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On Sanskruti and harmonic indices of a certain graph structure

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ABSTRACT

Graph theory is a delightful playground for the evaluation of proof, techniques in Discrete Mathematics and its results have applications in several areas of sciences. For a molecular graph, a numeric quantity that characterizes the complete formation of a graph is called a topological index. Topological indices are most helpful in the field of isomer discrimination, chemical validation, QSAR, QSPR, and pharmaceutical drug design. There are certain types of topological indices like distance-based, degree-based and counting related topological indices. In our work, we calculate and analyze the degree-based topological indices like $M_r(G)$, GA(G), $OGA_r(G)$, GAII(G), SK, SK_1, SK_2, H(G), $H_r(G)$, $\lambda(G)$, $\lambda_r(G)$, F(G), $GA_5(G)$ and the Sanskruti index for the web Graph. Furthermore, we give closed analytic results of these indices.

1. Introduction

Mathematics reveals specifics inherent in the natural world, e.g., rotating symmetry of plants and fractals the structure of arteries in the human being organization. Applications of mathematics can be established not just in biology, chemistry, physics, and computer science but also in art, music, design, and architecture. Graph theory is a branch of mathematics. Our journey into this theory starts with a puzzle that was solved by Leonhard Euler in 1735 and with his solution he laid the organization of what is now known as graph theory (West, 1996).

Graph theory is usually working in the study of networks, patterns, electric circuits, Scheduling and routings as diverse as waste collection. This hypothesis has established significant applications in electrical and civil engineering, operational research, physics, chemistry, computer science, architecture, communication science, anthropology, sociology, genetics, linguistics, economics, and psychology. A graph is an ordered pair G(V, E) comprising a set V of vertices or points together with set E of edges. A graph will be identified by a numeric range, a polynomial, a matrix that represents the complete graph or a sequence of numbers, and these representations are meant to be individually outlined for that graph. A simple graph G consists of a non-empty finite set V(G) of elements called vertices (or nodes), and a finite set E(G) of distinct unordered pairs of distinct elements of V(G) called edges. The number of vertices that are connected to a fixed vertex v is called the degree of a vertex v. In a graph, a set of vertices that adjoining to vertex m is known as the neighborhood of vertex m. Basically, in a neighborhood graph, the neighborhood from vertex m of any graph generated the subgraph (Christofides, 1975).

In the current study, we are apprehensive about the factor of graph theory that can be useful in topological indices. In the field of mathematics, the study of spatial associations and geometric properties which are unaltered by the constant transform of size and structure of the figure. In further expressions the mathematical objects by doing a little transform in size or structure which remains unaltered and a geometrical structural observation of these objects usually known as a topology. Any function on the graph which does not build upon numbering of its vertices is a molecular descriptor. This is also called a topological index. Basically, topological indices are numerical descriptors. Topological indices are significant tools for analyzing various physicochemical properties of molecules without performing any testing (Hayat and Imran, 2014).

Some most important types of topological indices of graphs are distance-based topological indices, degree-based topological indices and spectrum based topological indices. One of the most investigated categories of topological indices used in mathematical chemistry is called degree-based

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topological indices, which are defined in terms of the degrees of the vertices of a graph. Thus, we can write the definition of such a topological index in the form given as:

$TI(G)=\sum_{pq\in(G)} F(d(p),d(q))$

where G=(V(G), E(G)) is a simple, undirected, connected graph and d(u) denotes the degree of the vertex u. Topological indices also called molecular descriptors are presented to explain the physicochemical properties of molecules. Graph theory provides a gateway for chemists and scientists to focus on the topological descriptors of molecular graphs. Molecular compounds can be modeled by using the graph-theoretic method. These topological descriptors provide a better way to understand and to predict the properties and bioactivities of compounds (Hayat and Imran, 2015).

A vast number of topological indices exists in the literature but, in this paper, we pay our attention only to degree-based topological indices. These are the topological indices that depend on the only degree of the graph. We have only discussed degree-based topological indices like First General Zagreb Index, Geometric-Arithmetic Index, Ordinary Geometric-Arithmetic Index, Multiplicative Geometric-Arithmetic Index, SK indices, Harmonic Index, General Version of Harmonic Index, Sum Connectivity Index, General Sum Connectivity Index, Forgotten Topological Index and Fifth version of Geometric-Arithmetic Index and Sanskrit Index for the Web graph. Fig. 1 shows the Web graph.

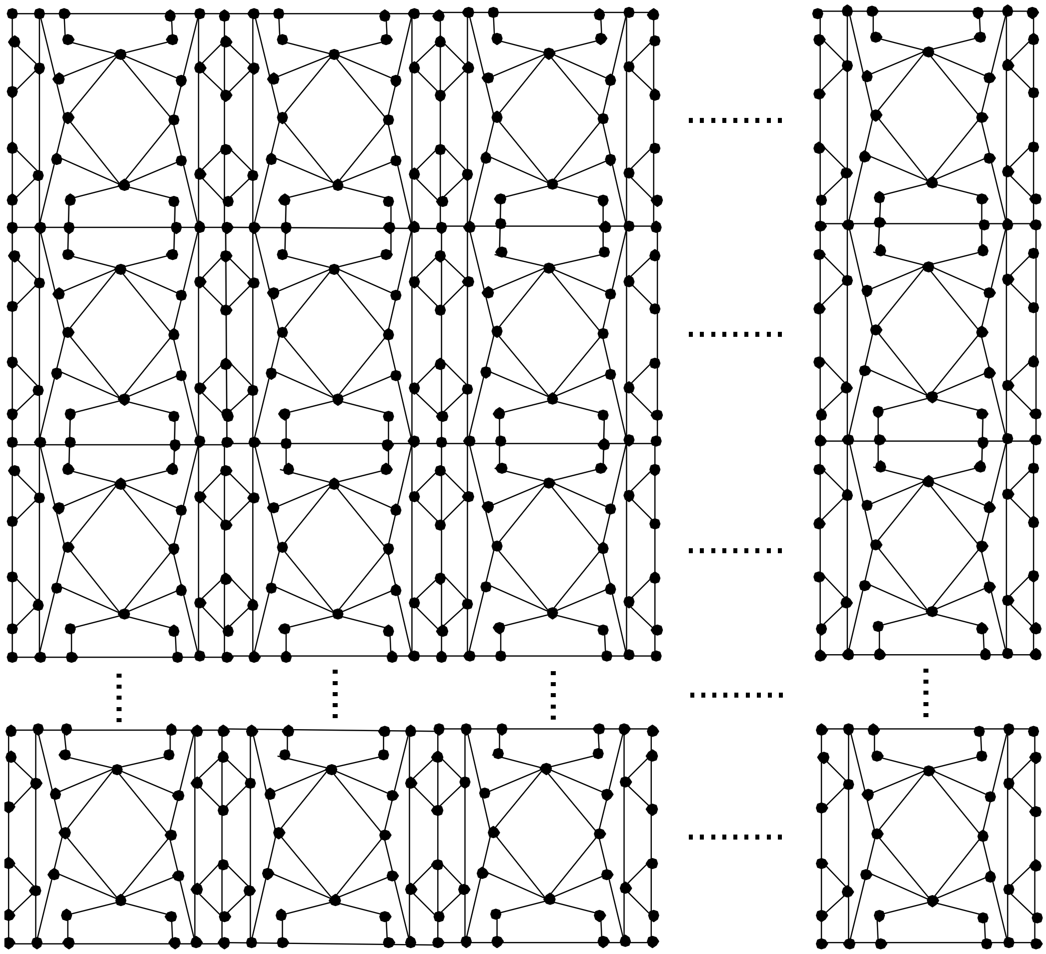
Fig. 1: Web graph

Li and Zhao (2004) proposed the first general Zagreb index:

$$M_r(G)=\sum_{p\in V(G)} (d_p)^r \quad (1)$$

Geometric-arithmetic (GA) index was defined by Vukičević and Furtula (2009) and compared (GA) index by the well-known Randic index (Das, 2010). For a connected graph G the Geometric-arithmetic (GA) index is defined as:

$$GA(G)=\sum_{pq\in E(G)} \left(\frac{2\sqrt{d_p.d_q}}{d_p+d_q}\right) \quad (2)$$

