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Determination of angular acceleration of the rotating driving member of planar mechanisms by the method of guessing

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ABSTRACT

In this paper, we present a novel method for finding an unknown angular acceleration of the driving member of the planar hinged-arm mechanism, which is based on the reduction of the mechanism and quite arbitrary guessing the value of angular acceleration. Using this method, it is possible to directly determine the angular acceleration of the driving rotary member of the mechanism without the need to calculate the kinematic characteristics of the other members. The presented method can be applied to all planar mechanisms. The procedure used in this method is much shorter than in the case of the general laws of system dynamics. The solution obtained by this method is independent of the assumed initial solution, with the exception that the assumed solution cannot be zero.

1. Introduction

One of the fundamental problems in the theory of mechanisms is the determination of the acceleration of individual members of the mechanism, as well as unknown forces in the joints, depending on the forces acting on the mechanism. To solve such problems, the general laws of body dynamics are generally applied (Beer and Johnston, 1997; Goldstein, 1991; Pytel and Kiusalaas, 1996; Chaudhary and Chaudhary, 2016).

The aim of this work is to present a new method in which the angular acceleration of the driving rotary member the mechanism can be determined directly, independently of the force determination in the kinematic pairs of the mechanism. To do this, we will use a method of mechanism reduction while arbitrary guessing the unknown angular acceleration of the driving member. This new method can generally be applied to all planar mechanisms.

2. Primary and secondary acceleration of the mechanism

The acceleration of an arbitrary point $K_{j,i}$ of the planar mechanism (i-th point of the j-th member of the mechanism) with one degree of freedom of motion is:

$$\vec{a}_{j,i} = \frac{d^2\vec{\rho}_{ji}}{dt^2} = \frac{d^2\vec{\rho}_{ji}}{d\varphi_1^2}\dot{\varphi}_1^2 + \frac{d\vec{\rho}_{ji}}{d\varphi_1}\ddot{\varphi}_1, \tag{1}$$

where, $\vec{\rho}_{ji}$- is position vector (radius vector) of point, K_{ji}, $\dot{\varphi}_1$, $\ddot{\varphi}_1$ are projections of the angular speed and angular acceleration of the driving rotary member of the axis mechanism which is perpendicular to the plane (axis z).

From the Eq. 1, it is seen that the acceleration of a point depends, in general, on the angular velocity of the driving member (primary acceleration),

$$\vec{a}_{ji}^{pr} = \frac{d^2\vec{\rho}_{ji}}{d\varphi_1^2}\dot{\varphi}_1^2 \tag{2}$$

and from the angular acceleration of the driving member (secondary acceleration),

$$\vec{a}_{ji}^{sec} = \frac{d\vec{\rho}_{ji}}{d\varphi_1}\ddot{\varphi}_1 . \tag{3}$$

It can be seen that the quotient between the secondary acceleration of a point of the mechanism and the acceleration of the propulsion member $\vec{a}_{ji}^{sec}/\ddot{\varphi}_1$ is not dependent on the kinematic state of the mechanism. As a matter of fact, it actually depends on the position of the mechanism (member $d\vec{\rho}_{ji}/d\varphi_1$).

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3. Use of a reduced mechanism for dynamic mechanism analysis

A reduced mechanism of a single lever mechanism with one degree of freedom of motion is obtained when the members of the mechanism are drawn parallel to their actual positions at certain proportions, where the poles of the current rotation of these members are placed in a single point (a pole of the reduced mechanism) (Gramblička et al., 2017; Voloder and Kljuno, 2018). As an example, it is shown in Fig. 1 a four-joint mechanism and its reduced mechanism.

According to the theorem of Zhukovsky (Artobolevskij, 1988), for a pole of the reduced mechanism, the main moment of all forces acting on the mechanism is equal to zero:

$$\vec{M}_R^* = 0 \tag{4}$$

In case of movement of the mechanism, this condition can be satisfied if we add the real forces to the inertia forces:

$$\vec{M}_R^* + \vec{M}_R^{*,in,pr} + \vec{M}_R^{*,in,sec} = 0, \tag{5}$$

where, $\vec{M}_R^{*,in,pr}$- is the main moment due to the primary inertial forces acting on the mechanism, in the relationship to the pole of the reduced mechanism; $\vec{M}_R^{*,in,sec}$ is the main moment due to the secondary inertial forces acting on the mechanism, in the relationship to the pole of the reduced mechanism.

