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Generalized thermoelasticity with fractional order strain of infinite medium with a cylindrical cavity

A. K. Khamis [1], A. A. El-Bary [2,*], Hamdy M. Youssef [3], Allal Bakali [1]

[1]*Departement of Mathematics, Faculty of Science, Northern Border University, Arar, Saudi Arabia*
[2]*Basic and Applied Science Institute, Arab Academy for Science and Technology, Alexandria, Egypt*
[3]*Departement of Mechanics, Faculty of Engineering and Islamic Architecture, Umm Al Qura University, Makkah, Saudi Arabia*

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ABSTRACT

In this paper, a problem of thermoelastic interactions in a homogenous isotropic thermoelastic infinite medium with a cylindrical cavity. The bounding surface of the cavity is thermally shocked and connected to a rigid body to prevent any deformation. The governing equations are taken in the context of generalized thermoelasticity with fractional order strain theory. The analytical solutions with the direct approach in the Laplace transform domain have been obtained. The numerical results for the temperatures increment, the strain, the displacement, and the stress are represented graphically with the various value of the fractional-order parameter to stand on its effect on all the studied state functions. The fractional-order parameter has significant effects on the strain, the displacement, and the stress distribution, while its effect on the temperature distribution is minimal.

1. Introduction

The first model applying fractional calculus with the idea that it is the order of the derivative of the deformation that characterizes the material's behavior was introduced by Magin and Royston (2010). In this model, the order of the derivative is zero for a Hookean solid and to one for a Newtonian fluid. Elastic and viscoelastic materials occupy the intermediate range with a fractional order parameter between zero and one (Magin and Royston, 2010).

Cartilage is a sensitive tissue that provides the lining of the joints in the body. Cartilage and tissue reveal a multi-scale architecture that spans a wide range of proteoglycan molecules and collagen to families of twisted macromolecular fibers and fibrils, and to the network of cells and an extracellular matrix that form layers in the connective tissue. The challenge for the bioengineer is to develop multi-scale modeling tools that predict the macro-scale mechanical performance of cartilage from micro-scale models so that this new model will help them (Magin and Royston, 2010). Fractional order Voigt models performed better compared to the integer-order models so, the development reported here will help in better understanding the thermoelastic properties of human soft tissue and may lead to improved diagnostic applications (Magin and Royston, 2010).

Youssef (2016) derived a new theory of thermoelasticity based on fraction order of strain, which is considered as a new modification to Duhamel-Neumann of stress-strain relation. After setting the equations which govern this theory, Youssef (2016) solved the first applications of thermoelasticity with fractional order strain for an isotropic, homogenous, one dimensional, and thermoelastic half-space based on different models of one-temperature thermoelasticity of Biot, Lord-Shulman, Green-Lindsay and Green-Naghdi type II.

Youssef (2005a; 2006a; 2006b; 2009; 2010; 2013) solved many applications of Thermoelasticity of infinite thermoelastic medium with cylindrical cavity (Youssef, 2005a; 2006a; 2006b; 2009; 2010; 2013; Ezzat and Youssef, 2013).

The theory of electro-magneto-thermo-viscoelasticity has aroused much interest in many industrial applications, particularly in a nuclear device, where there exists a primary magnetic field. Various investigations have been carried out by considering the interaction between magnetic, thermal, and strain fields. Analyses of such problems also influence various applications in biomedical engineering as well as in different geometric studies.

* Corresponding Author.
Email Address: aaelbary@aast.edu (A. A. El-Bary)
https://doi.org/10.21833/ijaas.2020.07.013
Corresponding author's ORCID profile:
https://orcid.org/0000-0002-8846-0487

Ezzat and El-Bary (2017c) have studied two temperature theories in generalized magneto-Thermo-viscoelasticity.

Many Applications of state space approach are developed for a different types of problems in Thermoelasticity (Ezzat et al., 2009; 2010; 2015a; 2015b; 2017; Ezzat and El-Bary, 2009; 2012; 2014; 2015a; 2015b; 2016a; 2016b; 2017a; 2017b; 2017c; 2018a; 2018b; El-Karamany et al., 2018; Youssef and El-Bary, 2018). Some researcher considered one temperature and other discussed two temperatures.