Eliasi and Iranmanesh (2011) presented the ordinary geometric-arithmetic index as the addition of geometric-arithmetic index which was defined as:

$$OGA_r(G)=\sum_{pq\in E(G)} \left(\frac{2\sqrt{d_p.d_q}}{d_p+d_q}\right)^r \quad (3)$$

Multiplicative geometric-arithmetic index (Zhou and Trinajstić, 2010) was stated as follows:

$$GAII(G)=\prod_{pq\in E(G)} \frac{2\sqrt{d_p.d_q}}{d_p+d_q} \quad (4)$$

For a connected graph G, SK indices were (Jie et al., 2017) defined as follows:

$$SK(G) = \sum_{pq \in E(G)} \left(\frac{d_p + d_q}{2}\right) \quad (5)$$

$$SK_1 = \sum_{pq \in E(G)} \left(\frac{d_p . d_q}{2}\right) \quad (6)$$

$$SK_2 = \sum_{pq \in E(G)} \left(\frac{d_p + d_q}{2}\right)^2 \quad (7)$$

The harmonic index (Zhong, 2012) which was described by,

$$H(G) = \sum_{pq \in E(G)} \left(\frac{2}{d_p + d_q}\right) \quad (8)$$

Yan et al. (2015) introduced the general version of the harmonic index which was defined by,

$$H_r(G) = \sum_{pq \in E(G)} \left(\frac{2}{d_p + d_q}\right)^r \quad (9)$$

Lučić et al. (2009) defined the sum-connectivity index (λ) which was formulated by,

$$\lambda(G) = \sum_{pq \in E(G)} \left(\frac{1}{\sqrt{d_p + d_q}}\right) \quad (10)$$

A few years ago, Zhou and Trinajstić (2010) introduced the general sum connectivity index as follows:

$$\lambda_r(G) = \sum_{pq \in E(G)} (d_p + d_q)^r \quad (11)$$

Furtula and Gutman (2015) reinvestigated the forgotten topological index or (F-index) and it is denoted by F(G),

$$F(G) = \sum_{pq \in E(G)} (d_p^2 + d_q^2) \quad (12)$$

Graovoc et al. (2011) introduced the fifth version of geometric-arithmetic index GA_5 and is defined as

$$GA_5(G) = \sum_{pq \in E(G)} \left(\frac{2\sqrt{S_p . S_q}}{S_p + S_q}\right) \quad (13)$$

Hosamani (2017) formulated the Sanskruti index S(G) of a general graph G. It is defined as

$$S(G) = \sum_{pq \in E(G)} \left(\frac{S_p . S_q}{S_p + S_q - 2}\right)^3 \quad (14)$$

2. Results on the web graph

Theorem 2.1: Let G be a graph with m ≥ 2 and n ≥ 2. Then

$$M_r(G) = 2^{r+2}(mn + n + 4\,m + 2) + 2 \times 3^r(2\,mn + 7\,n + 5\,m + 8) + 4^r(13\,mn + 8\,n + 14\,m + 10) + 2 \times 6^r(2\,mn + 2\,n + m + 1).$$

Proof: Let G be a graph and r is a real number. Then by using Table 1 we have,

$$M_r(G) = 2^{r+2}(mn + n + 4\,m + 2) + 2 \times 3^r(2\,mn + 7\,n + 5\,m + 8) + 4^r(13\,mn + 8\,n + 14\,m + 10) + 2 \times 6^r(2\,mn + 2\,n + m + 1).$$

Table 1: The vertex partition of graph g based on the degree of vertices

Degree of vertex	No. of vertices	Degree of vertex	No. of vertices
2	4mn+4n+4m+8	3	4mn+14n+10m+6
4	13mn+8n+14m+10	6	4mn+4n+2m+2
Total	48mn+53m+53n+58		

Theorem 2.2: Let G be a graph with m ≥ 2 edges and n ≥ 2 vertices.

1. GA(G)= $2/5\,\sqrt{6}(4\,m + 8) + 2/3\,\sqrt{2}(4\,mn + 4\,n + 4) + 1/2\,\sqrt{3}(4\,mn + 4\,n + 4\,m + 4) + 13\,n + 19\,m + 14 + 4/7\,\sqrt{3}(4\,mn + 12\,n + 18\,m + 20) + 2/3\,\sqrt{2} \times (8\,mn + 10\,n + 4\,m + 4) + 16\,mn + 2/5\,\sqrt{6}(12\,mn + 10\,n + 4\,m + 4)$.
2. $OGA_r(G) = 4\,m + 8 \times \left(2/5\,\sqrt{6}\right)^r + 4\,mn + 4\,n + 4 \times \left(2/3\,\sqrt{2}\right)^r + 4\,mn + 4\,n + 4\,m + 4 \times \left(1/2\,\sqrt{3}\right)^r + 13\,n + 19\,m + 14 + 4\,mn + 12\,n + 18\,m + 20 \times \left(4/7\,\sqrt{3}\right)^r + 8\,mn + 10\,n + 4\,m + 4 \times \left(2/3\,\sqrt{2}\right)^r + 16\,mn + 12\,mn + 10 + 4\,m + 4 \times \left(2/5\,\sqrt{6}\right)^r$.
3. $GAII(G) = \left(\frac{2}{5}\sqrt{6}\right)^{4\,m+8} + \left(\frac{2}{3}\sqrt{2}\right)^{4\,mn+4\,n+4} + \left(\frac{1}{2}\sqrt{3}\right)^{4\,mn+4\,n+4\,m+4} + 2 \quad + (4/7\,\sqrt{3})^{4\,mn+12\,n+18\,m+20} + (2/3\,\sqrt{2})^{8\,mn+10\,n+4\,m+4} + (2/5\,\sqrt{6})^{12\,mn+10\,n+4\,m+4}$
4. $SK(G) = 204 + 207\,n + 202\,mn + 201\,m.$
5. $SK_1 = 352 + 391\,n + 408\,mn + 373\,m.$
6. $SK_2 = 744 + \frac{1675}{2}\,n + 867\,mn + \frac{1561}{2}\,m.$
7. $H(G) = \frac{3545}{252}\,n + \frac{3671}{315}\,mn + \frac{18079}{1260}\,m + \frac{2155}{126}.$
8. $H_r(G) = 4\,m + 8 \times \left(\frac{2}{5}\right)^r + 4\,mn + 4\,n + 4 \cdot \left(\frac{1}{3}\right)^r + 4(\,mn + \,n + \,m + 1) \times \left(\frac{1}{4}\right)^r + 10\,n + 2\,m + 8 \times \left(\frac{1}{3}\right)^r + 4\,mn + 12\,n + 18\,m + 20 \times \left(\frac{2}{7}\right)^r + 8\,mn + 10\,n + 4\,m + 4 \times \left(\frac{2}{9}\right)^r + 16\,mn + 3\,n + 17\,m + 6 \times \left(\frac{1}{4}\right)^r + 12\,mn + 10\,n + 4\,m + 4 \times \left(\frac{1}{5}\right)^r.$
9. $\lambda(G) = \frac{4}{5}\sqrt{5}(m + 2) + \frac{4}{6}\sqrt{6}(mn + n + 1) + \sqrt{2}(\,mn + n + m + 1) + \frac{1}{6}\sqrt{6}(10\,n + 2\,m + 8) + \frac{1}{7}\sqrt{7}(4\,mn + 12\,n + 18\,m + 20) + 8/3\,mn + 10/3\,n + 4/3\,m + 4/3 + 1/4\,\sqrt{2}(16\,mn + 3\,n + 17\,m + 6) + 1/10\,\sqrt{10}(12\,mn + 10\,n + 4\,m + 4).$
10. $\lambda_r(G) \ = 5^r \times (4\,m + 8) + 4 \times 6^r(mn + n + 1) + 4 \times 8^r(\,mn + \,n + \,m + 1) \ + 10\,n + 2\,m + 8 \times 6^r + 4\,mn + 12\,n + 18\,m + 20 \times 7^r + 8\,mn + 10\,n + 4\,m + 4 \times 9^r + 16\,mn + 3\,n + 17\,m + 6 \times 8^r + 12\,mn + 10\,n + 4\,m + 4 \times 10^r.$
11. $F(G) = 1568 + 1786\,n + 1836\,mn + 1630\,m.$

