yield the following conditions for dividends, prices, and labor incomes:

$$d_t(z)^j = \frac{p_t(z)^j y_t(z)^j}{\epsilon}, \qquad p_t(z)^j = \frac{\epsilon}{\epsilon-1}\frac{w_t^j}{Z_t^j},$$

$$w_t L_t(z)^j = p_t(z)^j y_t(z)^j \frac{\epsilon-1}{\epsilon}, \qquad j = T, N,$$

where, prices are in units of the tradable consumption basket. Aggregate over tradable and non-tradable sectors to get the total dividends, prices, and labor income payments in each sector:

$$d_t^j = \frac{p_t^j y_t^j}{\epsilon} \qquad p_t^j = \frac{\epsilon}{\epsilon-1}\frac{w_t}{z_t^j}$$
$$w_t L_t^j = p_t^j y_t^j \frac{\epsilon-1}{\epsilon} \qquad j = T, N$$

3. Solving for the optimal portfolio

Combining Eqs. 4 and 5 from the consumers' first-order conditions, we have:

$$E_t\Big[\frac{C_t^{\frac{1}{\theta}-\frac{1}{\sigma}}}{(C_t^T)^{\frac{1}{\theta}}} r_{1t+1}\Big] = E_t\Big[\frac{C_t^{\frac{1}{\theta}-\frac{1}{\sigma}}}{(C_t^T)^{\frac{1}{\theta}}} r_{2t+1}\Big]$$

Denote that $\hat{x}_t$ is the log deviation of the variable x_t from its steady state. To solve for the optimal portfolio, I follow Devereux and Sutherland (2011) and take a second-order Taylor expansion of the above equation around the steady-state, which yields the following equation:

$$E_t[\hat{r}_{xt+1} + \tfrac{1}{2}(\hat{r}_{1t+1}^2 - \hat{r}_{2t+1}^2) - \tfrac{1}{\theta}\hat{C}_{t+1}^T \hat{r}_{xt+1} + (\tfrac{1}{\theta} - \tfrac{1}{\sigma})\hat{C}_{t+1}\hat{r}_{xt+1}] = 0 + 0(\epsilon^3)$$

where, $0(\epsilon^3)$ is a residual which contains all terms of order higher than two, which can be ignored in a second-order approximation. r_{xt} is the return differential between home and foreign stocks: $r_{xt} \equiv r_{1t} - r_{2t}$ and $\hat{r}_{xt} \equiv \hat{r}_{1t+1} - \hat{r}_{2t+1}$. Applying a similar procedure to the foreign first-order conditions gives us:

$$E_t[\hat{r}_{xt+1} + \tfrac{1}{2}(\hat{r}_{1t+1}^2 - \hat{r}_{2t+1}^2) - \tfrac{1}{\theta}\hat{C}_{t+1}^{T*} \hat{r}_{xt+1} + (\tfrac{1}{\theta} - \tfrac{1}{\sigma})\hat{C}_{t+1}^{*}\hat{r}_{xt+1}] = 0 + 0(\epsilon^3)$$

One can rearrange the above two equations to get the following equations:

$$E_t\left[\{-\tfrac{1}{\theta}(\hat{C}_{t+1}^T - \hat{C}_{t+1}^{T*}) + (\tfrac{1}{\theta} - \tfrac{1}{\sigma})(\hat{C}_{t+1} - \hat{C}_{t+1}^{*})\}\hat{r}_{xt+1}\right] = 0 + 0(\epsilon^3) \tag{6}$$

$$E_t[\hat{r}_{xt+1}] = E_t[-\tfrac{1}{2}(\hat{r}_{1t+1}^2 - \hat{r}_{2t+1}^2) + \tfrac{1}{2\theta}(\hat{C}_{t+1}^T + \hat{C}_{t+1}^{T*})\hat{r}_{xt+1} - \tfrac{1}{2}(\tfrac{1}{\theta} - \tfrac{1}{\sigma})(\hat{C}_{t+1} + \hat{C}_{t+1}^{*})\hat{r}_{xt+1}] + 0(\epsilon^3) \tag{7}$$

Eq. 6 is the portfolio optimality condition. Note that when the size of the non-tradable sector is zero, and C_{t+1}=C_{t+1}^T, we get the equation in Devereux and Sutherland (2011): $E_t[(\hat{C}_{t+1} - \hat{C}_{t+1}^{*})\hat{r}_{xt+1}] = 0$. Eq. 7 indicates that up to first-order approximation $E_t[\hat{r}_{xt+1}] = 0$. This is the same result as in Devereux and Sutherland (2011).

Define $W_t = \alpha_{1t} + \alpha_{2t}$ to be total net claims of home agents on the foreign country at the end of period t (i.e. the net foreign assets of home agents). The log deviation of W_t is defined as: $\widehat{W}_t = (W_t - \overline{W})/p^H y^T$ where $\overline{W}$, p^H and y^T are initial steady-state values of home net foreign assets, tradable price, and tradable output respectively. Let $\bar{\alpha} = \alpha_1/\beta p^T y^T$. Combining home and foreign budget constraints, first-order conditions for asset holdings, and shock processes, one can derive the dynamics of tradable consumption differentials and net foreign assets (Detailed derivations are given in Appendix A, Appendix B, and Appendix C).

$$\widehat{W}_t = \tfrac{1}{\beta}\widehat{W}_{t-1} + \bar{\alpha}\hat{r}_{x,t} + \tfrac{1}{2}[(AB - E)(\hat{z}_t^T - \hat{z}_t^{T*}) - (G - AC)(\hat{C}_t^T - \hat{C}_t^{T*}) - (AD - F)(\hat{z}_t^N - \hat{z}_t^{N*})], \tag{8}$$

$$(1-\xi)^i(\hat{C}_t^T - \hat{C}_t^{T*}) + \frac{1-\rho^T}{1-\xi-\rho^T}[(1-\xi)^i - (\rho^T)^i]I(\hat{z}_t^T - \hat{z}_t^{T*}) - \frac{1-\rho^N}{1-\xi-\rho^N}[(1-\xi)^i - (\rho^N)^i]K(\hat{z}_t^N - \hat{z}_t^{N*}) = E_t[\hat{C}_{t+1}^T - \hat{C}_{t+1}^{T*}], \forall i \geq 0, \tag{9}$$

where,

$$A = \frac{(1-\omega)}{\epsilon} + \frac{\epsilon-1}{\epsilon}\frac{1}{a}[1 + \varphi - \frac{\varphi(\sigma-\theta)(1-a)}{\sigma}] - \frac{\epsilon-1}{\epsilon}(1-\theta)\frac{1-a}{a}$$
$$B = \frac{\sigma a(\omega-1)}{\varphi[\sigma-(\sigma-\theta)(1-a)]+\sigma(\omega a+\theta(1-a))},$$
$$C = \frac{\varphi+\sigma(1-a)}{\varphi[\sigma-(\sigma-\theta)(1-a)]+\sigma(\omega a+\theta(1-a))},$$
$$D = \frac{\sigma(1-a)(1-\theta)+\varphi(\sigma-\theta)(1-a)}{\varphi[\sigma-(\sigma-\theta)(1-a)]+\sigma(\omega a+\theta(1-a))},$$
$$E = \frac{1}{\epsilon}(1-\omega),$$
$$F = \left[\frac{\epsilon-1}{\epsilon a}\frac{\varphi(\sigma-\theta)(1-a)}{\sigma} + \frac{\epsilon-1}{\epsilon}\frac{1-a}{a}(1-\theta)\right],$$
$$G = \left[\frac{\epsilon-1}{\epsilon a}\frac{\varphi}{\sigma} + 1 + \frac{1-a}{a}\frac{\epsilon-1}{\epsilon}\right],$$
$$I = \frac{(\sigma-\theta)(\omega-1)a(1-a)}{\varphi+\omega a+\theta(1-a)+(1-a)(\sigma-\theta)(1-a)},$$
$$K = \frac{(\sigma-\theta)(1-a)[\sigma(1-a)+\sigma\omega a+\varphi[\sigma-(\sigma-\theta)(1-a)]]}{\sigma[\varphi+\omega a+\theta(1-a)+(1-a)(\sigma-\theta)(1-a)]},$$
$$\xi = \frac{\sigma\eta}{1+C(1-a)(a-\theta)}.$$

Without the non-tradable sector, $a = 1$, $I = 0$ and $K = 0$. When the stationary inducing device is removed, $\eta = 0$ and Eq. 8 becomes: $(\hat{C}_t^T - \hat{C}_t^{T*}) = E_t[\hat{C}_{t+1}^T - \hat{C}_{t+1}^{T*}], \forall i \geq 0$. The consumption differential is a random walk that jumps immediately to its long-run level on the impact of shocks, which is the same result as in Ghironi et al. (2009) and Devereux and Sutherland (2010). We can combine Eq. 8, Eq. 9, and the no-Ponzi condition to solve for the on-impact tradable consumption differential, $(\hat{C}_t^T - \hat{C}_t^{T*})$, as a function of technology shocks (Detailed derivations are given in Appendix D).

$$\hat{C}_t^T - \hat{C}_t^{T*} = \frac{2[1-\beta(1-\xi)]\widehat{W}_t}{\beta[(G-AC)-2\bar{\alpha}(1-\omega)(1-\beta)C]} + \frac{1-\beta(1-\xi)}{[(G-AC)-2\bar{\alpha}(1-\omega)(1-\beta)C]}\left[\left\{2\bar{\alpha}(1-\beta)(1-\omega)\left((B-1) + \frac{I\beta(1-\rho^T)C}{1-\beta(1-\xi)}\right) + (AB-E) - \frac{I\beta(1-\rho^T)}{1-\beta(1-\xi)}(G-AC)\right\}\frac{(\hat{z}_t^T - \hat{z}_t^{T*})}{(1-\beta\rho^T)} - \left\{2\bar{\alpha}(1-\beta)(1-\omega)\left(D + \frac{K\beta(1-\rho^N)C}{1-\beta(1-\xi)}\right) + (AD-F) - \frac{K\beta(1-\rho^N)}{1-\beta(1-\xi)}(G-AC)\right\}\frac{(\hat{z}_t^N - \hat{z}_t^{N*})}{(1-\beta\rho^N)}\right]. \tag{10}$$