In the present investigation, we study the induced temperature and stress fields in a thermoelastic infinite body with a cylindrical cavity in one dimensional under the purview of the theory of thermoelasticity with fraction order strain. The medium continuum is made of an isotropic, homogeneous, and thermoelastic material. The surface of the bounding plane of the cavity is affected by thermal shock and is connected to a rigid body to avoid the radial deformation. The derived solutions are computed numerically for copper, and the results are presented in graphical form.

1.1. The governing equations

Consider a perfect conducting elastic infinite body with cylindrical cavity occupy the region $R \leq r < \infty$ of an isotropic homogeneous medium whose state can be expressed in terms of the space variable r and the time variable t such that all of the field functions vanish at infinity (Youssef, 2005a; 2006a; 2006b; 2009; 2010).

We use a cylindrical system of coordinates (r, ψ, z) with the z-axis lying along the axis of the cylinder. Due to symmetry, the problem is one-dimensional with all the functions considered depending on the radial distance r and the time t. It is assumed that there are no body forces in the medium and initially quiescent. Thus, the field equations in cylindrical one-dimensional with fractional order strain can be written as (Youssef, 2005a; 2006a; 2006b; 2009; 2010):

The equation of motion:

$$(\lambda+2\mu)(1+\tau^{\alpha}D_t^{\alpha})\frac{\partial e}{\partial r}-\gamma\frac{\partial T}{\partial r}=\rho\frac{\partial^2 u}{\partial t^2}. \qquad (1)$$

The heat equation:

$$\nabla^2\theta=\left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)\left[\frac{\rho C_E}{K}\theta+\frac{T_o\gamma}{K}(1+\tau^{\alpha}D_t^{\alpha})e\right]. \qquad (2)$$

The constitutive relations will take the forms:

$$\sigma_{rr}=2\mu(1+\tau^{\alpha}D_t^{\alpha})\frac{\partial u}{\partial r}+\lambda(1+\tau^{\alpha}D_t^{\alpha})e-\gamma\theta \qquad (3)$$

$$\sigma_{\psi\psi}=2\mu(1+\tau^{\alpha}D_t^{\alpha})\frac{u}{r}+\lambda(1+\tau^{\alpha}D_t^{\alpha})e-\gamma\theta \qquad (4)$$

$$\sigma_{zz}=\lambda(1+\tau^{\alpha}D_t^{\alpha})e-\gamma\theta \qquad (5)$$

and,

$$\sigma_{zr}=\sigma_{\psi r}=\sigma_{zz}=0 \qquad (6)$$

where e is the volume dilatation and satisfies the relation:

$$e=\frac{1}{r}\frac{\partial(ru)}{\partial r} \qquad (7)$$

and,

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}.$$

In the above equations, we apply the definition of the Riemann–Liouville fractional integral $I^{\alpha}f(t)$ written in a convolution type form Povstenko (2015):

$$I^{\alpha}f(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(t-\upsilon)^{\alpha-1}f(\upsilon)d\upsilon, t>0, \alpha>0 \qquad (8)$$

which gives Caputo fractional derivatives in the form:

$$D_t^{\alpha}f(t)=I^{-\alpha}f(t)=\frac{1}{\Gamma(1-\alpha)}\int_0^t(t-\upsilon)^{-\alpha}f'(\upsilon)d\upsilon, t>0, 1>\alpha>0 \qquad (9)$$

For convenience, we will use the following non-dimensional variables (Youssef, 2005a; 2006a; 2006b; 2009; 2010):

$$r'=c_o\eta r,\ u'=c_o\eta u,\ t'=c_o^2\eta t,\ \tau_o'=c_o^2\eta\tau_o,\ \tau'=c_o^2\eta\tau,\ R'=c_o\eta R,\ \theta'=\frac{\theta}{T_o},\ \sigma'=\frac{\sigma}{\mu},$$

where $c_o^2=\frac{\lambda+2\mu}{\rho}$ and $\eta=\frac{\rho C_E}{K}$.