Proof: Let G be the graph where we define $E(p, q)$ denotes the number of edges connecting the vertices of degree d_p and d_q. The two-dimensional structure of the given graph contains only edges.

$E_{\{2,3\}} = \{pq \in E(G[m, n]) | d_p = 2, d_q = 3\},$
$E_{\{2,4\}} = \{pq \in E(G[m, n]) | d_p = 2, d_q = 4\},$
$E_{\{2,6\}} = \{pq \in E(G[m, n]) | d_p = 2, d_q = 6\},$
$E_{\{3,3\}} = \{pq \in E(G[m, n]) | d_p = 3, d_q = 3\},$
$E_{\{3,4\}} = \{pq \in E(G[m, n]) | d_p = 3, d_q = 4\},$
$E_{\{3,6\}} = \{pq \in E(G[m, n]) | d_p = 3, d_q = 6\},$
$E_{\{4,4\}} = \{pq \in E(G[m, n]) | d_p = 4, d_q = 4\},$

and

$E_{\{4,6\}} = \{pq \in E(G[m,n]) | d_p = 4, d_q = 6\}$,

now,

$E_{\{2,3\}}| = 4m + 8$
$|E_{\{2,4\}}| = 4mn + 4n + 4$
$|E_{\{2,6\}}| = 4mn + 4n + 4m + 4$
$|E_{\{3,3\}}| = 10n + 2m + 8$
$|E_{\{3,4\}}| = 4mn + 12n + 18m + 20$
$|E_{\{3,6\}}| = 8mn + 10n + 4m + 4$
$|E_{\{4,4\}}| = 16mn + 3n + 17m + 6$

and,

$|E_{\{4,6\}}| = 12mn + 10n + 4m + 4.$

1. The geometric-arithmetic index is defined as:

$$GA(G) = \sum_{pq \in E(G)} \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right).$$

This implies that,

$$GA(G) = |E_{\{2,3\}}| \times \left(\frac{2\sqrt{2\times3}}{2+3}\right) + |E_{\{2,4\}}| \times \left(\frac{2\sqrt{2\times4}}{2+4}\right) + |E_{\{2,6\}}| \times \left(\frac{2\sqrt{2\times6}}{2+6}\right) + |E_{\{3,3\}}| \times \left(\frac{2\sqrt{3\times3}}{3+3}\right) + |E_{\{3,4\}}| \times \left(\frac{2\sqrt{3\times4}}{3+4}\right) + |E_{\{3,6\}}| \times \left(\frac{2\sqrt{3\times6}}{3+6}\right) + |E_{\{4,4\}}| \times \left(\frac{2\sqrt{4\times4}}{4+4}\right) + |E_{\{4,6\}}| \times \left(\frac{2\sqrt{4\times6}}{4+6}\right).$$

Now we get,

$$GA(G) = (4m + 8) \times \left(\frac{2\sqrt{2\times3}}{2+3}\right) + 4(mn + n + 1) \times \left(\frac{2\sqrt{2\times4}}{2+4}\right) + 4(mn + n + m + 1) \times \left(\frac{2\sqrt{2\times6}}{2+6}\right) + (10n + 2m + 8) \times \left(\frac{2\sqrt{3\times3}}{3+3}\right) + (4mn + 12n + 18m + 20) \times \left(\frac{2\sqrt{3\times4}}{3+4}\right) + (8mn + 10n + 4m + 4) \times \left(\frac{2\sqrt{3\times6}}{3+6}\right) + (16mn + 3n + 17m + 6).$$

After simplification we get,

$$GA(G) = 2/5\,\sqrt{6}(4\,m + 8) + 2/3\,\sqrt{2}(4\,mn + 4\,n + 4) + 2\,\sqrt{3}(\,mn +\, n +\, m + 1) + 13\,n + 19\,m + 14 + 4/7\,\sqrt{3}(4\,mn + 12\,n + 18\,m + 20) + 2/3\,\sqrt{2} \times (8\,mn + 10\,n + 4\,m + 4) + 16\,mn + 2/5\,\sqrt{6}(12\,mn + 10\,n + 4\,m + 4).$$

2. The ordinary geometric index is defined as,

$$OGA_r G = \sum_{pq \in E(G)} \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right)^r.$$

This implies that,

$$OGA_r(G) = |E_{\{2,3\}}| \times \left(\frac{2\sqrt{2\times3}}{2+3}\right)^r + |E_{\{2,4\}}| \times \left(\frac{2\sqrt{2\times4}}{2+4}\right)^r + |E_{\{2,6\}}| \times \left(\frac{2\sqrt{2\times6}}{2+6}\right)^r + |E_{\{3,3\}}| \times \left(\frac{2\sqrt{3\times3}}{3+3}\right)^r + |E_{\{3,4\}}| \times \left(\frac{2\sqrt{3\times4}}{3+4}\right)^r + |E_{\{3,6\}}| \times \left(\frac{2\sqrt{3\times6}}{3+6}\right)^r + |E_{\{4,4\}}| \times \left(\frac{2\sqrt{4\times4}}{4+4}\right)^r + |E_{\{4,6\}}| \times \left(\frac{2\sqrt{4\times6}}{4+6}\right)^r.$$

Now we get,

$$OGA_r(G) = (4m + 8) \times \left(\frac{2\sqrt{2\times3}}{2+3}\right)^r + 4(mn + n + 1) \times \left(\frac{2\sqrt{2\times4}}{2+4}\right)^r + 4(mn + n + m + 1) \times \left(\frac{2\sqrt{2\times6}}{2+6}\right)^r + (10n + 2m + 8) \times \left(\frac{2\sqrt{3\times3}}{3+3}\right)^r + (4mn + 12n + 18m + 20) \times \left(\frac{2\sqrt{3\times4}}{3+4}\right)^r + (8mn + 10n + 4m + 4) \times \left(\frac{2\sqrt{3\times6}}{3+6}\right)^r + (16mn + 3n + 17m + 6) \times \left(\frac{2\sqrt{4\times4}}{4+4}\right)^r + (12mn + 10n + 4m + 4) \times \left(\frac{2\sqrt{4\times6}}{4+6}\right)^r.$$