With the solution for $(\hat{C}_t^T - \hat{C}_t^{T*})$, and hence the dynamics of $(\hat{C}_{t+i}^T - \hat{C}_{t+i}^{T*})$, according to Eq. 9, we can solve for the on-impact return differential (Detailed derivations are given in Appendix E):

$$\hat{r}_{xt} = \frac{(1-\beta)(1-\omega)}{[(G-AC)-2\bar{\alpha}(1-\omega)(1-\beta)C]}\left\{[(B-1)(G-AC)+C(AB-E)]\frac{e_t^T}{1-\beta\rho^T} - [D(G-AC)+C(AD-F)]\frac{e_t^N}{1-\beta\rho^N}\right\}. \quad (11)$$

Without the non-tradable sector, $a = 1$ and the solution for the return differential coincides with the result in Ghironi et al. (2009) (The solution coincides with the case in Ghironi et al. (2009) when the government expenditure is zero and countries are symmetric):

$$\hat{r}_{xt} = \frac{\sigma(1+\varphi)(1-\beta)(\omega-1)}{(1-\beta\rho)[\varphi(\omega-1)+\sigma(\varphi+\omega)-2\bar{\alpha}(1-\omega)(1-\beta)\varphi]}e_t^T.$$

When $a = 1$ and $\varphi = 0$, or labor is in elastically supplied, the return differential is:

$$\hat{r}_{xt} = \frac{1-\beta}{1-\beta\rho}\left[\frac{(\omega-1)e_t^T}{\omega}\right],$$

which is similar to the solution found in Devereux and Sutherland (2010) (In my model, innovation to the dividend differential at time t when $a = 1$ and $\phi = 0$ is $(\omega-1)e_t^T/\omega$). Combining Eqs. 6, 10 and 11 gives the solution for α_1:

$$\alpha_1 = \frac{\beta\rho y}{2(1-\omega)(1-\beta)}\frac{-\left\{\frac{\Psi_1\Omega_1\iota}{(1-\beta\rho^T)^2}+\frac{\Psi_2\Omega_2}{(1-\beta\rho^N)^2}\right\}+\left\{\frac{(\Psi_1\Omega_2+\Psi_2\Omega_1)\rho^{TN}\sqrt{\iota}}{(1-\beta\rho^T)(1-\beta\rho^N)}\right\}}{\left\{\frac{\phi_1\Omega_1\iota}{(1-\beta\rho^T)^2}+\frac{\phi_2\Omega_2}{(1-\beta\rho^N)^2}\right\}-\left\{\frac{(\phi_1\Omega_2+\phi_2\Omega_1)\rho^{TN}\sqrt{\iota}}{(1-\beta\rho^T)(1-\beta\rho^N)}\right\}}$$

where p and y are steady-state relative price and output of home tradable sector in units of tradable consumption basket, and:

$$\phi_1 = B - 1 - IC\frac{1-\beta}{1-\beta(1-\xi)},$$
$$\phi_2 = D - KC\frac{1-\beta}{1-\beta(1-\xi)},$$
$$\Psi_1 = (AB-E) + I(G-AC)\frac{1-\beta}{1-\beta(1-\xi)},$$
$$\Psi_2 = (AD-F) + K(G-AC)\frac{1-\beta}{1-\beta(1-\xi)},$$
$$\Omega_1 = (B-1)(G-AC) + C(AB-E),$$
$$\Omega_2 = D(G-AC) + C(AD-F),$$

The total value of home equity in the tradable sector is $\beta py/((1-\beta)\epsilon)$. Therefore, the proportion of home equity in the tradable sector held by home households, δ^T, is given by:

$$\delta^T = \frac{\frac{\beta py}{(1-\beta)\epsilon}+\alpha_1}{\frac{\beta py}{(1-\beta)\epsilon}} = 1 + \left[\frac{1}{2(1-\omega)}\frac{-\left\{\frac{\Psi_1\Omega_1\iota}{(1-\beta\rho^T)^2}+\frac{\Psi_2\Omega_2}{(1-\beta\rho^N)^2}\right\}+\left\{\frac{(\Psi_1\Omega_2+\Psi_2\Omega_1)\rho^{TN}\sqrt{\iota}}{(1-\beta\rho^T)(1-\beta\rho^N)}\right\}}{\left\{\frac{\phi_1\Omega_1\iota}{(1-\beta\rho^T)^2}+\frac{\phi_2\Omega_2}{(1-\beta\rho^N)^2}\right\}-\left\{\frac{(\phi_1\Omega_2+\phi_2\Omega_1)\rho^{TN}\sqrt{\iota}}{(1-\beta\rho^T)(1-\beta\rho^N)}\right\}}\right]\epsilon.$$

4. Return to human capital

In order to calculate the return to human capital, I suppose that each agent in each country can trade the claim on the human capital to other agents of the same country. The human capital is defined as (Since the elasticity of the discount factor with respect to consumption is extremely small, $\eta = 0.001$, the result is not much different from defining human capital as the summation of the stream of wage income discounted by γt):

$$H_t = \sum_{i=0}^{\infty}\beta^i w_{t+i+1}.$$

The return on such claim is thus:

$$r_{ht} = \frac{H_t + w_t}{H_{t-1}}.$$

In equilibrium, every agent in one country will hold the same amount of human capital: $H_t^i = H_t^j$, $\forall i,j$ in the same country. Therefore, $H_t^i = H_t^j = 0$. The innovation to the human capital return differential can be expressed as (Detailed derivations are given in Appendix F):

$$\hat{r}_{xt}^h - E_{t-1}[\hat{r}_{xt}^h] = \frac{(1-\beta)}{[(G-AC)-2\bar{\alpha}(1-\omega)(1-\beta)C]}\left\{[B(G-AC) - C2\bar{\alpha}(1-\beta)(1-\omega) + C(AB-E)]\frac{e_t^T}{1-\beta\rho^T} - [D(G-AC) + C(AD-F)]\frac{e_t^N}{1-\beta\rho^N}\right\}, \quad (12)$$

where, $\hat{r}_{xt}^h = \hat{r}_t^h - \hat{r}_t^{h*}$.

5. The optimal portfolios

5.1. Benchmark calibration

I pick $\epsilon = 2.8$, which implies that the labor income share is 64% of the total output. I pick the elasticity of substitution between home and foreign trades $\omega = 1.8$. Backus et al. (1994) estimated this parameter to be approximately 1.5. Lai and Trefler (2002) estimated it to be 12 from disaggregated data. Similar to Tesar (1993), the elasticity of substitution between tradable and non-tradable goods is $\theta = 0.44$. I assume the coefficient of relative risk aversion (CRRA) $1/\sigma = 0.2$. The usual value of CRRA used in the business cycle literature is 1 or 2. However, there are empirical papers that estimate much lower values. Mankiw et al. (1985) estimated $1/\sigma$ to be in the range of 0.09 and 0.51. Amano and Wirjanto (1996) estimated $1/\sigma$ can be as low as 0.124. Pesenti and Wincoop (2002) found it to be 0.02. Thus, the value of $1/\sigma = 0.2$ is still within the range found in the empirical literature. I pick α to be 0.3, which approximately corresponds to the trade volume of the U.S in 2011 (Trade data from Bureau of Economic Analysis. GDP data from IMF). The discount factor is set to 0.95, corresponding to the annual return of 5%. Following King and Rebelo (1999) and Ghironi et al. (2009), the Frisch elasticity of labor supply is $\varphi = 4$. The autocorrelation coefficient of shocks is $\rho^T = \rho^N = 0.99$. Ireland (2001) estimated technology shock autocorrelation coefficient and find values as high as 0.9983 for quarterly data, corresponding to the value of 0.993 for annual data. The variance ratio of tradable and non-tradable sector relative technology shocks is 1.4 in Stockman and Tesar (1995), 2.5 in Corsetti et al. (2008), and 7.2 in Benigno and Thoenissen (2008). I set the variance ratio $\iota = 4$, which is within the range of the estimated. The corresponding correlation of shocks is 0.35, 0.01, and 0.34 in these papers respectively. I set the correlation of shocks to be 0.25.

5.2. Complete market

There are two assets in the model: The home and foreign equities of the tradable sector. The financial market is complete when there are only two shocks: home and foreign tradable sector technology shocks. This is the case when either the size of the non-tradable sector is 0, or the non-tradable sector relative technology shock variance is zeros.

5.2.1. Complete market without the non-tradable sector

When the size of the non-tradable sector is 0, $a = 1$ and the proportion of home equity held by home households becomes: $\delta^T = 1 - \epsilon/2$. The solution coincides with Ghironi et al. (2009) and Devereux and Sutherland (2011) (The solution without the non-tradable sector coincides with the special case of Devereux and Sutherland (2011) when capital is perfectly correlated with labor income). When $\epsilon = 2.8$, 64% of output is distributed toward labor income, the optimal portfolio is $\delta^T = \delta^T_{CM1} = -0.4$, and home agents should short sell home equity. This solution also coincided with Baxter and Jermann (1997). A foreign biased portfolio is optimal to hedge non-traded labor income risk.

5.2.2. Complete market with the non-tradable sector and non-tradable sector relative technology shock variance is zero

When $\sigma^N = 0, \iota = \infty$, the optimal portfolio is $\delta^T = \delta^T_{CM2} = -0.59$ given the benchmark calibration for the rest of the parameters. The optimal portfolio, in this case, consists of slightly less home equity compared to the case of the complete market without non-tradable goods. The intuition is in Fig. 6, which shows the impulse responses when the tradable sector relative technology shock hits.

When the non-tradable sector is present, favorable home tradable sector relative technology shock raises home productivity in the tradable sector. High home wage in the home tradable sector draws a fraction of home labor in the non-tradable sector toward the tradable sector, decreasing home non-tradable output and consumption. Home tradable consumption decreases on impact relative to foreign consumption to equalize to the marginal utility of tradable consumption across countries. Thus, home agents should hold less home equity because the on-impact consumption, in this case, is smaller, relative to the case of a complete market without non-tradable goods. The total consumption differential is highly correlated with the real exchange rate, which is consistent with the prediction of Backus and Smith (1993).