Here, moment $\vec{M}_{ji}^*$ of the force $\vec{F}_{ji}$ for the pole of the reduced mechanism P^* is equal to the product of the reduction factor of the member j on which given force and torque act for the pole of rotation of the observed member of the real mechanism P^*- shown in Fig. 2 (Hufnagl, 1984):

$$\vec{M}_{ji}^* = \beta_j \cdot \vec{M}_{ji} \tag{6}$$

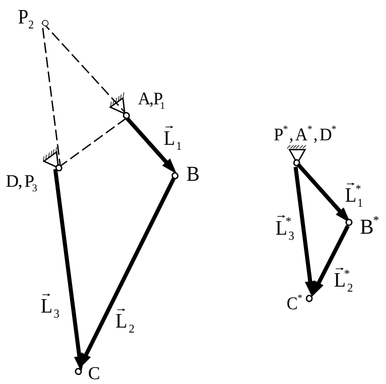

Fig. 1: Four-joint planar mechanism and its reduced mechanism

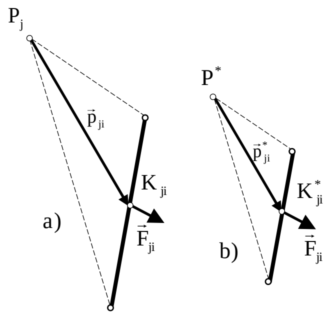

Fig. 2: The definition of torque for a pole of reduced mechanism

4. The method of guessing

The main moment due to the secondary inertial forces for the pole of the reduced mechanism is

$$\vec{M}_R^{*,in,sec} = \sum_j \sum_i [\vec{p}_{ji}^*, \vec{F}_{ji}^{in,sec}] = -\sum_j \sum_i m_{ji} [\vec{p}_{ji}^*, \vec{a}_{ji}^{sec}], \tag{7}$$

where, $\vec{p}_{ji}^*$- is the radius of the vector of the point K_{ji}^* in which the force $\vec{F}_{ji}$ acts concerning the pole of the reduced mechanism P^*. According to Eq. 7 and Eq. 3, we have

$$\vec{M}_R^{*,in,sec} = -\sum_j \sum_i m_{ji} \left[\vec{p}_{ji}^*, \frac{d\vec{\rho}_{ji}}{d\varphi_1} \ddot{\varphi}_1\right]. \tag{8}$$

In the same way, for some guessed acceleration $\vec{\varepsilon}_1^{guess}$ of member 1 of the mechanism, the moment of reduced mechanism due to secondary inertial forces will be:

$$\vec{M}_R^{*,in,sec,guess} = -\sum_j \sum_i m_{ji} \left[\vec{p}_{ji}^*, \frac{d\vec{\rho}_{ji}}{d\varphi_1} \ddot{\varphi}_1^{guess}\right]. \tag{9}$$

Now, according to (8) and (9), we have:

$$\vec{M}_R^{*,in,sec} = \frac{\ddot{\varphi}_1}{\ddot{\varphi}_1^{guess}} \cdot \vec{M}_R^{*,in,sec,guess}. \tag{10}$$

Incorporating the obtained Eq. 10 into Eq. 5, we get:

$$\vec{M}_R^* + \vec{M}_R^{*,in,pr} + \frac{\ddot{\varphi}_1}{\ddot{\varphi}_1^{guess}} \cdot \vec{M}_R^{*,in,sec,guess} = 0, \tag{11}$$

from which we can determine the unknown acceleration of driving member 1 as,

$$\ddot{\varphi}_1 = -\frac{\vec{M}_R^* + \vec{M}_R^{*,in,pr}}{\vec{M}_R^{*,in,sec,guess}} \cdot \ddot{\varphi}_1^{guess}. \tag{12}$$

The obtained expression represents the quotient of two collinear vectors, which represents a quotient of the projection of these vectors on the axis z, vertical to the plane of the mechanism,

$$\ddot{\varphi}_1 = -\frac{M_{R,z}^* + M_{R,z}^{*,in,pr}}{M_{R,z}^{*,in,sec,guess}} \cdot \ddot{\varphi}_1^{guess}. \tag{13}$$

In case the drive member is not rotational, index 1 can refer to any rotary member of the mechanism and in this way, we can determine the angular acceleration of that member.

It is important to emphasize that the assumed angular ($\ddot{\varphi}_1^{guess}$) can have any value other than zero. If this assumed value were equal to zero, an indefinite solution would be obtained for $\ddot{\varphi}_1$. The following example is presented to illustrate the application of the above method, t.

5. Example

The lever 1 in the horizontal plane in Fig. 3 has at any given moment the angular velocity ω_1, with an angular moment M_1 acting on it. A slider 2 of negligible mass, which is hinged to the lever 3, can slide on the lever. For an arbitrary angle of the lever ϕ, determine the acceleration of the lever by applying general laws of dynamics, as well as by the method of guessing.

