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Calculus of new intuitionistic fuzzy generator: In generated intuitionistic fuzzy sets and its applications in medical diagnosis

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ABSTRACT

In this research paper, we defined the generated intuitionistic fuzzy set. We generated an intuitionistic fuzzy generator to define the generated intuitionistic fuzzy set. The generated intuitionistic fuzzy set is a generalization of the intuitionistic fuzzy set. We proved some basic properties of generated intuitionistic fuzzy set in the context of intuitionistic fuzzy sets; some results are proved by using the notion of newly generated intuitionistic fuzzy set and constructed intuitionistic fuzzy generator. A mathematical approach for the application of the generated intuitionistic fuzzy set is also given in this paper.

1. Introduction

The concept of fuzzy logic was given by Zadeh (1965), that takes into account the membership grade only. The membership function considers the grade of favorable cases and gives birth to the linguistic variables. The fuzzy logic in terms of the linguistic variable is a marvelous tool to deal with uncertainty. Due to the linguistic terminology of fuzzy logic, it plays a vital role in decision making that becomes fuzzy decision making about the jobs that possess current uncertainty. But where the membership grade is inadequate to define the current uncertainty due to the non-consideration of unfavorable cases, then it gave birth to a new logic, which consists of favorable, unfavorable and some other function to define the complete information about any object introduced by Atanassov (1986) further modified in Atanassov (1999). Intuitionistic fuzzy sets work on a theory, which considers favorable and unfavorable cases together and also defines the membership degree and non-membership degree of an element regarding its belongings, not belongings, and some other part, the sum of these three values always lies between 0 and 1. If the characteristics of an element are defined by membership and non-membership completely, then this concept turns into the concept of a vague set given by Gau and Buehrer (1993). The notion of vague logic and intuitionistic fuzzy logic is the same. However, in real-life applications, the linguistic negation does meet the requirement of the logical negation, while selecting the membership grade. There may be some kind of hesitation function in constructing the membership function as well as non-membership function. The membership function may be triangular, trapezoidal, exponential, Gaussian, bell-shaped, or any other function. So, due to this hesitation part, non-membership grade is less than or equal to the standard fuzzy complement of the membership grade. Cause of this, different approaches have been explained in defining the membership functions. There are many applications of intuitionistic fuzzy sets in the medical field given by De et al. (2001), and Szmidt and Kacprzyk (2001).

In constructing an intuitionistic fuzzy set (IFS), generators based on fuzzy logic are used. Fuzzy generators are fuzzy complements, and fuzzy complement functional is used to construct the fuzzy compliment given by Chaira (2019) which is defined as follow;

$$\emptyset\big(\mu(x)\big) = \xi^{-1}(\xi(1) - \xi(\mu(x))) \quad (1)$$

where $\mu(x)$ denotes the membership grade of the IFS and ξ is an increasing function with the condition that$\xi(0) = 0$, Sugeno (1977) used an increasing function as follows;

$$\xi\big(\mu(x)\big) = \frac{1}{\lambda}\log\big(1 + \lambda.\mu(x)\big), \lambda \geq 0$$

Then using the above function with fuzzy complement functional defined by Eq. 1, we get;

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$$\emptyset(\mu(x)) = \xi^{-1}\left(\frac{1}{\lambda}\log(1+\lambda) - \frac{1}{\lambda}\log(1+\lambda.\mu(x))\right)$$

as

$$\xi(1) = \frac{1}{\lambda}\log(1+\lambda)$$
$$= \xi^{-1}(\frac{1}{\lambda}\log(\frac{1+\lambda}{1+\lambda.\mu(x)}))$$

On behalf of Sugeno-type intuitionistic fuzzy complement, IFS become;

$$IFS = \langle\left(x, \mu(x), \frac{1-\mu(x)}{1+\lambda\mu(x)}\right): x \in X, \lambda \geq 0\rangle$$

Yager (1980) also suggested fuzzy complement functional for $0 < \delta \leq 1$ as follows;

$$\emptyset(\mu(x)) = (1-(\mu(x))^{\delta})^{\frac{1}{\delta}}$$

This gives birth to the Yager-type intuitionistic fuzzy set defined as;

$$IFS = \langle\left(x, \mu(x), (1-\mu(x)^{\delta})^{\frac{1}{\delta}}\right): x \in X\rangle$$

Other intuitionistic fuzzy sets have also been studied based on various intuitionistic fuzzy generators by Chaira (2019). Bustince et al. (2000), also extended intuitionistic fuzzy sets and their application in fuzzy complements based on intuitionistic fuzzy sets. Chaira (2014), gave an intuitionistic fuzzy generator as follows;

$$\emptyset(\mu(x)) = (\frac{1-\mu(x)}{1+(e^{\lambda}-1)\mu(x)}), \qquad \lambda \geq 0$$

with using a monotonically increasing function ξ, for any $\lambda \geq 0$ as;

$$\xi(\mu(x)) = \frac{1}{\lambda}\log(1 - \mu(x)(1-e^{\lambda}))$$

and the intuitionistic fuzzy set using the defined intuitionistic fuzzy generator as follows;

$$IFS = \langle\left(x, \mu(x), \frac{1-\mu(x)}{1+(e^{\lambda}-1)\mu(x)}\right): x \in X, \lambda \geq 0\rangle$$

This intuitionistic fuzzy generator is based on the exponential distribution. But it involves only one parameter λ. For better intuitionistic fuzzy generator, we need magnitude also, i.e., our requirement needs two parameters. So, we will generate the other parameter for magnitude.