5.2.3. The optimal portfolio as a function of labor share

The blue and green lines in Fig. 7 show the relationship between δ^T and labor income share. When the financial market is complete, the optimal portfolios are highly negatively correlated with labor income share. When the labor share increases, home agents need more foreign equities to hedge the home non-traded labor income risk. Thus, the optimal portfolio δ^T consists of less home equity.

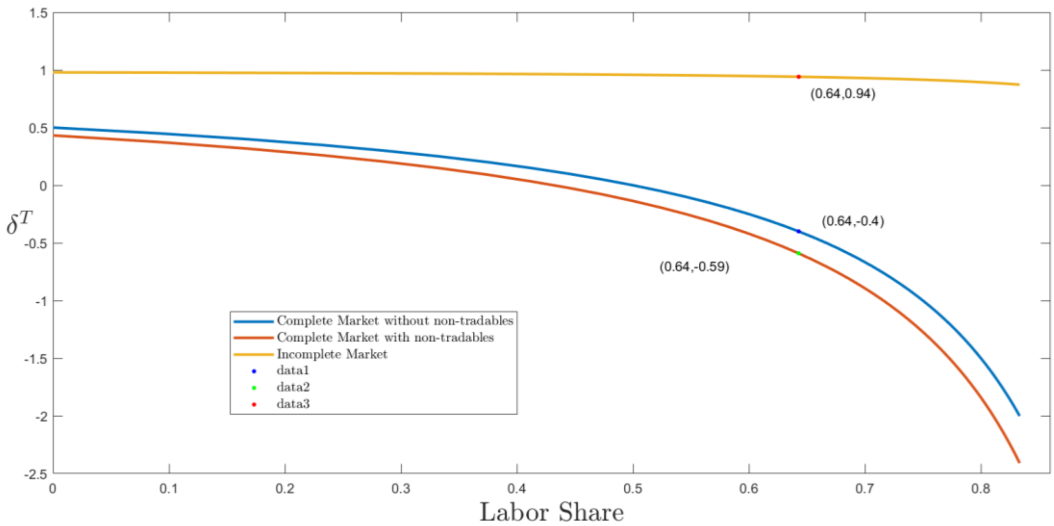

Fig. 7: Optimal portfolio and labor share

5.3. Incomplete market

With the presence of the non-tradable sector and non-tradable sector relative technology shocks, the financial market is incomplete. The optimal portfolio, given the benchmark parameters, consists of 94% home equity, despite that labor income accounts for 64% of output and domestic human capital return is highly correlated with domestic equity return. In my model, the unconditional correlation of $(\hat{r}_{xt+1} - E_t[\hat{r}_{xt+1}])$ and $(\hat{r}^h_{xt+1} - E_t[\hat{r}^h_{xt+1}])$ generated by the simulation is 0.77.

5.3.1. The optimal portfolio as a function of labor share

The red line in Fig. 7 shows the relationship between δ^T and the labor income share for the case of an incomplete market. The change of δ^T is small when the labor income share changes.

5.3.2. What generates home bias and its insensitivity to the change in the labor income share?

When the labor income share=0, my model generates home biased equity portfolios, which are the vertical intercept of the red line in Fig. 7. When the labor income share=0, the result is intuitive, given the incomplete financial market and the complementary relationship between tradable and non-tradable goods, as also observed in Pesenti and van Wincoop (2002) and Hnatkovska (2010). When the financial market is incomplete, home agents cannot fully ensure against non-tradable sector relative technology shocks. When the home non-tradable sector relative technology shock hits and home non-tradable output and consumption increase, the home marginal utility of tradable consumption is high since tradable and non-tradable goods are complements. The mobile labor market generates output co-movement across sectors, increasing the home tradable sector equity return. Home tradable sector equity is therefore a desirable asset to hedge home non-tradable sector relative technology shocks.

When the labor income shares increases, on one hand, home agents would like to hold more foreign equity of the tradable sector to hedge the positive correlation between domestic equity return and labor income generated by tradable sector relative technology shocks. On the other hand, as capital share decreases, more tradable sector equity is needed to provide the same claim of a fraction of tradable output. As a result, home agents have an incentive to hold more home equity to hedge non-tradable sector relative technology shocks. The incentives to hedge tradable and non-tradable sector relative technology shocks pull home bias in opposite directions. Consequently, the change in home bias is small when the labor income shares change.

5.4. The optimal portfolio as a function of the size of the tradable sector

Fig. 8 shows the optimal portfolio δ^T as a function of the size of the tradable sector a. The horizontal asymptote is at $\delta^T = \delta^T_{CM1} = -0.4$. When $a = 1$, we have the optimal portfolio of the complete market case without the non-tradable sector. To gain intuition on why δ^T decreases when a increases, log linearize Eq. 1:

$$\hat{C}_t^T\left[\frac{a(\sigma-\theta)-\sigma}{\theta\sigma}\right]+\hat{C}_t^N\left[\frac{(1-a)(\sigma-\theta)}{\theta\sigma}\right]=\hat{\lambda}_t\,.$$

The left-hand side is the log-linear of home agents' marginal utility of tradable consumption. The higher a is, the larger $\frac{a(\sigma-\theta)-\sigma}{\theta\sigma}$, and the more impact a given deviation of tradable consumption, $\hat{C}_t^T$, has on the marginal utility of tradable consumption. Thus, the tradable consumption risk increases. Similarly, the higher a, the smaller $\frac{(1-a)(\sigma-\theta)}{\theta\sigma}$, and the less impact a given deviation of non-tradable consumption, $\hat{C}_t^N$, has on the marginal utility of tradable consumption. Thus, the non-tradable consumption risk decreases. Consequently, home agents hold more foreign equity because the incentive to hedge tradable sector relative technology shocks is dominant.

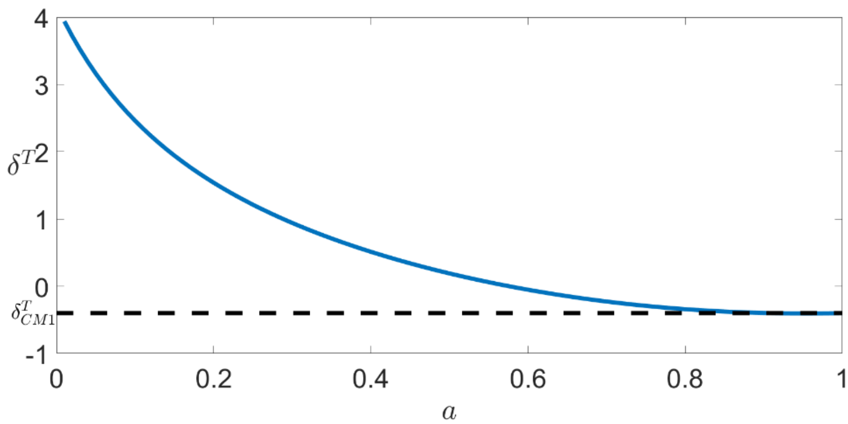

Fig. 8: Optimal portfolio and the size of the tradable sector

5.5. The optimal portfolio as a function of the variance ratio

Fig. 9 shows the optimal portfolio δ^T as a function of the variance ratio ι. The optimal portfolio decreases as ι increases. When ι increases, the non-tradable sector relative technology shock variance becomes smaller, relative to that of the tradable sector. Home agents then tilt their portfolios toward foreign equity to hedge the tradable sector relative technology shocks. The horizontal asymptote is at $\delta^T = \delta^T_{CM2} = -0.59$. When ι approaches ∞, the non-tradable sector relative technology shock variance becomes infinitesimally small relative to that of the tradable sector, and δ^T approaches the optimal portfolio as in the case of the complete market with the non-tradable sector.

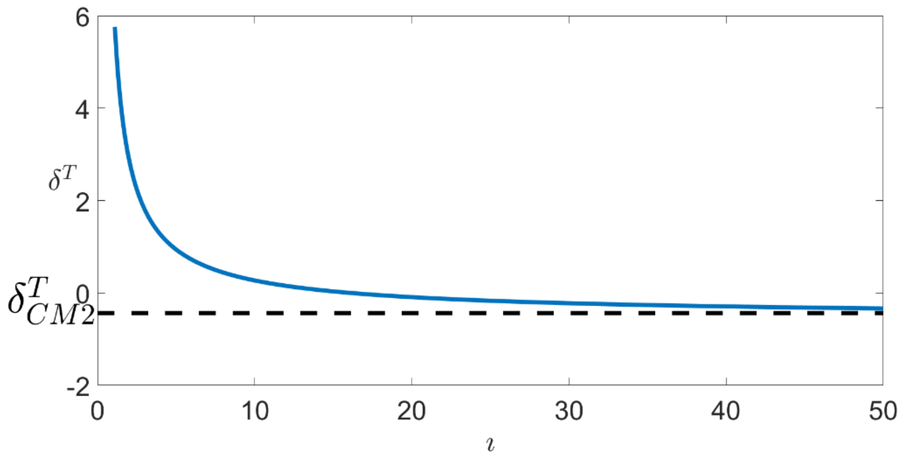

Fig. 9: Optimal portfolio and variance ratio of relative technology shocks

5.6. The optimal portfolio as a function of σ and ω

Fig. 10 shows the optimal portfolio δ^T as a function of σ and ω. Keeping σ constant, δ^T increases as ω decreases. The closer ω is to 1, the more volatile the term of trade is and the more risk-sharing it provides when tradable sector relative productivity shocks hit. Thus, the tradable sector risk becomes smaller and home agents hold a more home-biased portfolio. This result is similar to Cole and Obstfeld (1991). Keeping ω constant, δ^T increases as σ increases. The intuition also comes from the log-linear version of the marginal utility of tradable consumption:

$$\hat{C}_t^T\left[\frac{a(\sigma-\theta)-\sigma}{\theta\sigma}\right]+\hat{C}_t^N\left[\frac{(1-a)(\sigma-\theta)}{\theta\sigma}\right]=\hat{\lambda}_t\,,$$

or

$$\hat{C}_t^T\left[\frac{a(\sigma-\theta)-\sigma}{\theta\sigma}\right]+\hat{C}_t^N(1-a)\left[\frac{1}{\theta}-\frac{1}{\sigma}\right]=\hat{\lambda}_t\,.$$

The smaller σ is, the more impact a given deviation $\hat{C}_t^N$ has on the marginal utility of tradable consumption. Thus, the smaller σ is, the ”riskier” the non-tradable sector relative productivity shocks are, and a more home-biased portfolio is needed to hedge these shocks.