Eqs. 1-5 take the following forms (the primes are suppressed for simplicity):

$$(1+\tau^{\alpha}D_t^{\alpha})\nabla^2 e-b\nabla^2\theta=\frac{\partial^2 e}{\partial t^2} \qquad (10)$$

$$\nabla^2\theta=\left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)[\theta+\varepsilon(1+\tau^{\alpha}D_t^{\alpha})e] \qquad (11)$$

$$\sigma_{rr}=\beta^2(1+\tau^{\alpha}D_t^{\alpha})e-2(1+\tau^{\alpha}D_t^{\alpha})\frac{u}{r}-\varepsilon_1\theta \qquad (12)$$

$$\sigma_{\psi\psi}=\beta^2(1+\tau^{\alpha}D_t^{\alpha})e-2(1+\tau^{\alpha}D_t^{\alpha})\frac{\partial u}{\partial r}-\varepsilon_1\theta \qquad (13)$$

$$\sigma_{zz}=(\beta^2-2)(1+\tau^{\alpha}D_t^{\alpha})e-\varepsilon_1\theta \qquad (14)$$

where, $\gamma=(3\lambda+2\mu)\alpha_T$, $\varepsilon_1=\frac{\gamma T_o}{\mu}$, $\varepsilon=\frac{\gamma}{\rho C_E}$, $\beta=\left(\frac{\lambda+2\mu}{\mu}\right)^{\frac{1}{2}}$, $b=\frac{\varepsilon_1}{\beta^2}$.

2. The governing equation in the Laplace transform domain

We apply the Laplace transform defined as:

$$\ell\{f(t)\}=\bar{f}(s)=\int_0^{\infty}f(t)e^{-st}dt. \qquad (15)$$

For both sides of Eqs. 10-14 when all the state functions are initially at rest, hence we obtain:

$$(1+\tau^{\alpha}s^{\alpha})\nabla^2\bar{e}-b\nabla^2\bar{\theta}=s^2\bar{e} \qquad (16)$$

$$\nabla^2\bar{\theta}=(s+\tau_o s^2)[\bar{\theta}+\varepsilon(1+\tau^{\alpha}s^{\alpha})\bar{e}] \qquad (17)$$

$$\bar{\sigma}_{rr}=\beta^2(1+\tau^{\alpha}s^{\alpha})\bar{e}-2(1+\tau^{\alpha}s^{\alpha})\frac{\bar{u}}{r}-\varepsilon_1\bar{\theta} \qquad (18)$$

$$\bar{\sigma}_{\psi\psi}=\beta^2(1+\tau^{\alpha}s^{\alpha})\bar{e}-2(1+\tau^{\alpha}s^{\alpha})\frac{\partial\bar{u}}{\partial r}-\varepsilon_1\bar{\theta} \qquad (19)$$

$$\bar{\sigma}_{zz}=(\beta^2-2)(1+\tau^{\alpha}s^{\alpha})\bar{e}-\varepsilon_1\bar{\theta} \qquad (20)$$

and,

$$\bar{e} = \frac{1}{r}\frac{\partial(r\bar{u})}{\partial r} = \frac{\bar{u}}{r} + \frac{\partial \bar{u}}{\partial r} \quad (21)$$

The Laplace transform of the fractional derivative is defined as (Povstenko, 2015):

$$\ell\{D_t^{\alpha} f(t)\} = s^{\alpha}\bar{f}(s) - D_t^{\alpha} I^{1-\alpha} f(0^+), 0 < \alpha < 1. \quad (22)$$

We can re-write Eq. 17 in the form:

$$(\nabla^2 - \alpha_1)\bar{\theta} = \alpha_2 \bar{e} \quad (23)$$

where

$$\alpha_1 = (s + \tau_o s^2), \alpha_2 = \varepsilon\alpha_1(1 + \tau^{\alpha} s^{\alpha}).$$