After simplification we obtain,

$$OGA_r(G) = 4\,m + 8 \times \left(\tfrac{2}{5}\sqrt{6}\right)^r + 4\,mn + 4\,n + 4 \times \left(\tfrac{2}{3}\sqrt{2}\right)^r + 4\,mn + 4\,n + 4\,m + 4 \times \left(\tfrac{1}{2}\sqrt{3}\right)^r + 13\,n + 19\,m + 14 + 4\,mn + 12\,n + 18\,m + 20 \times \left(\tfrac{4}{7}\sqrt{3}\right)^r + 8\,mn + 10\,n + 4\,m + 4 \times \left(\tfrac{2}{3}\sqrt{2}\right)^r + 16\,mn + 12\,mn + 10\,n + 4\,m + 4 \times \left(\tfrac{2}{5}\sqrt{6}\right)^r.$$

3. Multiplicative Geometric-arithmetic index is defined as:

$$GAII(G) = \prod_{pq \in E(G)} \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right).$$

This implies that,

$$GAII(G) = \prod_{pq \in E_1(G)} \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right) \times \prod_{pq \in E_2(G)} \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right) \times \prod_{pq \in E_3(G)} \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right) \times \prod_{pq \in E_4(G)} \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right) \times \prod_{pq \in E_4(G)} \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right) \times \prod_{pq \in E_5(G)} \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right) \times \prod_{pq \in E_6(G)} \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right) \times \prod_{pq \in E_7(G)} \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right) \times \prod_{pq \in E_8(G)} \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right).$$

$$GAII(G) = \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right)^{|E_1(G)|} \times \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right)^{|E_2(G)|} \times \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right)^{|E_3(G)|} \times \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right)^{|E_4(G)|} \times \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right)^{|E_5(G)|} \times \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right)^{|E_6(G)|} \times \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right)^{|E_7(G)|} \times \left(\frac{2\sqrt{d_p \times d_q}}{d_p + d_q}\right)^{|E_8(G)|}.$$

$$GAII(G) = \left(\frac{2\sqrt{2\times3}}{2+3}\right)^{4m+8} \times \left(\frac{2\sqrt{2\times4}}{2+4}\right)^{4mn+4n+4} \times \left(\frac{2\sqrt{2\times6}}{2+6}\right)^{4mn+4n+4m+4} \times \left(\frac{2\sqrt{3\times3}}{3+3}\right)^{10n+2m+8} \times \left(\frac{2\sqrt{3\times4}}{3+4}\right)^{4mn+12n+18m+20} \times \left(\frac{2\sqrt{3\times6}}{3+6}\right)^{8mn+10n+4m+4} \times \left(\frac{2\sqrt{4\times4}}{4+4}\right)^{16mn+3n+17m+6} \times \left(\frac{2\sqrt{4\times6}}{4+6}\right)^{12mn+10n+4m+4}.$$

After simplification, we obtained that,

$$GAII(G) = (2/5\,\sqrt{6})^{4\,m+8} + (2/3\,\sqrt{2})^{4\,mn+4\,n+4} + (1/2\,\sqrt{3})^{4\,mn+4\,n+4\,m+4} + 2 + (4/7\,\sqrt{3})^{4\,mn+12\,n+18\,m+20} + (2/3\,\sqrt{2})^{8\,mn+10\,n+4\,m+4} + (2/5\,\sqrt{6})^{12\,mn+10\,n+4\,m+4}.$$

Which completes the proof.

4. The SK index is defined as follows:

$SK(G) = \sum_{pq\in E(G)} \left(\frac{d_p+d_q}{2}\right)$.

This implies that,

$SK(G) = |E_{\{2,3\}}| \times \left(\frac{2+3}{2}\right) + |E_{\{2,4\}}| \times \left(\frac{2+4}{2}\right) + |E_{\{2,6\}}| \times \left(\frac{2+6}{2}\right) + |E_{\{3,3\}}| \times \left(\frac{3+3}{2}\right) + |E_{\{3,4\}}| \times \left(\frac{3+4}{2}\right) + |E_{\{3,6\}}| \times \left(\frac{3+6}{2}\right) + |E_{\{4,4\}}| \times \left(\frac{4+4}{2}\right) + |E_{\{4,6\}}| \times \left(\frac{4+6}{2}\right)$.

Now we get,

$SK(G) = (4m + 8) \times \left(\frac{2+3}{2}\right) + (4mn + 4n + 4) \times \left(\frac{2+4}{2}\right) + (4mn + 4n + 4m + 4) \times \left(\frac{2+6}{2}\right) + (10n + 2m + 8) \times \left(\frac{3+3}{2}\right) + (4mn + 12n + 18m + 20) \times \left(\frac{3+4}{2}\right) + (8mn + 10n + 4m + 4) \times \left(\frac{3+6}{2}\right) + (16mn + 3n + 17m + 6) \times \left(\frac{4+4}{2}\right) + (12mn + 10n + 4m + 4) \times \left(\frac{4+6}{2}\right)$.

After simplification we obtain,

$SK(G) = 204 + 207\, n + 202\, mn + 201\, m$.

Which completes the proof.

5. The SK_1 index is defined as:

$SK_1 = \sum_{pq\in E(G)} \left(\frac{d_p\times d_q}{2}\right)$.

This implies that,

$SK_1(G) = |E_{\{2,3\}}| \times \left(\frac{2\times3}{2}\right) + |E_{\{2,4\}}| \times \left(\frac{2\times4}{2}\right) + |E_{\{2,6\}}| \times \left(\frac{2\times6}{2}\right) + |E_{\{3,3\}}| \times \left(\frac{3\times3}{2}\right) + |E_{\{3,4\}}| \times \left(\frac{3\times4}{2}\right) + |E_{\{3,6\}}| \times \left(\frac{3\times6}{2}\right) + |E_{\{4,4\}}| \times \left(\frac{4\times4}{2}\right) + |E_{\{4,6\}}| \times \left(\frac{4\times6}{2}\right)$.

Now we get,

$SK_1(G) = (4m + 8) \times \left(\frac{2\times3}{2}\right) + (4mn + 4n + 4) \times \left(\frac{2\times4}{2}\right) + (4mn + 4n + 4m + 4) \times \left(\frac{2\times6}{2}\right) + (10n + 2m + 8) \times \left(\frac{3\times3}{2}\right) + (4mn + 12n + 18m + 20) \times \left(\frac{3\times4}{2}\right) + (8mn + 10n + 4m + 4) \times \left(\frac{3\times6}{2}\right) + (16mn + 3n + 17m + 6) \times \left(\frac{4\times4}{2}\right) + (12mn + 10n + 4m + 4) \times \left(\frac{4\times6}{2}\right)$.

After simplification we obtain,

$SK_1 = 352 + 391\, n + 408\, mn + 373\, m$.

Hence proved.

6. The SK_2 index is defined as:

$SK_2 = \sum_{pq\in E(G)} \left(\frac{d_p+d_q}{2}\right)^2$.