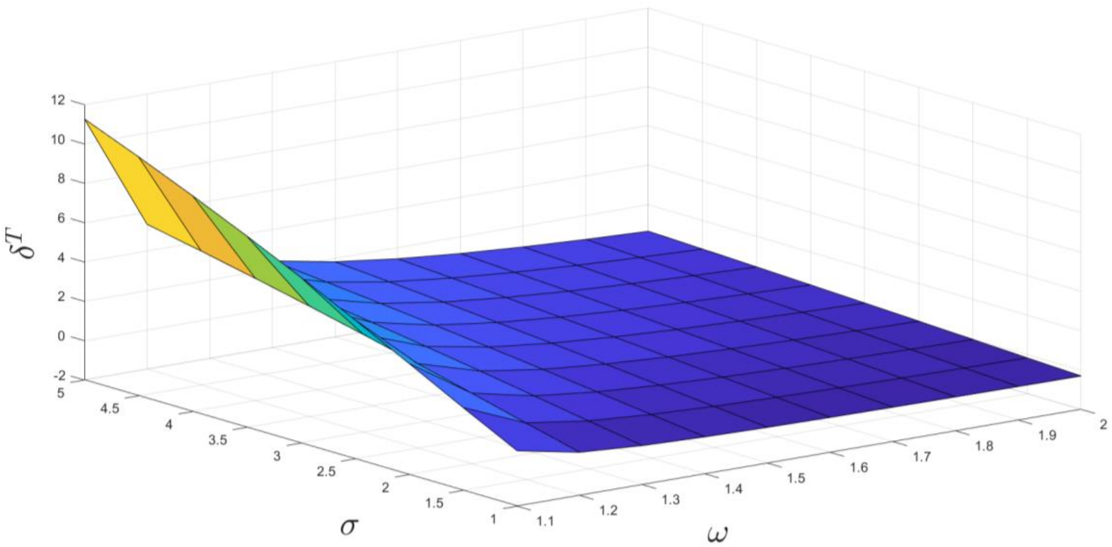

Fig. 10: Optimal portfolio as a function of σ and ω

6. Macroeconomics dynamics and consumption differential-real exchange rate correlation

6.1. Tradable sector relative technology shock

Fig. 11 shows the impulse responses when the tradable sector relative technology shock hits. Higher home technology increases the on-impact equity return differential. Subsequent equity return differentials are 0, as indicated by Eq. 7. Higher home productivity in the tradable sector increases the home wage above foreign wage. Home agents, enjoying higher labor income and portfolio income, increase their consumptions and asset positions. The relative price of non-tradable goods, which depends on relative wage, also jumps. Since the tradable consumption baskets are the same in both countries, the real exchange rate only depends on the relative price of non-tradable. Thus, the real exchange rate drops on impact. The total consumption differential and real exchange rate move in opposite directions following tradable sector relative technology shocks.

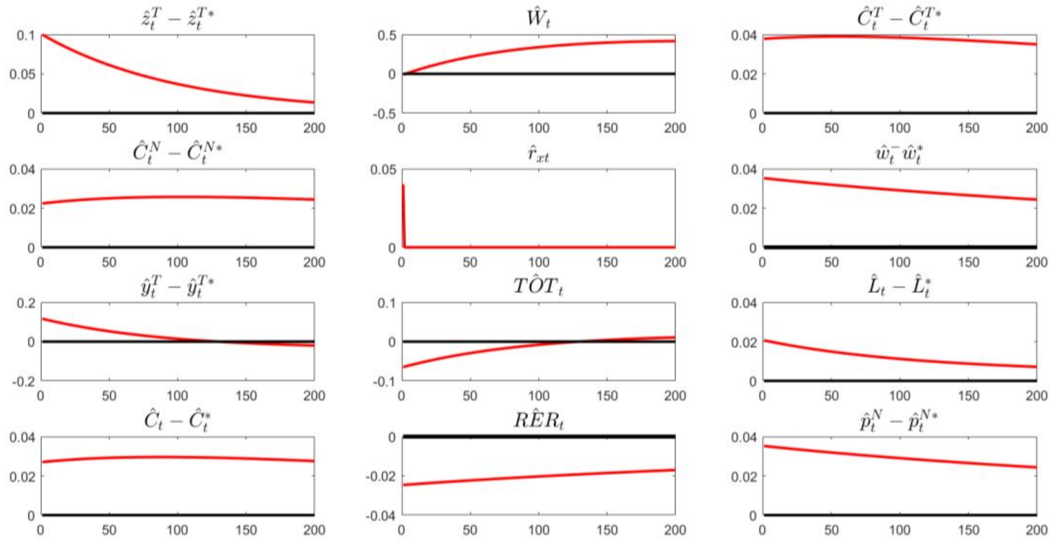

Fig. 11: Incomplete market, tradable sector relative technology shock

6.2. Non-tradable sector relative technology shock

Fig. 12 shows the impulse responses when the non-tradable sector relative technology shock hits. Due to higher home productivity in the non-tradable sector, home non-tradable output and consumption increase above foreign levels. Thus, home tradable and total consumptions also increase above foreign levels on impact. However, since the optimal portfolio does not fully insure against the non-tradable sector relative technology shocks, home agents have to spend their permanent wealth to support the higher home consumptions on impact and few periods thereafter. The home net foreign asset position deteriorates. With less wealth, home agents, in the long run, have to consume less and supply more labor at a lower wage, keeping the relative price of non-tradable at a lower level. Therefore, the real exchange rate is higher in the long run. The total consumption differential and the real exchange rate move in opposite directions in the long run.

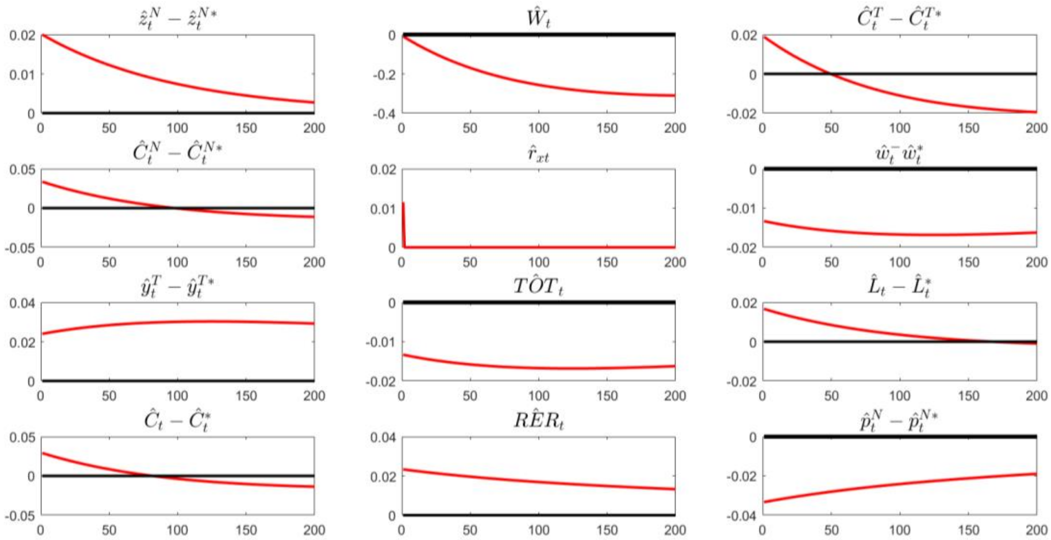

Fig. 12: Incomplete market, non-tradable sector relative technology shock

6.3. Simulation and consumption differential-Real exchange rate correlation

I generate two series of shocks e_t^T and e_t^N for 100 periods. The shocks are drawn from a normal distribution with a variance-covariance matrix described in section 3. I then feed the shocks to the model and generate the time series for variables of the model. I HP-filter these series with smoothing parameter $\lambda = 6.25$ and calculate the correlation of the cyclical components of the consumption differential and the real exchange rate. I repeat the process 1000 times and take the average correlation. The unconditional correlation generates by my model is 0.

Fig. 13 shows the unconditional correlation of consumption differential-Real exchange rate for different values of σ and ω. Keeping σ constant, the correlation changes little when ω changes. When ω increases, the term of trade is less volatile and provides less risk-sharing. Thus, for a given optimal portfolio, the volatility generated by tradable sector relative technology shocks is higher, which tends to decrease the unconditional correlation. However, as ω increases, the optimal portfolio becomes less home-biased, the jump in consumption differential due to higher home wealth generated by favorable tradable productivity shocks becomes smaller. Thus, the volatility generated by the non-tradable sector dominates, which increases the unconditional correlation. As a result, the correlation changes little when ω changes. Keeping ω constant, when σ increases, the correlation becomes more negative. As σ increases, the portfolio becomes more home biased and the volatilities generated by tradable sector relative technology shocks are larger, which decreases the correlation.