Substituting from Eq. 23 into the Eq. 16, we get:

$$(\nabla^2 - \alpha_3)\bar{e} = \alpha_4 \bar{\theta} \quad (24)$$

where

$$\alpha_3 = \frac{(s^2 + b\alpha_2)}{(1 + \tau^{\alpha} s^{\alpha})}, \alpha_4 = \frac{b\alpha_1}{(1 + \tau^{\alpha} s^{\alpha})}.$$

Eliminating $\bar{e}$from Eqs. 23 and 24, we obtain:

$$[\nabla^4 - (\alpha_1 + \alpha_3)\nabla^2 + \alpha_1\alpha_3 - \alpha_2\alpha_4]\bar{\theta} = 0. \quad (25)$$

Similarly, we can show that $\bar{e}$ satisfies the following equation:

$$[\nabla^4 - (\alpha_1 + \alpha_3)\nabla^2 + \alpha_1\alpha_3 - \alpha_2\alpha_4]\bar{e} = 0. \quad (26)$$

The finite solutions of Eqs. 25 and 26 at infinity take the forms (Youssef, 2005a; 2006a; 2006b; 2009; 2010):

$$\bar{\theta} = \sum_{i=1}^{2} A_i (p_i^2 - \alpha_3) K_0(p_i r) \quad (27)$$

and,

$$\bar{e} = \sum_{i=1}^{2} B_i K_0(p_i r) \quad (28)$$

where and K_0 (*) is the modified Bessel function of the second kind of order zero.

The constants A_1, A_2, B_1, and B_2, depending on the parameter of the Laplace transform s, while p_1^2 and p_2^2 are the roots of the characteristic equation:

$$p^4 - Lp^2 + M = 0 \quad (29)$$

where, $L = \alpha_1 + \alpha_3$ and $M = \alpha_1\alpha_3 - \alpha_2\alpha_4$

Using Eq. 24, we obtain:

$$B_i = \alpha_4 A_i, i = 1,2 \quad (30)$$

Hence, we have,

$$\bar{e} = \alpha_4 \sum_{i=1}^{2} A_i K_0(p_i r). \quad (31)$$

Using Eq. 21 and Eq. 31, we obtain:

$$\bar{u} = -\alpha_4 \sum_{i=1}^{2} \frac{A_i}{p_i} K_1(p_i r) \quad (32)$$

where K_1(*) is the modified Bessel function of the second kind of order one.

Within deriving Eq. 32, we used the relation of the Bessel function as follows:

$$\int z K_0(z) dz = -z K_1(z) \quad (33)$$

To complete the solution in the Laplace transform domain, we will consider the bounding plane of the cavity of the cylinder r=R is subjected to thermal shock and without deformation as follows:

$$\theta(R, t) = \theta_o U(t) \quad (34)$$

where $U(t)$is the unit step function and θ_0 is constant. After using the Laplace transform, we have:

$$\bar{\theta}(R, s) = \frac{\theta_o}{s}. \quad (35)$$

No deformation on the bounding plane of the cavity, which gives:

$$e(R, t) = 0 \quad (36)$$

moreover, after using Laplace transform, we have:

$$\bar{e}(R, s) = 0. \quad (37)$$

Applying the last two conditions leads to the following system of equations:

$$\sum_{i=1}^{2} A_i (p_i^2 - \alpha_3) K_0(p_i R) = \frac{\theta_o}{s} \quad (38)$$

$$\sum_{i=1}^{2} A_i K_0(p_i R) = 0. \quad (39)$$

Solving the system we obtain:

$$A_1 = \frac{\theta_o}{s(p_1^2 - p_2^2) K_0(p_1 R)}, A_2 = -\frac{\theta_o}{s(p_1^2 - p_2^2) K_0(p_2 R)}. \quad (40)$$