This implies that,

$SK_2 = |E_{\{2,3\}}| \times \left(\frac{2+3}{2}\right)^2 + |E_{\{2,4\}}| \times \left(\frac{2+4}{2}\right)^2 + |E_{\{2,6\}}| \times \left(\frac{2+6}{2}\right)^2 + |E_{\{3,3\}}| \times \left(\frac{3+3}{2}\right)^2 + |E_{\{3,4\}}| \times \left(\frac{3+4}{2}\right)^2 + |E_{\{3,6\}}| \times \left(\frac{3+6}{2}\right)^2 + |E_{\{4,4\}}| \times \left(\frac{4+4}{2}\right)^2 + |E_{\{4,6\}}| \times \left(\frac{4+6}{2}\right)^2$.

Now we get,

$SK_2 = (4m + 8) \times \left(\frac{2+3}{2}\right)^2 + (4mn + 4n + 4) \times \left(\frac{2+4}{2}\right)^2 + (4mn + 4n + 4m + 4) \times \left(\frac{2+6}{2}\right)^2 + (10n + 2m + 8) \times \left(\frac{3+3}{2}\right)^2 + (4mn + 12n + 18m + 20) \times \left(\frac{3+4}{2}\right)^2 + (8mn + 10n + 4m + 4) \times \left(\frac{3+6}{2}\right)^2 + (16mn + 3n + 17m + 6) \times \left(\frac{4+4}{2}\right)^2 + (12mn + 10n + 4m + 4) \times \left(\frac{4+6}{2}\right)^2$.

After simplification we obtain,

$SK_2 = 744 + \frac{1675}{2}\, n + 867\, mn + \frac{1561}{2}\, m$.

Hence proved.

7. The Harmonic index is defined as:

$H(G) = \sum_{pq\in E(G)} \left(\frac{2}{d_p+d_q}\right)$.

This implies that,

$H(G) = |E_{\{2,3\}}| \times \left(\frac{2}{2+3}\right) + |E_{\{2,4\}}| \times + \left(\frac{2}{2+4}\right) |E_{\{2,6\}}| \times \left(\frac{2}{2+6}\right) + |E_{\{3,3\}}| \times \left(\frac{2}{3+3}\right) + |E_{\{3,4\}}| \times \left(\frac{2}{3+4}\right) + |E_{\{3,6\}}| \times \left(\frac{2}{3+6}\right) + |E_{\{4,4\}}| \times \left(\frac{2}{4+4}\right) + |E_{\{4,6\}}| \times \left(\frac{2}{4+6}\right)$.

Now we get,

$H(G) = (4m + 8) \times \left(\frac{2}{2+3}\right) + (4mn + 4n + 4) \times \left(\frac{2}{2+4}\right) + (4mn + 4n + 4m + 4) \times \left(\frac{2}{2+6}\right) + (10n + 2m + 8) \times \left(\frac{2}{3+3}\right) + (4mn + 12n + 18m + 20) \times \left(\frac{2}{3+4}\right) + (8mn + 10n + 4m + 4) \times \left(\frac{2}{3+6}\right) + (16mn + 3n + 17m + 6) \times \left(\frac{2}{4+4}\right) + (12mn + 10n + 4m + 4) \times \left(\frac{2}{4+6}\right)$.

After simplification we obtain,

$H(G) = \frac{3545}{252}\, n + \frac{3671}{315}\, mn + \frac{18079}{1260}\, m + \frac{2155}{126}$.

Hence Proved.

8. The General Harmonic index is defined as:

$H_r(G) = \sum_{pq\in E(G)} \left(\frac{2}{d_p+d_q}\right)^r$.

This implies that,

$H_r(G) = |E_{\{2,3\}}| \times \left(\frac{2}{2+3}\right)^r + |E_{\{2,4\}}| \times + \left(\frac{2}{2+4}\right)^r + |E_{\{2,6\}}| \times \left(\frac{2}{2+6}\right)^r + |E_{\{3,3\}}| \times \left(\frac{2}{3+3}\right)^r + |E_{\{3,4\}}| \times \left(\frac{2}{3+4}\right)^r + |E_{\{3,6\}}| \times \left(\frac{2}{3+6}\right)^r + |E_{\{4,4\}}| \times \left(\frac{2}{4+4}\right)^r + |E_{\{4,6\}}| \times \left(\frac{2}{4+6}\right)^r$.

Now we get,

$H_r(G) = (4m + 8) \times \left(\frac{2}{2+3}\right)^r + (4mn + 4n + 4) \times \left(\frac{2}{2+4}\right)^r (4mn + 4n + 4m + 4) \times \left(\frac{2}{2+6}\right)^r + (10n + 2m + 8) \times \left(\frac{2}{3+3}\right)^r + (4mn + 12n + 18m + 20) \times \left(\frac{2}{3+4}\right)^r +$

$(8mn+10n+4m+4)\times\left(\frac{2}{3+6}\right)^r+(16mn+3n+17m+6)\times\left(\frac{2}{4+4}\right)^r+(12mn+10n+4m+4)\times\left(\frac{2}{4+6}\right)^r.$

After simplification we obtain,

$H_r(G)=4\,m+8\times\left(\frac{2}{5}\right)^r+4\,mn+4\,n+4\times\left(\frac{1}{3}\right)^r+4(\,mn+\,n+\,m+1)\times(1/4)^r+10\,n+2\,m+8\times(1/3)^r+4\,mn+12\,n+18\,m+20\times(2/7)^r+8\,mn+10\,n+4\,m+4\times(2/9)^r+16\,mn+3\,n+17\,m+6\times(1/4)^r+12\,mn+10\,n+4\,m+4\times(1/5)^r.$

Hence proved.

9. The sum- connectivity index is defined as:

$\lambda(G)=\sum_{pq\in E(G)}\left(\frac{1}{\sqrt{d_p+d_q}}\right).$

This implies that,

$\lambda(G)=|E_{\{2,3\}}|\times\left(\frac{1}{\sqrt{2+3}}\right)+|E_{\{2,4\}}|\times\left(\frac{1}{\sqrt{2+4}}\right)+|E_{\{2,6\}}|\times\left(\frac{1}{\sqrt{2+6}}\right)+|E_{\{3,3\}}|\times\left(\frac{1}{\sqrt{3+3}}\right)+|E_{\{3,4\}}|\times\left(\frac{1}{\sqrt{3+4}}\right)+|E_{\{3,6\}}|\times\left(\frac{1}{\sqrt{3+6}}\right)+|E_{\{4,4\}}|\times\left(\frac{1}{\sqrt{4+4}}\right)+|E_{\{4,6\}}|\times\left(\frac{1}{\sqrt{4+6}}\right).$

Now we get,

$\lambda(G)=(4m+8)\times\left(\frac{1}{\sqrt{2+3}}\right)+(4mn+4n+4)\times\left(\frac{1}{\sqrt{2+4}}\right)+4(mn+n+m+1)\times\left(\frac{1}{\sqrt{2+6}}\right)+(10n+2m+8)\times\left(\frac{1}{\sqrt{3+3}}\right)+(4mn+12n+18m+20)\times\left(\frac{1}{\sqrt{3+4}}\right)+(8mn+10n+4m+4)\times\left(\frac{1}{\sqrt{3+6}}\right)+(16mn+3n+17m+6)\times\left(\frac{1}{\sqrt{4+4}}\right)+(12mn+10n+4m+4)\times\left(\frac{1}{\sqrt{4+6}}\right).$

After simplification we obtain,

$\lambda(G)=\frac{4}{5}\sqrt{5}(m+2)+\frac{4}{6}\sqrt{6}(mn+n+1)+\sqrt{2}(\,mn+n+m+1)+\frac{1}{6}\sqrt{6}(10\,n+2\,m+8)+\frac{1}{7}\sqrt{7}(4\,mn+12\,n+18\,m+20)+8/3\,mn+10/3\,n+4/3\,m+4/3+1/4\,\sqrt{2}(16\,mn+3\,n+17\,m+6)+1/10\,\sqrt{10}(12\,mn+10\,n+4\,m+4).$

Which completes the proof.