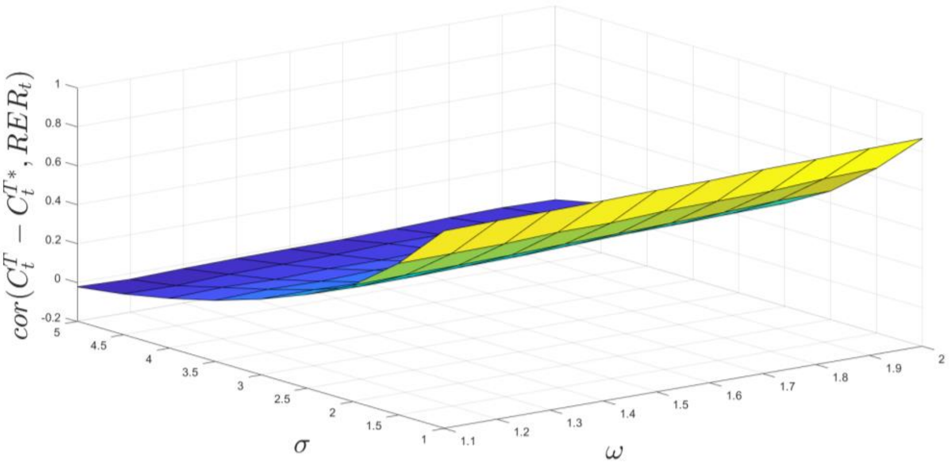

Fig. 13: Unconditional correlation of consumption differential-real exchange rate as a function of σ and ω

Fig. 14 shows the unconditional correlation of consumption differential-Real exchange rate for different values of δ^T and ω. δ^T in this case is not calculated from the model but rather, given exogenously. As one can see, for higher values of ω, keeping ω fixed, the unconditional correlation greatly differs with different values of the steady-state portfolio. Previous literature only considers the implied unconditional correlation of consumption differential-real exchange rate for given shock processes and concludes that such model and shock processes can generate a correlation that matches data. However, it is possible that the steady-state portfolio implied by such shock processes will change the correlation. Thus, by jointly incorporating the home equity bias puzzle and the Backus Smith puzzle, not only do I generate a home equity biased portfolio, but I also convincingly generate a low consumption differential-real exchange rate correlation.

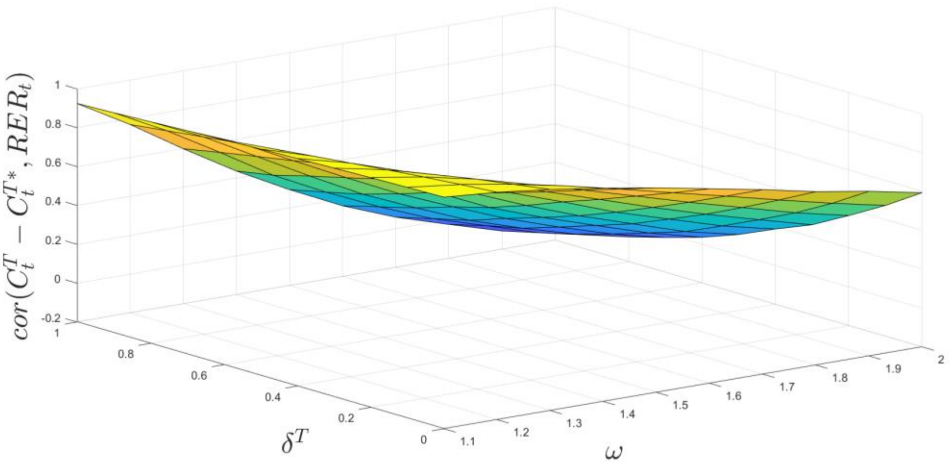

Fig. 14: Unconditional correlation of consumption differential-real exchange rate as a function of δ^T and ω

7. Conclusion

My paper has been written to explain two features of the international equity home bias puzzle. First, the equity home bias exists in every country worldwide, despite the non-traded labor income that implies optimal foreign biased portfolio. Second, the equity home bias is not negatively correlated with the fraction of labor income, which is the implication in a standard model when the labor income and equity return are positively correlated. My model generates a large home equity biased portfolio, despite the presence of non-traded human capital and the strong positive correlation of its return with domestic equity return.

My model also generates a zero unconditional correlation of consumption differential-real exchange rate. I find that the correlation depends on the coefficient of relative risk aversion, $1/\sigma$, and elasticity of substitution between home and foreign tradable goods, ω. Keeping σ constant, the correlation changes little when ω changes. Keeping ω constant, the correlation decreases when $1/\sigma$ decreases. Previous literature considers the Backus-Smith puzzle in an incomplete market setting without jointly solving for the optimal portfolio. I show that the steady-state portfolio can greatly change the result of the correlation. It is possible to apply a given model to shock processes that can generate the consumption differential-real exchange rate correlation, and for the results to match existing data. However, once the optimal portfolio implied by such shocks is considered, the correlation can be greatly different. By jointly considering the two puzzles together, I can convincingly prove that by using my model and the optimal portfolio implied by the model to analyze shock processes, it is possible to generate a low correlation.

Appendix A: The model in steady-state

In the steady-state, $p^T = p^{T*} = p^{\mathrm{N}} = p^{\mathrm{N}*} = p = 1, y^T = y^{T*} = y = C^T = C^{T*} = L^T = L^{T*} = a,\ C^H = C^F = C^{H*} = C^{F*} = \frac{a}{2}, y^N = C^N = y^{N*} = C^{N*} = L^N = L^{N*} = 1 - a, w = w^* = \frac{\epsilon-1}{\epsilon}, d^T = d^{T^*} = \frac{a}{\epsilon}, r_1 = r_2 = \frac{1}{\beta}, r_x = 0, W = W^* = 0.$

Appendix B: Derivation for Eq. 8

From the definition of Wt, we have $W_t = -W_t{}^*$ and $\widehat{W}_t = -\widehat{W}_t{}^*$ combining the log-linear version of the home and foreign budget constraints gives:

$$2\widehat{W}_t = 2\bar{\alpha}\hat{r}_{x,t} + \frac{2}{\beta}\widehat{W}_{t-1} + \frac{1}{\epsilon}(\hat{d}_t^T - \hat{d}_t^{T*}) + \frac{Lw}{py}(\widehat{w}_t - \widehat{w}_t^*) - \frac{C^T}{py}(\hat{C}_t^T - \hat{C}_t^{T*}) - \frac{\epsilon-1}{\epsilon}\frac{p^N C^N}{py}[(\hat{C}_t^N - \hat{C}_t^{N*}) + (\hat{p}_t^T - \hat{p}_t^{T*})].$$

We will express variable differentials as functions of technology, tradable consumption, and wage differentials. From the consumer first-order conditions for consumption, Eq. 1 and Eq. 2, and the firmoptimal pricing equation in the non-tradable sector, we have: $\frac{1}{\theta}(\hat{C}_t^T - \hat{C}_t^N) = \hat{p}_t^N = \widehat{w}_t - \hat{z}_t^N$. Thus, non-tradable consumption and price differentials can be written as:

$$\hat{C}_t^N - \hat{C}_t^{N*} = [(\hat{C}_t^T - \hat{C}_t^{T*}) - \theta[(\widehat{w}_t - \widehat{w}_t^*) - (\hat{z}_t^N - \hat{z}_t^{N*})],$$
$$\hat{p}_t^N - \hat{p}_t^{N*} = (\widehat{w}_t - \widehat{w}_t^*) - (\hat{z}_t^N - \hat{z}_t^{N*}).$$

Log linearizing the firm optimal pricing equation in the tradable sector gives the equation for the term of the trade (TOT): $T\hat{O}T = \hat{p}_t^T - \hat{p}_t^{T*} = (\widehat{w}_t - \widehat{w}_t^*) - (\hat{z}_t^T - \hat{z}_t^{T*})$. Demands for home tradable goods from home and foreign households are: $C_t^H = \frac{1}{2}(p_t^T)^{-\omega}C_t^T$ and $C_t^{H*} = \frac{1}{2}(p_t^T)^{-\omega}C_t^{T*}$ respectively. Market clearing condition for home tradable goods ensures: $y_t^T = C_t^H + C_t^{H^*}$. Combining the three equations above with equations for firms' dividends and prices, one can express the tradable sector dividend and output differentials as functions of tradable sector technology and wage differentials:

$$\hat{d}_t^T - \hat{d}_t^{T^*} = (1-\omega)[(\widehat{w}_t - \widehat{w}_t^*) - (\hat{z}_t^T - \hat{z}_t^{T*})],$$
$$\hat{y}_t^T - \hat{y}_t^{T^*} = -\omega[(\widehat{w}_t - \widehat{w}_t^*) - (\hat{z}_t^T - \hat{z}_t^{T*})].$$

Combining first-order conditions for tradable consumption and leisure, one can derive labor supply differential as a function of consumption, wage, and non-tradable sector technology differentials:

$$\hat{L}_t - \hat{L}_t^* = -\frac{\varphi}{\sigma}\left(\hat{C}_t^T - \hat{C}_t^{T^*}\right) + \varphi\left(1 - \frac{(\sigma-\theta)(1-a)}{\sigma}\right)(\widehat{w}_t - \widehat{w}_t^*) + \frac{\varphi(\sigma-\theta)(1-a)}{\sigma}\left(\hat{z}_t^N - \hat{z}_t^{N^*}\right).$$

We can now express $\widehat{W}_t$ as a function of technology, wage, and tradable consumption differentials:

$$2\widehat{W}_t = \frac{2}{\beta}\widehat{W}_{t-1} + 2\bar{\alpha}\hat{r}_{x,t} + \left[\frac{(1-\omega)}{\epsilon} + \frac{Lw}{py}\left(1 + \varphi - \frac{\varphi(\sigma-\theta)(1-a)}{\sigma}\right) - \frac{(\epsilon-1)}{\epsilon}\frac{(1-\theta)p^N C^N}{py}\right](\widehat{w}_t - \widehat{w}_t^*) - \frac{1}{\epsilon}(1-\omega)(\hat{z}_t^T - \hat{z}_t^{T*}) + \left(\frac{Lw}{py}\frac{\varphi(\sigma-\theta)(1-a)}{\sigma} + \frac{\epsilon-1}{\epsilon}\frac{p^N C^N}{py}(1-\theta)\right)(\hat{z}_t^N - \hat{z}_t^{N^*}) - \left(\frac{Lw}{py}\frac{\varphi}{\sigma} + \frac{C^T + p^N S^{\frac{\epsilon-1}{\epsilon}}}{py}\right)(\hat{C}_t^T - \hat{C}_t^{T^*}).$$