Then, we have heat distribution in the form:

$$\bar{\theta} = \frac{\theta_o}{s(p_1^2 - p_2^2)} \begin{bmatrix} \text{and} \frac{(p_1^2 - \alpha_3)}{K_0(p_1 R)} K_0(p_1 r) - \\ \text{and} \frac{(p_2^2 - \alpha_3)}{K_0(p_2 R)} K_0(p_2 r) \end{bmatrix} \quad (41)$$

and the deformation takes the form:

$$\bar{e} = \frac{\alpha_4 \theta_o}{s(p_1^2 - p_2^2)} \left[\frac{K_0(p_1 r)}{K_0(p_1 R)} - \frac{K_0(p_2 r)}{K_0(p_2 R)}\right]. \quad (42)$$

The displacement takes the form:

$$\bar{u} = -\frac{\alpha_4 \theta_o}{s(p_1^2 - p_2^2)} \left(\frac{K_1(p_1 r)}{p_1 K_0(p_1 R)} - \frac{K_1(p_2 r)}{p_2 K_0(p_2 R)}\right). \quad (43)$$

By substituting from Eqs. 41-43 into Eqs. 18-20, we can get the stress components in the Laplace transform domain.

To determine the conductive and thermal temperature, displacement and stress distributions in the time domain, the Riemann-sum approximation method is used to obtain the numerical results. In this method, any function in the Laplace domain can be inverted to the time domain as (Tzou, 1995):

$$f(t) = \frac{e^{\kappa t}}{t}\left[\frac{1}{2}\bar{f}(\kappa) + \mathrm{Re}\sum_{n=1}^{N}(-1)^n \bar{f}\left(\kappa + \frac{in\pi}{t}\right)\right] \quad (44)$$

where, Re is the real part, and i is an imaginary number unit. For faster convergence, numerous numerical experiments have shown that the value of κ satisfies the relation $\kappa t \approx 4.7$ (Tzou, 1995).

3. Numerical results and discussion

To illustrate the analytical procedure presented earlier, we now consider a numerical example for which computational results are given. For this purpose, copper is taken as the thermoelastic material for which we take the following values of the different physical constants:

$K = 386 \mathrm{kgmk^{-1}s^{-3}}$, $\alpha_T = 1.78\ (10)^{-5}\mathrm{k}^{-1}$, $T_o = 293\ \mathrm{k}$, $\rho = 8954 \mathrm{kgm^{-3}}$, $C_E = 383.1 \mathrm{m^2k^{-1}s^{-2}}$,
$\mu = 3.86\ (10)^{10}\mathrm{kgm^{-1}s^{-2}}$, $\lambda = 7.76\ (10)^{10}\mathrm{kgm^{-1}s^{-2}}$.

From the above values, we get the non-dimensional values of the problem as (Youssef, 2005a; 2005b; 2006a; 2006b; 2009; 2010; 2013):

$b = 0.01041$, $\varepsilon_1 = 0.0417232$, $\varepsilon = 1.618$, $\beta^2 = 4$, $R = 1.0$, $\theta_0 = 1.0$, $\tau_o = 0.02$, $\tau = 0.01$.

The numerical results of temperature increment, the strain, the displacement, and the stress distributions have been illustrated for a wide range of the dimensionless radial distance $r (R \leq r \leq 2.0)$ when the radius of the cylindrical cavity $R = 1.0$ at the instant value of dimensionless time $t = 0.02$.

The calculations have been carried out for various values of fractional order parameter $\alpha = (0.1, 0.5, 0.9)$ to stand on the effect of this parameter on all the studied functions.

Fig. 1 shows the temperature increment distribution, and we found that the effect of the fractional-order parameter has a little effect where the curves of the three cases almost coincide.

Fig. 2 represents the strain distribution, and we found that the fractional-order parameter has a significant effect. When the value of the fractional-order parameter increases, the values of the strain increase until they reach the maximum values, and then the situation is reversed.

Fig. 3 represents the displacement distribution, and we found that the fractional-order parameter has a significant effect. When the value of the fractional-order parameter increases, the values of the displacement increase until they reach the intersection point, and then the situation is reversed.