The following data are given: Geometric measures h and ϕ; the mass m_3; the moment of inertia of lever 1 for the axis of rotation J_{1A}.

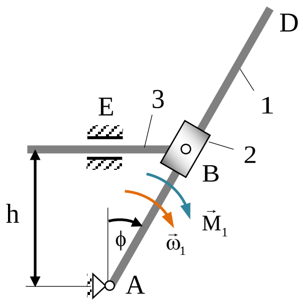

Fig. 3: Mechanism of the given example

5.1. A classic solution by applying general laws of dynamics

For lever 1, we will apply the law on changing the momentum of the motion of the system (Fig. 4).

$$J_{1A} \cdot \varepsilon_1 = M_1 - F_{B(2,1)} \cdot \frac{h}{cos\varphi}. \tag{14}$$

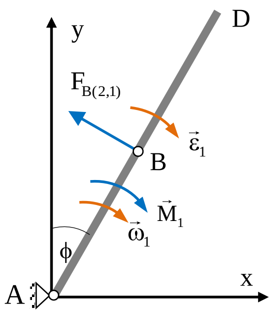

Fig. 4: Use of the law on changing moment of motion for point A on member 1

By applying the law of movement of the center of inertia to the slider 2 (Fig. 5), we get:

$$F_{B(1,2)} \cdot cos\emptyset - X_{B(3,2)} = m_2 \cdot a_2. \tag{15}$$

However, since,

$$m_2 = 0,$$

becomes,

$$F_{B(1,2)} \cdot cos\emptyset - X_{B(3,2)} = 0. \tag{16}$$

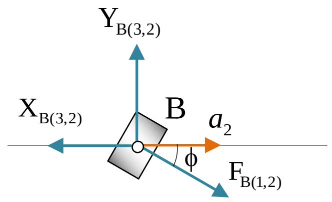

Fig. 5: Use of the law of the motion of the center of inertia on member 2

By applying the law of the center of inertia on slider 3 (Fig. 6) we get:

$$m_3 \cdot a_3 = X_{B(2,3)} \tag{17}$$

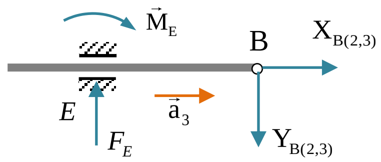

Fig. 6: Use of the law of the motion of the center of inertia on member 3

For point B, we can write,

$$X_B = h \cdot tg\emptyset, \tag{18}$$

so the velocity becomes,

$$\dot{x}_B = \frac{h}{cos^2\emptyset} \cdot \omega_1, \tag{19}$$

and acceleration,

$$a_3 = \ddot{x}_B = h \cdot \frac{-2cos\emptyset \cdot (-sin\emptyset)}{cos^4\emptyset} \cdot \omega_1^2 + \frac{h}{cos^2\emptyset} \cdot \varepsilon_1 = h \cdot \frac{2sin\emptyset}{cos^3\emptyset} \omega_1^2 + \frac{h}{cos^2\emptyset} \cdot \varepsilon_1. \tag{20}$$

From (17) we get:

$$m_3 \left(h \cdot \frac{2sin\emptyset}{cos^3\emptyset} \omega_1^2 + \frac{h}{cos^2\emptyset} \cdot \varepsilon_1 \right) = X_{B(2,3)}, \tag{21}$$

where,

$$F_{B(1,2)} = F_{B(2,1)}, X_{B(2,3)} = X_{B(3,2)}.$$

From Eqs. 14, 15 and 20, we can determine unknown ε_1, $F_{B(1,2)}$ and $X_{B(2,3)}$.

Eq. 16 gives:

$$X_{B(3,2)} = F_{B(1,2)} \cdot cos\emptyset, \tag{22}$$

so by incorporating Eq. 22 into Eq. 20, we get:

$$m_3 \left(h \cdot \frac{2sin\emptyset}{cos^3\emptyset} \omega_1^2 + \frac{h}{cos^2\emptyset} \cdot \varepsilon_1 \right) = F_{B(1,2)} \cdot cos\emptyset. \tag{23}$$

By combining Eq. 15 and 23, we get:

$$\frac{m_3}{cos\emptyset}\left(h\cdot\frac{2sin\emptyset}{cos^3\emptyset}\omega_1^2+\frac{h}{cos^2\emptyset}\cdot\varepsilon_1\right)=\frac{cos\emptyset}{h}(M_1-J_{1A}\cdot\varepsilon_1)$$

and finally,

$$\varepsilon_1=\frac{M_1cos^4\emptyset-2tg\emptyset\cdot m_3\cdot h^2\cdot\omega_1^2}{m_3h^2+cos^4\emptyset\cdot J_{1A}} \tag{24}$$

5.2. The solution of the problem by the method of guessing

Fig. 7 shows the reduced mechanism of the given mechanism, with the reduction factor of member 1 $\beta_1=1$.