To further solve for the dynamics, we need to solve for the wage differential. The wage differential is determined from the labor supply and demand equations. The total labor demand is the sum of labor demands in the tradable and non-tradable sectors. The labor demand equation is:

$$\hat{L}_t - \hat{L}_t^* = \frac{L^T}{L}\left\{-\omega\left[(\widehat{w}_t - \widehat{w}_t{}^*) - \left(\hat{z}_t^T - \hat{z}_t^{T^*}\right)\right] - \left(\hat{z}_t^T - \hat{z}_t^{T^*}\right)\right\} + \frac{L^N}{L}\left\{\left(\hat{C}_t^T - \hat{C}_t^{T^*}\right) - \theta\left[(\widehat{w}_t - \widehat{w}_t{}^*) - \left(\hat{z}_t^N - \hat{z}_t^{N^*}\right)\right] - \left(\hat{z}_t^N - \hat{z}_t^{N^*}\right)\right\}$$

Combining labor demand and labor supply equations, we can solve for wage differential as a function of tradable consumption and technology differentials:

$$\widehat{w}_t - \widehat{w}_t{}^* = \frac{\varphi L + \sigma L^N}{\varphi L[\sigma-(\sigma-\theta)(1-a)] + \sigma(\omega L^T + \theta L^N)}\left(\hat{C}_t^T - \hat{C}_t^{T^*}\right) + \frac{\sigma L^T(\omega-1)}{\varphi L[\sigma-(\sigma-\theta)(1-a)] + \sigma(\omega L^T + \theta L^N)}\left(\hat{z}_t^T - \hat{z}_t^{T^*}\right) + \frac{\sigma L^N(\theta-1) - \varphi(\sigma-\theta)(1-a)L}{\varphi L[\sigma-(\sigma-\theta)(1-a)] + \sigma(\omega L^T + \theta L^N)}\left(\hat{z}_t^N - \hat{z}_t^{N^*}\right).$$

Plug the equation for wage differential into the equation for $\widehat{W}_t$, one can express the dynamics of net foreign assets as a function of tradable consumption and technology differentials:

$$2\widehat{W}_t = \frac{2}{\beta}\widehat{W}_{t-1} + 2\bar{\alpha}\hat{r}_{x,t} + \left[\frac{(1-\omega)}{\epsilon} + \frac{Lw}{py}\left(1+\varphi - \frac{\varphi(\sigma-\theta)(1-a)}{\sigma}\right) - \frac{(\epsilon-1)}{\epsilon}\frac{(1-\theta)p^N C^N}{py}\right]\left[\frac{\varphi L+\sigma L^N}{\varphi L[\sigma-(\sigma-\theta)(1-a)]+\sigma(\omega L^T+\theta L^N)}\left(\hat{C}_t^T - \hat{C}_t^{T^*}\right) + \frac{\sigma L^T(\omega-1)}{\varphi L[\sigma-(\sigma-\theta)(1-a)]+\sigma(\omega L^T+\theta L^N)}\left(\hat{z}_t^T - \hat{z}_t^{T^*}\right) - \frac{\sigma L^N(1-\theta)}{\varphi L[\sigma-(\sigma-\theta)(1-a)]+\sigma(\omega L^T+\theta L^N)}\left(\hat{z}_t^N - \hat{z}_t^{N^*}\right)\right] - \frac{1}{\epsilon}(1-\omega)(\hat{z}_t^T - \hat{z}_t^{T*}) + \left(\frac{Lw}{py}\frac{\varphi(\sigma-\theta)(1-a)}{\sigma} + \frac{\epsilon-1}{\epsilon}\frac{p^N C^N}{py}(1-\theta)\right)(\hat{z}_t^N - \hat{z}_t^{N^*}) - \left(\frac{Lw}{py}\frac{\varphi}{\sigma} + \frac{C^T + p^N S^{\frac{\epsilon-1}{\epsilon}}}{py}\right)\left(\hat{C}_t^T - \hat{C}_t^{T^*}\right).$$

Let:

$$A = \frac{(1-\omega)}{\epsilon} + \frac{Lw}{py}\left(1+\varphi-\frac{\varphi(\sigma-\theta)(1-a)}{\sigma}\right) - \frac{(\epsilon-1)}{\epsilon}\frac{(1-\theta)p^N S}{py}$$
$$= \frac{(1-\omega)}{\epsilon} + \frac{\epsilon-1}{\epsilon}\frac{1}{a}\left[1+\varphi-\frac{\varphi(\sigma-\theta)(1-a)}{\sigma}\right] - \frac{(\epsilon-1)}{\epsilon}(1-\theta)\frac{1-a}{a},$$
$$B = \frac{\sigma L^T(\omega-1)}{\varphi L[\sigma-(\sigma-\theta)(1-a)]+\sigma(\omega L^T+\theta L^N)} = \frac{\sigma a(\omega-1)}{\varphi[\sigma-(\sigma-\theta)(1-a)]+\sigma(\omega a+\theta(1-a))},$$
$$C = \frac{\varphi L+\sigma L^N}{\varphi L[\sigma-(\sigma-\theta)(1-a)]+\sigma(\omega L^T+\theta L^N)} = \frac{\varphi+\sigma(1-a)}{\varphi[\sigma-(\sigma-\theta)(1-a)]+\sigma(\omega a+\theta(1-a))},$$
$$D = \frac{\sigma L^N(\theta-1)+\varphi(\sigma-\theta)(1-a)L}{\varphi L[\sigma-(\sigma-\theta)(1-a)]+\sigma(\omega L^T+\theta L^N)} = \frac{\sigma(1-a)(1-\theta)+\varphi(\sigma-\theta)(1-a)}{\varphi[\sigma-(\sigma-\theta)(1-a)]+\sigma(\omega a+\theta(1-a))},$$
$$E = \frac{1}{\epsilon}(1-\omega)$$
$$F = \left[\frac{Lw}{py}\frac{\varphi(\sigma-\theta)(1-a)}{\sigma} + \frac{\epsilon-1}{\epsilon}\frac{p^N S}{py}(1-\theta)\right] = \left[\frac{\epsilon-1}{\epsilon a}\frac{\varphi(\sigma-\theta)(1-a)}{\sigma} + \frac{\epsilon-1}{\epsilon}\frac{1-a}{a}(1-\theta)\right],$$
$$G = \left[\frac{Lw}{py}\frac{\varphi}{\sigma} + \frac{C^T+p^N S^{\frac{\epsilon-1}{\epsilon}}}{py}\right] = \left[\frac{\epsilon-1}{\epsilon a}\frac{\varphi}{\sigma} + 1 + \frac{1-a}{a}\frac{\epsilon-1}{\epsilon}\right].$$

A, B, C, D, F, G are simply constants that depend on parameters. One can rewrite $\widehat{W}_t$ as:

$$2\widehat{W}_t = \frac{2}{\beta}\widehat{W}_{t-1} + 2\bar{\alpha}\hat{r}_{x,t} + A\left[B\left(\hat{z}_t^T - \hat{z}_t^{T^*}\right) + C\left(\hat{C}_t^T - \hat{C}_t^{T^*}\right) - D\left(\hat{z}_t^N - \hat{z}_t^{N^*}\right)\right] - E\left(\hat{z}_t^T - \hat{z}_t^{T^*}\right) + F\left(\hat{z}_t^N - \hat{z}_t^{N^*}\right) - G\left(\hat{C}_t^T - \hat{C}_t^{T^*}\right).$$

Dividing both sides by 2, we can get Eq. 8.

Appendix C: Derivation for Eq. 9

The total consumption differential can be written as a function of tradable and non-tradable consumption differentials. Substitute the non-tradable consumption differential with the function of wage and technology differentials, we can get the following equation for the total consumption differential:

$$\hat{C}_t - \hat{C}_t^* = \left(\hat{C}_t^T - \hat{C}_t^{T^*}\right) - (1-a)\theta\left[\left(\widehat{w}_t - \widehat{w}_t^{\,*}\right) - \left(\hat{z}_t^N - \hat{z}_t^{N^*}\right)\right].$$

From Appendix B, the wage differential can be written as:

$$\widehat{w}_t - \widehat{w}_t^{\,*} = C\left(\hat{C}_t^T - \hat{C}_t^{T^*}\right) + B\left(\hat{z}_t^T - \hat{z}_t^{T^*}\right) D\left(\hat{z}_t^N - \hat{z}_t^{N^*}\right)$$

Combining the above two equations with the log-linear version of Eq. 1 and Eq. 4, shock processes and the discount factor process, one can get:

$$(1-\xi)\left(\hat{C}_t^T - \hat{C}_t^{T^*}\right) + (1-p^T)I\left(\hat{z}_t^T - \hat{z}_t^{T^*}\right) - (1-p^N)K\left(\hat{z}_t^N - \hat{z}_t^{N^*}\right) = E_t\left[\left(\hat{C}_{t+1}^T - \hat{C}_{t+1}^{T^*}\right)\right],$$

where:

$$I = \frac{(\sigma-\theta)(\omega-1)a(1-a)}{\varphi+\omega a+\theta(1-a)+(1-a)(\sigma-\theta)(1-a)},$$
$$K = \frac{(\sigma-\theta)(1-a)[\sigma(1-a)+\sigma\omega a+\varphi[\sigma-(\sigma-\theta)(1-a)]]}{\sigma[\varphi+\omega a+\theta(1-a)+(1-a)(\sigma-\theta)(1-a)]},$$
$$\xi = \frac{\sigma\eta}{1+C(1-a)(\sigma-\theta)}.$$

The general formula is:

$$(1-\xi)^i\left(\hat{C}_t^T - \hat{C}_t^{T^*}\right) + \frac{1-p^T}{1-\xi-p^T}\left[(1-\xi)^i - (p^T)^i\right]I\left(\hat{z}_t^T - \hat{z}_t^{T^*}\right) - \frac{1-p^N}{1-\xi-p^N}\left[(1-\xi)^i - (p^N)^i\right]K\left(\hat{z}_t^N - \hat{z}_t^{N^*}\right) = E_t\left[\left(\hat{C}_{t+i}^T - \hat{C}_{t+i}^{T^*}\right)\right].$$