Fig. 4 represents the stress distribution, and we found that the fractional-order parameter has a significant effect. When the value of the fractional-order parameter increases, the values of the stress increase.

4. Conclusion

The paper was dealing with a problem of a thermoelastic homogenous isotropic infinite medium with a cylindrical cavity when the bounding surface of the cavity is thermally shocked and connected to a rigid body to prevent any deformation. The governing equations of the model have been taken in the context of generalized thermoelasticity with fractional order strain theory. The numerical results for the temperatures increment, the strain, the displacement, and the stress are represented graphically with the various value of the fractional-order parameter. The fractional-order parameter has impacts on the strain, the displacement, and the stress distribution, while its effect on the temperature distribution is minimal.

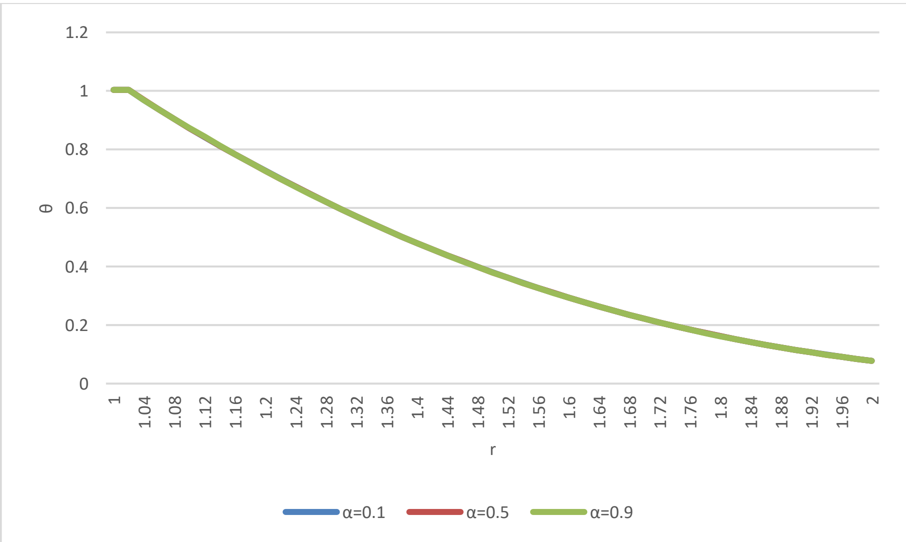

Fig. 1: The temperature increment distribution with various values of fractional order