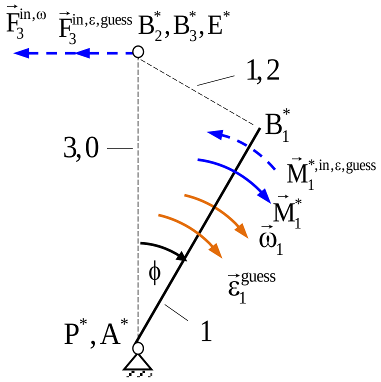

Fig. 7: The reduced mechanism of the given mechanism

According to Eq. 20, the primary component of acceleration of point B is:

$$\ddot{x}_B^{pr}=h\cdot\frac{2sin\emptyset}{cos^3\emptyset}\cdot\omega_1^2, \tag{25}$$

so the primary inertial force of member 3 is:

$$F_{B3}^{in,pr}=m_3\,\ddot{x}_B^{pr}=m_3h\cdot\frac{2sin\emptyset}{cos^3\emptyset}\cdot\omega_1^2. \tag{26}$$

The secondary inertial force is given by,

$$\ddot{x}_B^{sec}=\frac{h}{cos^2\emptyset}\cdot\varepsilon_1, \tag{27}$$

so the secondary inertial force of member 3 is:

$$F_{B3}^{in,sec}=m_3\,\ddot{x}_B^{sec}=m_3\cdot\frac{h}{cos^2\emptyset}\varepsilon_1. \tag{28}$$

If we arbitrary suppose that $\ddot{\emptyset}_1^{guess}=1\,s^{-2}$, then,

$$F_3^{in,sec,gues}=m_3\cdot\frac{h}{cos^2\emptyset}. \tag{29}$$

From the reduced mechanism in Fig. 7. We can write

$$\overline{P^*B_2^*}=\frac{\overline{P^*B_1^*}}{cos\emptyset}=\frac{\alpha_1\cdot\overline{AB}}{cos\emptyset}=\frac{1\cdot\frac{h}{cos\emptyset}}{cos\emptyset}=\frac{h}{cos^2\emptyset}. \tag{30}$$

Incorporating the obtained values into the Eq. 14, we get:

$$\ddot{\emptyset}_1=-\frac{M_{1,z}^*+F_{B3}^{in,pr}\cdot\overline{P^*B_2^*}}{M_{1,z}^{*,in,sec,guess}+F_3^{in,sec,guess}\cdot\overline{P^*B_2^*}}\cdot 1=$$

$$-\frac{-\beta_1\cdot M_1+F_3^{in,pr}\cdot\overline{P^*B_2^*}}{\beta_1\cdot J_{1A}\cdot\ddot{\emptyset}_1^{guess}+F_3^{in,sec,gues}\cdot\overline{P^*B_2^*}}=\frac{1\cdot M_1-m_3h\frac{2sin\emptyset}{cos^3\emptyset}\cdot\omega_1^2\cdot\frac{h}{cos^2\emptyset}}{1\cdot J_{1A}\cdot 1+m_3\cdot\frac{h}{cos^2\emptyset}\cdot\frac{h}{cos^2\emptyset}},$$

$$\ddot{\emptyset}_1=\frac{M_1\cdot cos^4\emptyset-2tg\emptyset\cdot m_3\cdot h^2\cdot\omega_1^2}{m_3\cdot h^2+cos^4\emptyset\cdot J_{1A}}. \tag{31}$$

From the above analysis, we see that the result is the same as that obtained by applying the general laws of dynamics (Eq. 24).

6. Conclusion

Using the primary and secondary acceleration of points of a planar mechanism, as well as the reduction of the mechanism, an expression is obtained for direct determination of the unknown acceleration of the driving rotational member of the mechanism by the novel method of guessing. In this way, if we do not want to, we do not have to determine the unknown characteristics of the motion of other members of the mechanism, as well as the hidden forces in the joints of the mechanism. The method can generally be applicable to all planar mechanisms.

By applying this novel method, the solution is obtained faster and more effective than using a classic method. This is demonstrated from the shown illustrative example.

Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

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