We prove it by induction. The statement is true for $i = 1$ since the general formula simply becomes:

$$(1-\xi)\left(\hat{C}_t^T - \hat{C}_t^{T^*}\right) + (1-p^T)I\left(\hat{z}_t^T - \hat{z}_t^{T^*}\right) - (1-p^N)K\left(\hat{z}_t^N - \hat{z}_t^{N^*}\right) = E_t\left[\left(\hat{C}_{t+1}^T - \hat{C}_{t+1}^{T^*}\right)\right],$$

which is shown above. Suppose the statement is true for $i = n$, and we have:

$$(1-\xi)^n\left(\hat{C}_t^T - \hat{C}_t^{T^*}\right) + \frac{1-p^T}{1-\xi-p^T}\left[(1-\xi)^n - (p^T)^n\right]I\left(\hat{z}_t^T - \hat{z}_t^{T^*}\right) - \frac{1-p^N}{1-\xi-p^N}\left[(1-\xi)^n - (p^N)^n\right]K\left(\hat{z}_t^N - \hat{z}_t^{N^*}\right) = E_t\left[\left(\hat{C}_{t+n}^T - \hat{C}_{t+n}^{T^*}\right)\right].$$

We will show that the statement is true for $i = n + 1$. The first order condition for period $t + n$ yields:

$$E_t\left[(1-\xi)\left(\hat{C}_{t+n}^T - \hat{C}_{t+n}^{T^*}\right) + (1-p^T)I\left(\hat{z}_{t+n}^T - \hat{z}_{t+n}^{T^*}\right) - (1-p^N)K\left(\hat{z}_{t+n}^N - \hat{z}_{t+n}^{N^*}\right)\right] = E_t\left[\hat{C}_{t+n+1}^T - \hat{C}_{t+n+1}^{T^*}\right]$$

We use the assumption that the statement is true for $i = n$ and substitute $E_t\left[\left(\hat{C}_{t+n}^T - \hat{C}_{t+n}^{T^*}\right)\right]$ with $(1-\xi)^n\left(\hat{C}_t^T - \hat{C}_t^{T^*}\right) + \frac{1-p^T}{1-\xi-p^T}\left[(1-\xi)^n - (p^T)^n\right]I\left(\hat{z}_t^T - \hat{z}_t^{T^*}\right) - \frac{1-p^N}{1-\xi-p^N}\left[(1-\xi)^n - (p^N)^n\right]K\left(\hat{z}_t^N - \hat{z}_t^{N^*}\right)$.

We also use the fact that $E_t\left[\hat{z}_{t+n}^j - \hat{z}_{t+n}^{j^*}\right] = \left(p^j\right)^n\left(\hat{z}_t^j - \hat{z}_t^{j^*}\right), j = T, N$:

$$(1-\xi)\left[(1-\xi)^n\left(\hat{C}_t^T - \hat{C}_t^{T^*}\right) + \frac{1-p^T}{1-\xi-p^T}\left[(1-\xi)^n - (p^T)^n\right]I\left(\hat{z}_t^T - \hat{z}_t^{T^*}\right) - \frac{1-p^N}{1-\xi-p^N}\left[(1-\xi)^n - (p^N)^n\right]K\left(\hat{z}_t^N - \hat{z}_t^{N^*}\right) + (1-p^T)(p^T)^n I\left(\hat{z}_t^T - \hat{z}_t^{T^*}\right) - (1-p^N)(p^N)^n K\left(\hat{z}_t^N - \hat{z}_t^{N^*}\right)\right] = E_t\left[\left(\hat{C}_{t+n+1}^T - \hat{C}_{t+n+1}^{T^*}\right)\right]$$

The above equation can be easily reduced to:

$(1-\xi)^{n+1}(\hat{C}_t^T-\hat{C}_t^{T^*})+\frac{1-p^T}{1-\xi-p^T}[(1-\xi)^{n+1}-(p^T)^{n+1}]I(\hat{z}_t^T-\hat{z}_t^{T^*})-\frac{1-p^N}{1-\xi-p^N}[(1-\xi)^{n+1}-(p^T)^{n+1}]K(\hat{z}_t^N-\hat{z}_t^{N^*})$
$=E_t[(\hat{C}_{t+n+1}^T-\hat{C}_{t+n+1}^{T^*})]$

Thus, the statement is true for $i=n+1$, and hence, our proof for the general formula for consumption dynamics.

Appendix D: Derivation for Eq. 10

In order to solve for the on-impact tradable consumption differential, we need to solve for the on-impact return differential as a function of tradable consumption and technology differentials. We start from the basic equation for the return of home assets:

$$r_{1t}=\frac{d_t^T+Z_{1t}}{Z_{1t-1}},$$

where, Z_{1t} is the price of the home assets in period t. The log-linear version of the above equation is:

$$\hat{r}_{1t}=(1-\beta)\hat{d}_t^T+\beta\hat{Z}_{1t}-\hat{Z}_{1t-1}.$$

The same equations hold for subsequent period returns:

$$\beta\hat{r}_{1t+1}=\beta(1-\beta)\hat{d}_{t+1}^T+\beta^2\hat{Z}_{1t+1}-\beta\hat{Z}_{1t}, \ldots etc.$$

Summing up all of the equations for returns of a home asset gives:

$$\hat{r}_{1t}=(1-\beta)[\hat{d}_t^T+\beta\hat{d}_{t+1}^T+\ldots]-\hat{Z}_{1t-1}.$$

Since the same equation applies for the returns of foreign assets, the return differential at time t is given by:

$$\hat{r}_{xt}=(1-\beta)E_t[(\hat{d}_t^T-\hat{d}_t^{T^*})+\beta(\hat{d}_{t+1}^T-\hat{d}_{t+1}^{T^*})+\ldots]-(\hat{Z}_{1t-1}-\hat{Z}_{2t-1}).$$

Replacing dividend differential with wage and technology differentials into the above equation gives:

$$\begin{aligned}\hat{r}_{xt}&=(1-\beta)E_t[(\hat{d}_t^T-\hat{d}_t^{T^*})+\beta(\hat{d}_{t+1}^T-\hat{d}_{t+1}^{T^*})+\ldots]-(\hat{Z}_{1t-1}-\hat{Z}_{2t-1})\\&=(1-\beta)(1-\omega)\sum_{i=0}^{\infty}\beta^iE_t[(\hat{w}_{t+i}-\hat{w}_{t+i}^*)-(\hat{z}_{t+i}^T-z_{t+i}^{T^*})]-(\hat{Z}_{1t-1}-\hat{Z}_{2t-1})\\&=(1-\beta)(1-\omega)\left[\frac{B-1}{1-\beta\rho^T}(\hat{z}_t^T-\hat{z}_t^{T^*})+\sum_{i=0}^{\infty}\beta^iE_tC(\hat{C}_{t+i}^T-\hat{C}_{t+i}^{T^*})-\frac{D}{1-\beta p^N}(\hat{z}_t^N-\hat{z}_t^{N^*})\right]-(\hat{Z}_{1t-1}-\hat{Z}_{2t-1}).\end{aligned}$$

From Eq. 8:

$$\begin{aligned}(G-AC)(\hat{C}_t^T-\hat{C}_t^{T^*})=\frac{2}{\beta}\widehat{W}_{t-1}-2\widehat{W}_t+2\bar{\alpha}\hat{r}_{x,t}\\+(AB-E)(\hat{z}_t^T-\hat{z}_t^{T^*})\\-(AD-F)(\hat{z}_t^N-\hat{z}_t^{N^*}).\end{aligned}$$

Similar equations hold for subsequent periods. Therefore:

$$\sum_{i=0}^{\infty}(G-AC)\beta^iE_t(\hat{C}_{t+i}^T-\hat{C}_{t+i}^{T^*})=\frac{2}{\beta}\widehat{W}_{t-1}+2\bar{\alpha}\hat{r}_{x,t}+\frac{AB-E}{1-\beta\rho^T}(\hat{z}_t^T-\hat{z}_t^{T^*})-\frac{AD-F}{1-\beta\rho^N}(\hat{z}_t^N-\hat{z}_t^{N^*}).$$

We use the no-Ponzi condition in the above summation. Plugging the equation for the on-impact return differential into the equation above gives:

$$\begin{aligned}&\sum_{i=0}^{\infty}(G-AC)\beta^iE_t(\hat{C}_{t+i}^T-\hat{C}_{t+i}^{T^*})=\\&\frac{2}{\beta}\widehat{W}_{t-1}+2\bar{\alpha}\left\{(1-\beta)(1-\omega)\left\{\frac{B-1}{1-\beta\rho^T}(\hat{z}_t^T-\hat{z}_t^{T^*})+C\sum_{i=0}^{\infty}\beta^iE_t(\hat{C}_{t+i}^T-\hat{C}_{t+i}^{T^*})-\frac{D}{1-\beta p^N}(\hat{z}_t^N-\hat{z}_t^{N^*})\right\}\right\}+\\&\frac{AB-E}{1-\beta\rho^T}(\hat{z}_t^T-\hat{z}_t^{T^*})-\frac{AD-F}{1-\beta\rho^N}(\hat{z}_t^N-\hat{z}_t^{N^*}).\end{aligned}$$

From Eq. 9 that we prove in Appendix C, we can express $\sum_{i=0}^{\infty}\beta^iE_t(\hat{C}_{t+i}^T-\hat{C}_{t+i}^{T^*})$ as function of $(\hat{C}_t^T-\hat{C}_t^{T^*})$:

$$E_t\sum_{i=0}^{\infty}\beta^i(\hat{C}_{t+i}^T-\hat{C}_{t+i}^{T^*})=\frac{\hat{C}_t^T-\hat{C}_t^{T^*}}{1-\beta(1-\xi)}+\frac{\beta(1-\rho^T)}{[1-\beta(1-\xi)][1-\beta\rho^T]}I(\hat{z}_t^T-\hat{z}_t^{T^*})-\frac{\beta(1-\rho^N)}{[1-\beta(1-\xi)][1-\beta\rho^N]}K(\hat{z}_t^N-\hat{z}_t^{N^*}).$$

Combining the two equations above, we can get Eq. 10.