10. The general sum-connectivity index is defined as:

$\lambda_r(G)=\sum_{pq\in E(G)}\left(d_p+d_q\right)^r.$

This implies that,

$\lambda_r(G)=|E_{\{2,3\}}|\times(2+3)^r+|E_{\{2,4\}}|\times(2+4)^r+|E_{\{2,6\}}|\times(2+6)^r+|E_{\{3,3\}}|\times(3+3)^r+|E_{\{3,4\}}|\times(3+4)^r+|E_{\{3,6\}}|\times(3+6)^r+|E_{\{4,4\}}|\times(4+4)^r+|E_{\{4,6\}}|\times(4+6)^r.$

Now we get,

$\lambda_r(G)=(4m+8)\times(2+3)^r+4(mn+n+1)\times(2+4)^r+4(mn+n+m+1)\times(2+6)^r+(10n+2m+8)\times(3+3)^r+(4mn+12n+18m+20)\times(3+4)^r+(8mn+10n+4m+4)\times(3+6)^r+(16mn+3n+17m+6)\times(4+4)^r+(12mn+10n+4m+4)\times(4+4)^r.$

After simplification we obtain,

$\lambda_r(G)\ =5^r\times(4\,m+8)+4\times6^r(mn+n+1)+4\times8^r(\,mn+\,n+\,m+1)+10\,n+2\,m+8\times6^r+4\,mn+12\,n+18\,m+20\times7^r+8\,mn+10\,n+4\,m+4\times9^r+16\,mn+3\,n+17\,m+6\times8^r+12\,mn+10\,n+4\,m+4\times10^r.$

Hence Proved.

11. The F-index is defined as:

$F(G)=\sum_{pq\in E(G)}\,(d_p^2+d_q^2).$

This implies that,

$F(G)=|E_{\{2,3\}}|\times(2^2+3^2)+|E_{\{2,4\}}|\times(2^2+4^2)+|E_{\{2,6\}}|\times(2^2+6^2)+|E_{\{3,3\}}|\times(3^2+3^2)+|E_{\{3,4\}}|\times(3^2+4^2)+|E_{\{3,6\}}|\times(3^2+6^2)+|E_{\{4,4\}}|\times(4^2+4^2)+|E_{\{4,6\}}|\times(4^2+6^2).$

Now we get,

$F(G)=(4m+8)\times(2^2+3^2)+(4mn+4n+4)\times(2^2+4^2)+4(mn+n+m+1)\times(2^2+6^2)+(10n+2m+8)\times(3^2+3^2)+(4mn+12n+18m+20)\ \times(3^2+4^2)+(8mn+10n+4m+4)\times(3^2+6^2)+(16mn+3n+17m+6)\times(4^2+4^2)+(12mn+10n+4m+4)\times(4^2+6^2).$

After simplification we obtain,

$F(G)=1568+1786\,n+1836\,mn+1630\,m.$

Hence proved.

Theorem 2.3: In the next two theorems we calculated the GA_5 index and Sanskruti index:

1. $GA_5(G)=10+13\,n+10\,mn+\frac{3}{8}\,(8\,m+8)\sqrt{7}+\frac{8}{27}\,m\sqrt{182}+\frac{8}{11}\,n\sqrt{7}+\frac{12}{17}\,(16\,mn+8\,n+4\,m)\sqrt{2}+\frac{48}{31}\,m\sqrt{15}+\frac{2}{9}\,mn\,\sqrt{77}+\frac{4}{5}\,n\sqrt{6}+13\,m+\frac{4}{21}\,mn\,\sqrt{110}+\frac{6}{5}\,mn\,\sqrt{11}+\frac{16}{19}\,mn\,\sqrt{22}+\frac{64}{37}\,n\sqrt{21}+\frac{8}{9}\,mn\,\sqrt{5}+\frac{16}{13}\,\sqrt{42}+\frac{8}{11}\,n\sqrt{30}+\frac{1}{6}\,(4\,mn+4\,n+4)\sqrt{35}+\frac{32}{13}\,n\sqrt{10}+\frac{3}{7}\,(4\,mn+4\,n)\sqrt{5}+\frac{16}{25}\,m\sqrt{39}+\frac{8}{9}\,m\sqrt{5}+\frac{3}{2}\,\sqrt{7}+\frac{16}{19}\,\sqrt{21}+\frac{24}{19}\,\sqrt{10}+\frac{728}{345}\,\sqrt{14}+\frac{16}{7}\,\sqrt{3}+\frac{700}{319}\,m\sqrt{13}+\frac{2}{3}\,(4\,m+4)\sqrt{2}.$
2. $S(G)=27152.35\,n+35589.02\,mn+23930.48\,m+180321.56.$

Proof: There are thirty- three types of edges on the degree-based sum of neighbors vertices of each edge in the graph, We use this partition of edges to calculate Sanskruti index and GA_5 indices. The line division of neighbors of (m, n) of the general graph is shown in Table 2.

1. GA_5 index is defined as:

$$GA_5(G)=\sum_{\mathrm{pq}}\in E(G)\left(\frac{2\sqrt{S_p\times S_q}}{S_p+S_q}\right).$$

Table 2: The line division of neighbors of (m, n) of a general graph

(S_p, S_q), Where $pq \in E(G)$	No. of edges	(S_p, S_q), Where $pq \in E(G)$	No. of edges
(7,9)	4	(7,12)	4
(9,9)	6m+6	(9,10)	4
(9,12)	4	(9,13)	4m
(9,14)	4	(9,18)	4m+4
(10,10)	6n+2	(10,12)	4n
(10,14)	4(mn+n+1)	(10,16)	8n
(10,18)	4mn+4n	(12,13)	4m
(12,14)	8	(12,15)	2m
(12,21)	2n	(13,14)	4m
(13,16)	4m	(14,14)	mn+n
(14,16)	4	(14,18)	8m+8
(14,21)	2n	(14,22)	2mn
(15,16)	6m	(16,16)	3mn+2n+5m-2
(16,18)	16mn+8n+4m	(16,20)	2mn
(16,21)	8n	(16,22)	4mn
(18,18)	6mn+4n+2m+4	(18,22)	4mn
(20,22)	2mn		
Total	48mn+53(m+n)+58		