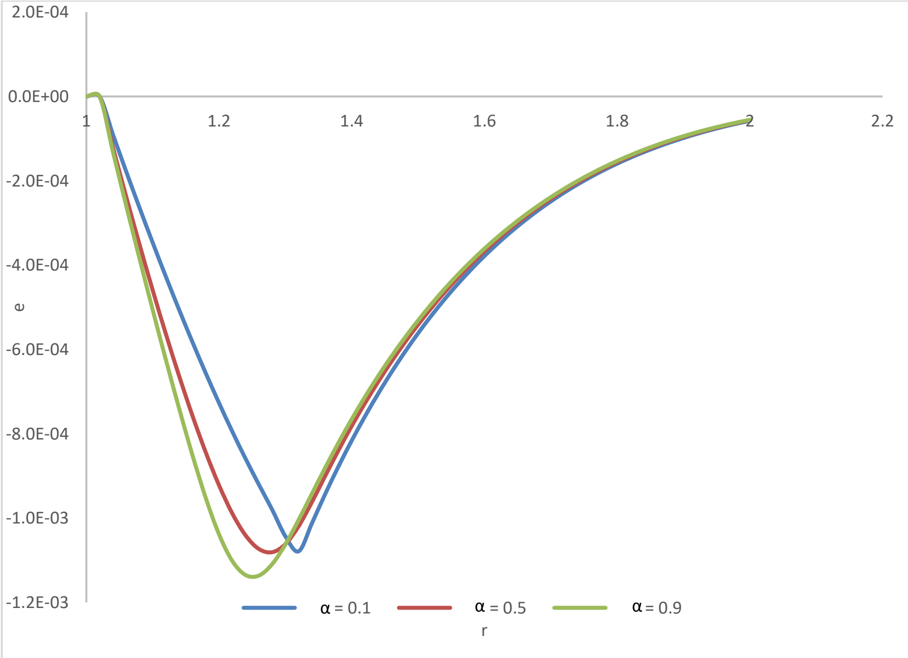

Fig. 2: The deformation distribution with various values of fractional order

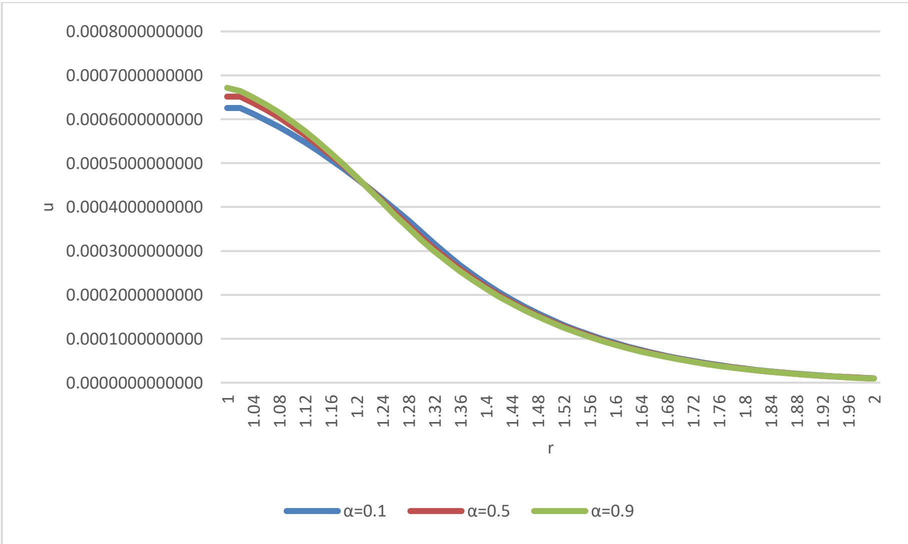

Fig. 3: The displacement distribution with various values of fractional order

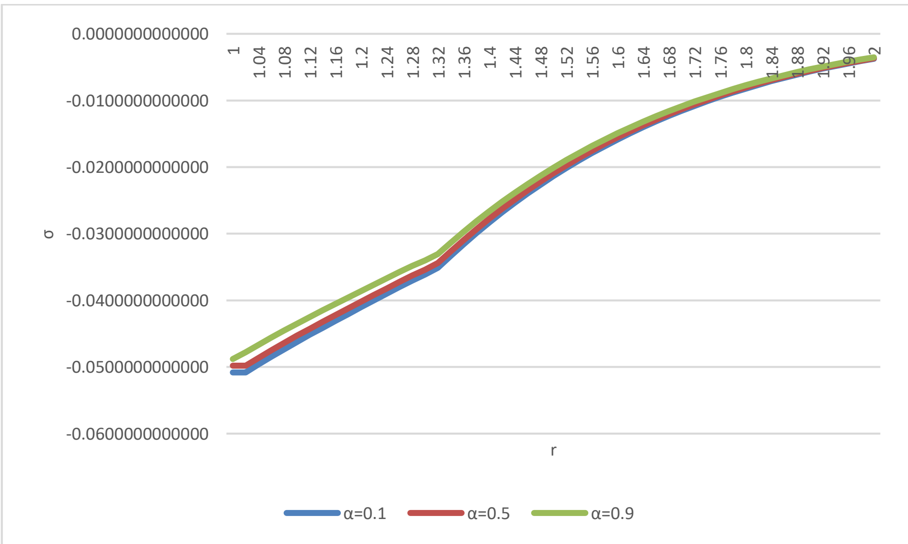

Fig. 4: The stress distribution with various values of fractional order

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Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

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