Appendix E: Derivation for Eq. 11

From the equation for return differential proved in Appendix C, we have:

$$\hat{r}_{xt}=(1-\beta)(1-\omega)\left\{\frac{B-1}{1-\beta\rho^T}(\hat{z}_t^T-\hat{z}_t^{T^*})+C\sum_{i=0}^{\infty}\beta^iE_t(\hat{C}_{t+i}^T-\hat{C}_{t+i}^{T^*})-\frac{D}{1-\beta p^N}(\hat{z}_t^N-\hat{z}_t^{N^*})\right\}-(\hat{Z}_{1t-1}-\hat{Z}_{2t-1}).$$

Similar to the step used in Appendix D, substitute $\sum_{i=0}^{\infty}\beta^iE_t(\hat{C}_{t+i}^T-\hat{C}_{t+i}^{T^*})$ with time t tradable consumption and technology differentials:

$$\begin{aligned}\hat{r}_{xt}&=(1-\beta)(1-\omega)\left\{\frac{B-1}{1-\beta\rho^T}(\hat{z}_t^T-\hat{z}_t^{T^*})+C\sum_{i=0}^{\infty}\beta^iE_t(\hat{C}_{t+i}^T-\hat{C}_{t+i}^{T^*})-\frac{D}{1-\beta p^N}(\hat{z}_t^N-\hat{z}_t^{N^*})\right\}-(\hat{Z}_{1t-1}-\hat{Z}_{2t-1}),\\&=(1-\beta)(1-\omega)\left\{\frac{B-1}{1-\beta\rho^T}(\hat{z}_t^T-\hat{z}_t^{T^*})+\frac{C}{[1-\beta(1-\xi)]}\left\{(\hat{C}_t^T-\hat{C}_t^{T^*})+\frac{\beta(1-\rho^T)}{(1-\beta\rho^T)}I_t-\frac{\beta(1-\rho^N)}{(1-\beta\rho^N)}K_t\right\}-\frac{D}{1-\beta p^N}(\hat{z}_t^N-\hat{z}_t^{N^*})\right\}-(\hat{Z}_{1t-1}-\hat{Z}_{2t-1}),\\&=(1-\beta)(1-\omega)\left\{\frac{B-1}{1-\beta\rho^T}(\hat{z}_t^T-\hat{z}_t^{T^*})+\frac{C}{[1-\beta(1-\xi)]}\frac{2[1-\beta(1-\xi)]\widehat{W}_{t-1}}{\beta[(G-AC)-2\bar{\alpha}(1-\omega)(1-\beta)C]}+\frac{C}{[1-\beta(1-\xi)]}\frac{1-\beta(1-\xi)}{[(G-AC)-2\bar{\alpha}(1-\omega)(1-\beta)C]}\left[\left\{2\bar{\alpha}(1-\beta)(1-\omega)\frac{B-1}{1-\beta\rho^T}+\frac{AB-E}{1-\beta\rho^T}\right\}(\hat{z}_t^T-\hat{z}_t^{T^*})-\left\{2\bar{\alpha}(1-\beta)(1-\omega)\frac{D}{1-\beta\rho^N}+\frac{AD-F}{1-\beta\rho^N}\right\}(\hat{z}_t^N-\hat{z}_t^{N^*})\right]-\frac{D}{1-\beta\rho^N}(\hat{z}_t^N-\hat{z}_t^{N^*})\right\}-(\hat{Z}_{1t-1}-\hat{Z}_{2t-1}).\end{aligned}$$

We use the trick $\hat{r}_{xt} = \hat{r}_{xt} - E_{t-1}[\hat{r}_{xt}]$ to get rid of terms containing W_{t-1} and $(\hat{Z}_{1t-1} - \hat{Z}_{2t-1})$ and $(\hat{z}_t^j - \hat{z}_t^{j^*}), i = T, N,$

$$\hat{r}_{xt} = (1-\beta)(1-\omega)\left\{(B-1)\frac{\hat{e}_t^T}{1-\beta\rho^T} + \frac{C}{[(G-AC)-2\bar{\alpha}(1-\omega)(1-\beta)C]}\left[\{2\bar{\alpha}(1-\beta)(1-\omega)(B-1) + (AB-E)\}\frac{\hat{e}_t^T}{1-\beta\rho^T} - \{2\bar{\alpha}(1-\beta)(1-\omega)D + (AD-F)\}\frac{\hat{e}_t^N}{1-\beta\rho^N}\right] - D\frac{\hat{e}_t^N}{1-\beta\rho^N}\right\}$$

$$= \frac{(1-\beta)(1-\omega)}{[(G-AC)-2\bar{\alpha}(1-\omega)(1-\beta)C]}\left\{(B-1)[(G-AC)-2\bar{\alpha}(1-\omega)(1-\beta)C]\frac{\hat{e}_t^T}{1-\beta\rho^T} + C\left[\{2\bar{\alpha}(1-\beta)(1-\omega)(B-1) + (AB-E)\}\frac{\hat{e}_t^T}{1-\beta\rho^T} - \{2\bar{\alpha}(1-\beta)(1-\omega)D + (AD-F)\}\frac{\hat{e}_t^N}{1-\beta\rho^N}\right] - D[(G-AC)-2\bar{\alpha}(1-\omega)(1-\beta)C]\frac{\hat{e}_t^N}{1-\beta\rho^N}\right\}$$

$$= \frac{(1-\beta)(1-\omega)}{[(G-AC)-2\bar{\alpha}(1-\omega)(1-\beta)C]}\left\{[(B-1)(G-AC) + C(AB-E)]\frac{\hat{e}_t^T}{1-\beta\rho^T} - [D(G-AC) + C(AD-F)]\frac{\hat{e}_t^N}{1-\beta\rho^N}\right\}$$

This is Eq. 11.

Appendix F: Derivation for Eq. 12

The steady-state values are $\bar{r}_h = \frac{1}{\beta}$ and $\bar{H} = \frac{\beta w}{1-\beta}$. One can log-linear the definition of human capital and its return:

$$(E_t - E_{t-1})\hat{H}_t = \frac{1-\beta}{\beta}\sum_{i=0}^{\infty}\beta^i (E_t - E_{t-1})\hat{w}_{t+1+i}$$
$$(E_t - E_{t-1})\hat{r}_{ht} = (1-\beta)(E_t - E_{t-1})\hat{w}_t + \beta(E_t - E_{t-1})\hat{H}_t$$

Innovation to return to human capital can then be expressed as innovation to wages:

$$(E_t - E_{t-1})\hat{r}_{ht} = (1-\beta)\sum_{i=0}^{\infty}\beta^i (E_t - E_{t-1})[\hat{w}_{t+i}]$$
$$(E_t - E_{t-1})(\hat{r}_{ht} - \hat{r}_{ht}^*) = (1-\beta)\sum_{i=0}^{\infty}\beta^i (E_t - E_{t-1})(\hat{w}_{t+i} - \hat{w}_{t+i}^*)$$

We previously showed that $\hat{w}_{t+i} - \hat{w}_{t+i}^* = B * (\hat{z}_{t+i}^T - \hat{z}_{t+i}^{T^*}) + C(\hat{C}_{t+i}^T - \hat{C}_{t+i}^{T^*}) - D(\hat{z}_{t+i}^N - \hat{z}_{t+i}^{N^*})$. Substitute in to find the formula for $\hat{r}_{xt}^h = \hat{r}_{ht} - \hat{r}_{ht}^*$:

$$(E_t - E_{t-1})\hat{r}_{xt}^h = (1-\beta)(E_t - E_{t-1})\sum_{i=0}^{\infty}\beta^i\{B(\hat{z}_{t+i}^T - \hat{z}_{t+i}^{T^*}) + C(\hat{C}_{t+i}^T - \hat{C}_{t+i}^{T^*}) - D(\hat{z}_{t+i}^N - \hat{z}_{t+i}^{N^*})\}$$

$$= (1-\beta)\left\{\frac{B}{1-\beta\rho^T}e_t^T + (E_t - E_{t-1})C\sum_{i=0}^{\infty}\beta^i(\hat{C}_{t+i}^T - \hat{C}_{t+i}^{T^*}) - \frac{D}{1-\beta\rho^N}e_t^N\right\}$$

$$= (1-\beta)\left\{\frac{B}{1-\beta\rho^T}e_t^T + C\left\{\frac{(E_t - E_{t-1})[\hat{c}_t^T - \hat{c}_t^{T^*}]}{1-\beta} + Ie_t^T\frac{\beta(1-\rho^T)}{(1-\beta)(1-\beta\rho^T)} - Ke_t^N\frac{\beta(1-\rho^N)}{(1-\beta)(1-\beta\rho^N)}\right\} - \frac{D}{1-\beta\rho^N}e_t^N\right\}$$

$$= (1-\beta)\left\{\frac{B}{1-\beta\rho^T}e_t^T + \frac{C}{1-\beta}\frac{1-\beta}{[(G-AC)-2\bar{\alpha}(1-\omega)(1-\beta)C]}\left[\left\{2\bar{\alpha}(1-\beta)(1-\omega)\frac{B-1}{1-\beta\rho^T} + \frac{AB-E}{1-\beta\rho^T}\right\}e_t^T - \left\{2\bar{\alpha}(1-\beta)(1-\omega)\frac{D}{1-\beta\rho^N} + \frac{AD-F}{1-\beta\rho^N}\right\}e_t^N\right] - \frac{D}{1-\beta\rho^N}e_t^N\right\}.$$

Innovation to human capital return differential becomes:

$$\hat{r}_{xt}^h - E_{t-1}[\hat{r}_{xt}^h] = \frac{(1-\beta)}{[(G-AC)-2\bar{\alpha}(1-\omega)(1-\beta)C]}\left\{[B(G-AC) - C2\bar{\alpha}(1-\omega)(1-\beta) + C(AB-E)]\frac{e_t^T}{1-\beta\rho^T} - [D(G-AC) + C(AD-F)]\frac{e_t^N}{1-\beta\rho^N}\right\},$$

which is Eq. 12 in the model.

Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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