This implies that,

$$GA_5(G) = |E_{\{7,9\}}| \times \left(\frac{2\sqrt{7\times9}}{7+9}\right) + |E_{\{7,12\}}| \times \left(\frac{2\sqrt{7\times12}}{7+12}\right) + |E_{\{9,9\}}| \times \left(\frac{2\sqrt{9\times9}}{9+9}\right) + |E_{\{9,10\}}| \times \left(\frac{2\sqrt{9\times10}}{9+10}\right) + |E_{\{9,12\}}| \times \left(\frac{2\sqrt{9\times12}}{9+12}\right) + |E_{\{9,13\}}| \times \left(\frac{2\sqrt{9\times13}}{9+13}\right) + |E_{\{9,14\}}| \times \left(\frac{2\sqrt{9\times14}}{9+14}\right) + |E_{\{9,18\}}| \times \left(\frac{2\sqrt{9\times18}}{9+18}\right) + |E_{\{10,10,\}}| \times \left(\frac{2\sqrt{10\times10}}{10+10}\right) + |E_{\{10,12,\}}| \times \left(\frac{2\sqrt{10\times12}}{10+12}\right) + |E_{\{10,14,\}}| \times \left(\frac{2\sqrt{10\times14}}{10+14}\right) + |E_{\{10,16\}}| \times \left(\frac{2\sqrt{10\times16}}{10+16}\right) + |E_{\{10,18\}}| \times \left(\frac{2\sqrt{10\times18}}{10+18}\right) + |E_{\{12,13\}}| \times \left(\frac{2\sqrt{12\times13}}{12+13}\right) + |E_{\{12,14\}}| \times \left(\frac{2\sqrt{12\times14}}{12+14}\right) + |E_{\{12,15\}}| \times \left(\frac{2\sqrt{12\times15}}{12+15}\right) + |E_{\{12,21\}}| \times \left(\frac{2\sqrt{12\times21}}{12+21}\right) + |E_{\{13,14\}}| \times \left(\frac{2\sqrt{13\times14}}{13+14}\right) + |E_{\{13,16\}}| \times \left(\frac{2\sqrt{13\times16}}{13+16}\right) + |E_{\{14,14\}}| \times \left(\frac{2\sqrt{14\times14}}{14+14}\right) + |E_{\{14,16\}}| \times \left(\frac{2\sqrt{14\times16}}{14+16}\right) + |E_{\{14,18\}}| \times \left(\frac{2\sqrt{14\times18}}{14+18}\right) + |E_{\{14,21,\}}| \times \left(\frac{2\sqrt{14\times21}}{14+21}\right) + |E_{\{14,22\}}| \times \left(\frac{2\sqrt{14\times22}}{14+22}\right) + |E_{\{15,16,\}}| \times \left(\frac{2\sqrt{15\times16}}{15+16}\right) + |E_{\{16,16,\}}| \times \left(\frac{2\sqrt{16\times16}}{16+16}\right) + |E_{\{16,18\}}| \times \left(\frac{2\sqrt{16\times18}}{16+18}\right) + |E_{\{16,20\}}| \times \left(\frac{2\sqrt{16\times20}}{16+20}\right) + |E_{\{16,21\}}| \times \left(\frac{2\sqrt{16\times21}}{16+21}\right) + |E_{\{16,22\}}| \times \left(\frac{2\sqrt{16\times22}}{16+22}\right) + |E_{\{18,18\}}| \times \left(\frac{2\sqrt{18\times18}}{18+18}\right) + |E_{\{18,22\}}| \times \left(\frac{2\sqrt{18\times22}}{18+22}\right) + |E_{\{20,22\}}| \times \left(\frac{2\sqrt{20\times22}}{20+22}\right).$$

Using Table 2 we get,

$$GA_5(G) = 4 \times \left(\frac{2\sqrt{7\times9}}{7+9}\right) + 4 \times \left(\frac{2\sqrt{7\times12}}{7+12}\right) + 6m + 6 \times \left(\frac{2\sqrt{9\times9}}{9+9}\right) + 4 \times \left(\frac{2\sqrt{9\times10}}{9+10}\right) + 4 \times \left(\frac{2\sqrt{9\times12}}{9+12}\right) + 4m \times \left(\frac{2\sqrt{9\times13}}{9+13}\right) + 4 \times \left(\frac{2\sqrt{9\times14}}{9+14}\right) + 4m + 4 \times \left(\frac{2\sqrt{9\times18}}{9+18}\right) + 6n + 2 \times \left(\frac{2\sqrt{10\times10}}{10+10}\right) + 4 \times \left(\frac{2\sqrt{10\times12}}{10+12}\right) + 4mn + 4n + 4 \times \left(\frac{2\sqrt{10\times14}}{10+14}\right) + 8n \times \left(\frac{2\sqrt{10\times16}}{10+16}\right) + 4mn + 4n \times \left(\frac{2\sqrt{10\times18}}{10+18}\right) + 4m \times \left(\frac{2\sqrt{12\times13}}{12+13}\right) + 8 \times \left(\frac{2\sqrt{12\times14}}{12+14}\right) + 2m \times \left(\frac{2\sqrt{12\times15}}{12+15}\right) + 2n \times \left(\frac{2\sqrt{12\times21}}{12+21}\right) + 4m \times \left(\frac{2\sqrt{13\times14}}{13+14}\right) + 4m \times \left(\frac{2\sqrt{13\times16}}{13+16}\right) + mn + n \times \left(\frac{2\sqrt{14\times14}}{14+14}\right) + \times \left(\frac{2\sqrt{14\times16}}{14+16}\right) + 8m + 8 \times \left(\frac{2\sqrt{14\times18}}{14+18}\right) + 2n \times \left(\frac{2\sqrt{14\times21}}{14+21}\right) + 2mn \times \left(\frac{2\sqrt{14\times22}}{14+22}\right) + 6m \times \left(\frac{2\sqrt{15\times16}}{15+16}\right) + 3mn + 2n + 5m - 2 \times \left(\frac{2\sqrt{16\times16}}{16+16}\right) + 16mn + 8n + 4m \times \left(\frac{2\sqrt{16\times18}}{16+18}\right) + 2mn \times \left(\frac{2\sqrt{16\times20}}{16+20}\right) + 8n \times \left(\frac{2\sqrt{16\times21}}{16+21}\right) + 4mn \times \left(\frac{2\sqrt{16\times22}}{16+22}\right) + 6mn + 4n + 2m + 4 \times \left(\frac{2\sqrt{18\times18}}{18+18}\right) + 4mn \times \left(\frac{2\sqrt{18\times22}}{18+22}\right) + 2mn \times \left(\frac{2\sqrt{20\times22}}{20+22}\right).$$

After simplification we have,

$$GA_5(G) = 10 + 13\,n + 10\,mn + \frac{3}{8}\,(8\,m + 8)\sqrt{7} + \frac{8}{27}\,m\sqrt{182} + \frac{8}{11}\,n\sqrt{7} + \frac{12}{17}\,(16\,mn + 8\,n + 4\,m)\sqrt{2} + \frac{48}{31}\,m\sqrt{15} + \frac{2}{9}\,mn\,\sqrt{77} + \frac{4}{5}\,n\sqrt{6} + 13\,m + \frac{4}{21}\,mn\,\sqrt{110} + \frac{6}{5}\,mn\,\sqrt{11} + \frac{16}{19}\,mn\,\sqrt{22} + \frac{64}{37}\,n\sqrt{21} + \frac{8}{9}\,mn\,\sqrt{5} + \frac{16}{13}\,\sqrt{42} + \frac{8}{11}\,n\sqrt{30} + \frac{1}{6}\,(4\,mn + 4\,n + 4)\sqrt{35} + \frac{32}{13}\,n\sqrt{10} + \frac{3}{7}\,(4\,mn + 4\,n)\sqrt{5} + \frac{16}{25}\,m\sqrt{39} + \frac{8}{9}\,m\sqrt{5} + \frac{3}{2}\,\sqrt{7} + \frac{16}{19}\,\sqrt{21} + \frac{24}{19}\,\sqrt{10} + \frac{728}{345}\,\sqrt{14} + \frac{16}{7}\,\sqrt{3} + \frac{700}{319}\,m\sqrt{13} + \frac{2}{3}\,(4\,m + 4)\sqrt{2}.$$

2. The Sanskruti index $S(G)$ of a molecular graph G is defined as:

$$S(G) = \sum_{pq\in E(G)} \left(\frac{S_p \times S_q}{S_p + S_q - 2}\right)^3.$$

This implies that,

$$S(G) = |E_{\{7,9\}}| \times \left(\frac{7\times9}{7+9-2}\right)^3 + |E_{\{7,12\}}| \times \left(\frac{7\times12}{7+12-2}\right)^3 + |E_{\{9,9\}}| \times \left(\frac{9\times9}{9+9-2}\right)^3 + |E_{\{9,10\}}| \times \left(\frac{9\times10}{9+10-2}\right)^3 + |E_{\{9,12\}}| \times \left(\frac{9\times12}{9+12-2}\right)^3 + |E_{\{9,13\}}| \times \left(\frac{9\times13}{9+13-2}\right)^3 + |E_{\{9,14\}}| \times \left(\frac{9\times14}{9+14-2}\right)^3 + |E_{\{9,18\}}| \times \left(\frac{9\times18}{9+18-2}\right)^3 + |E_{\{10,10,\}}| \times \left(\frac{10\times10}{10+10-2}\right)^3 + |E_{\{10,12,\}}| \times \left(\frac{10\times12}{10+12-2}\right)^3 + |E_{\{10,14,\}}| \times \left(\frac{10\times14}{10+14-2}\right)^3 + |E_{\{10,16\}}| \times \left(\frac{10\times16}{10+16-2}\right)^3 + |E_{\{10,18\}}| \times \left(\frac{10\times18}{10+18-2}\right)^3 + |E_{\{12,13\}}| \times \left(\frac{12\times13}{12+13-2}\right)^3 + |E_{\{12,14\}}| \times \left(\frac{12\times14}{12+14-2}\right)^3 + |E_{\{12,15\}}| \times \left(\frac{12\times15}{12+15-2}\right)^3 + |E_{\{12,21\}}| \times \left(\frac{12\times21}{12+21-2}\right)^3 + |E_{\{13,14\}}| \times \left(\frac{13\times14}{13+14-2}\right)^3 + |E_{\{13,16\}}| \times \left(\frac{13\times16}{13+16-2}\right)^3 + |E_{\{14,14\}}| \times \left(\frac{14\times14}{14+14-2}\right)^3 + |E_{\{14,16\}}| \times \left(\frac{14\times16}{14+16-2}\right)^3 + |E_{\{14,18\}}| \times \left(\frac{14\times18}{14+18-2}\right)^3 + |E_{\{14,21,\}}| \times \left(\frac{14\times21}{14+21-2}\right)^3 + |E_{\{14,22\}}| \times \left(\frac{14\times22}{14+22-2}\right)^3 + |E_{\{15,16,\}}| \times \left(\frac{15\times16}{15+16-2}\right)^3 + |E_{\{16,16,\}}| \times \left(\frac{16\times16}{16+16-2}\right)^3 + |E_{\{16,18\}}| \times \left(\frac{16\times18}{16+18-2}\right)^3 + |E_{\{16,20\}}| \times \left(\frac{16\times20}{16+20-2}\right)^3 + |E_{\{16,21\}}| \times \left(\frac{16\times21}{16+21-2}\right)^3 + |E_{\{16,22\}}| \times \left(\frac{16\times22}{16+22-2}\right)^3 + |E_{\{18,18\}}| \times \left(\frac{18\times18}{18+18-2}\right)^3 + |E_{\{18,22\}}| \times \left(\frac{18\times22}{18+22-2}\right)^3 + |E_{\{20,22\}}| \times \left(\frac{20\times22}{20+22-2}\right)^3.$$

Using Table 2 we get,

$$S(G) = 4\times\left(\frac{7\times 9}{7+9-2}\right)^3 + 4\times\left(\frac{7\times 12}{7+12-2}\right)^3 + 6m + 6\times\left(\frac{9\times 9}{9+9-2}\right)^3 + 4\times\left(\frac{9\times 10}{9+10-2}\right)^3 + 4\times\left(\frac{9\times 12}{9+12-2}\right)^3 + 4m\times\left(\frac{9\times 13}{9+13-2}\right)^3 + 4\times\left(\frac{9\times 14}{9+14-2}\right)^3 + 4m + 4\times\left(\frac{9\times 18}{9+18-2}\right)^3 + 6n + 2\times\left(\frac{10\times 10}{10+10-2}\right)^3 + 4n\times\left(\frac{10\times 12}{10+12-2}\right)^3 + 4mn + 4n + 4\times\left(\frac{10\times 14}{10+14-2}\right)^3 + 8n\times\left(\frac{10\times 16}{10+16-2}\right)^3 + 4mn + 4n\times\left(\frac{10\times 18}{10+18-2}\right)^3 + 4m\times\left(\frac{12\times 13}{12+13-2}\right)^3 + 8\times\left(\frac{12\times 14}{12+14-2}\right)^3 + 2m\times\left(\frac{12\times 15}{12+15-2}\right)^3 + 2n\times\left(\frac{12\times 21}{12+21-2}\right)^3 + 4m\times\left(\frac{13\times 14}{13+14-2}\right)^3 + 4m\times\left(\frac{13\times 16}{13+16-2}\right)^3 + mn + n\times\left(\frac{14\times 14}{14+14-2}\right)^3 + 4\times\left(\frac{14\times 16}{14+16-2}\right)^3 + 8m + 8\times\left(\frac{14\times 18}{14+18-2}\right)^3 + 2n\times\left(\frac{14\times 21}{14+21-2}\right)^3 + 2mn\times\left(\frac{14\times 22}{14+22-2}\right)^3 + 6m\times\left(\frac{15\times 16}{15+16-2}\right)^3 + 3mn + 2n + 5m - 2\times\left(\frac{16\times 16}{16+16-2}\right)^3 + 16mn + 8n + 4m\times\left(\frac{16\times 18}{16+18-2}\right)^3 + 2mn\times\left(\frac{16\times 20}{16+20-2}\right)^3 + 8n\times\left(\frac{16\times 21}{16+21-2}\right)^3 + 4mn\times\left(\frac{16\times 22}{16+22-2}\right)^3 + 6mn + 4n + 2m + 4\times\left(\frac{18\times 18}{18+18-2}\right)^3 + 4mn\times\left(\frac{18\times 22}{18+22-2}\right)^3 + 2mn\times\left(\frac{20\times 22}{20+22-2}\right)^3.$$

After simplification, we obtain that,

$$S(G) = 27152.35\,n + 35589.02\,mn + 23930.48\,m + 180321.56.$$

Which completes the proof.

3. Conclusion

In this paper, we determined the well popular topological indices of the web graph. We have determined and computed the closed formulas of the first general Zagreb index, geometric-arithmetic index GA, ordinary geometric-arithmetic index OGA_r, multiplicative geometric-arithmetic index GAII, SK, SK_1, SK_2, harmonic index, general version of Harmonic index $H_r(G)$, sum connectivity index $\lambda(G)$, general sum connectivity index $\lambda_r(G)$, Forgotten topological index F(G) for the web graph. We have determined and computed the closed formulas of the $GA_5(G)$ and Sanskruti index S(G) for the Web graph. These outcomes provide a significant contribution in graph theory and they give a superior foundation to appreciate the topology of the web graph. In the future, we are interested in studying and computing spectrum based topological indices of a web graph.

Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

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