In this present research paper, we suggested a new intuitionistic fuzzy generator, which is the generalization of Chaira (2014), which gives a wide range of constructing the membership function to cover up the more impreciseness. In this present research paper, we introduced two parameters in the construction of the intuitionistic fuzzy generator. Using this new generator, we will define the generated intuitionistic fuzzy set. The present work, in this research paper, is divided into six sections, in the second section of the research paper; we defined some basic definitions based on intuitionistic fuzzy generators. In the third section, we generalized Chaira's (2014) fuzzy generator by providing our intuitionistic fuzzy generator, and by using this generator, we defined the generalized intuitionistic fuzzy set. In the fourth section of the research paper, we will prove some results based on the proposed intuitionistic fuzzy generator and intuitionistic fuzzy set, while in the fifth section, we will give an approach for the applications of this generalized intuitionistic fuzzy generator in medical diagnosis and the last section of the work contains conclusion part.

2. Basic definitions

2.1. Fuzzy complement

Let $A = (x, \mu(x)): x \in X$ be any fuzzy set defined on a Universal set$X$, and then a fuzzy complement $'c'$ of a fuzzy set A (Klir and Yuan, 1995) is defined as;

$c: [0,1] \to [0,1]$

such that

$c(\mu(x)) = c.\mu(x)$,

which possesses four axioms;

a) Boundary condition i.e., $c(0) = 1$ and $c(1) = 0$,
b) Monotonicity, i.e., if $\leq b, then \quad c(a) \geq c(b), \quad \forall a, b \in [0,1]$.
c) c is continuous.
d) c is involutive, i.e., $c(c(a)) = a, \forall a \in [0,1]$.

2.2. Intuitionistic fuzzy complement

Let $A = \{(x, \mu(x), \nu(x)): x \in X\}$ be any intuitionistic fuzzy set defined on a Universal set X, then an intuitionistic fuzzy complement;

$c : [0,1] \to [0,1]$

such that $c(a) = c.a, \forall a \in \{\mu(x), \nu(x)\}$ i.e., we may extend the concept of a fuzzy complement over non-membership values because non-membership values for the IFS are computed by using intuitionistic fuzzy generators.

2.3. Intuitionistic fuzzy generator

Let $S = [0,1]$ be a closed unit length interval and a function $f(x): S \to S$ is called an intuitionistic fuzzy generator if;

a) f is monotonic,
b) $f(x) \leq 1 - x, \quad \forall x \in S$, (Bustince et al., 2000)
c) $f(0) \leq 1\ and f(1) = 0$.

This intuitionistic fuzzy generator has the following properties;

1. Equilibrium point: Any $x \in [0,1]$ is an equilibrium point of an intuitionistic fuzzy generator $f(x)$, if $f(x) = x$.
2. Duality of points: Let $f(x)$ be an intuitionistic fuzzy generator then x is the dual point of y with respect to f, if $f(x) + f(y) = x + y$, holds.
3. Involution of intuitionistic fuzzy generator: $f(x): [0,1] \to [0,1]$ be an intuitionistic fuzzy generator then, it is called involutive in [0, 1] is $f(f(x)) = x$, for all $x \in [0,1]$.

3. Construction of intuitionistic fuzzy generator and generated intuitionistic fuzzy set

For an intuitionistic fuzzy set, fuzzy complement functional has used to generate the fuzzy complement, which is defined by;

$$\emptyset(\mu(x)) = \xi^{-1}(\xi(1) - \xi(\mu(x)))$$

where ξ represents an increasing function with $\xi(0) = 0$.

Now, we are constructing an increasing function as follows;

$$\xi(\mu(x)) = \frac{1}{\lambda}\log\left(1 - \gamma(1 - e^{\lambda})\mu(x)\right) \forall \gamma \in [0,1]$$

and

$$\lambda \geq 0 \tag{2}$$

from Eq. 2, we have

$$\xi(1) = \frac{1}{\lambda}\log(1 - \gamma(1 - e^{\lambda}))$$

now,

$$\xi(1) - \xi(\mu(x)) = \frac{1}{\lambda}\log\left(1 - \gamma(1 - e^{\lambda})\right) - \frac{1}{\lambda}\log(1 - \gamma(1 - e^{\lambda})\mu(x)) = \frac{1}{\lambda}\log\left(\frac{1-\gamma(1-e^{\lambda})}{1-\gamma(1-e^{\lambda})\mu(x)}\right)$$

So,

$$\xi^{-1}\left(\xi(1) - \xi(\mu(x))\right) = \xi^{-1}\left(\frac{1}{\lambda}\log\left(\frac{1-\gamma(1-e^{\lambda})}{1-\gamma(1-e^{\lambda})\mu(x))}\right)\right) \tag{3}$$

by Eq. 2, we compute $\xi^{-1}(\mu(y)) = \frac{1-e^{x\lambda}}{\gamma(1-e^{\lambda})}$, by using this and from Eq. 3, we compute an intuitionistic fuzzy generator, which is as follows;

$$\emptyset(\mu(x)) = \xi^{-1}\left(\frac{1}{\lambda}\log\left(\frac{1-\gamma(1-e^{\lambda})}{1-\gamma(1-e^{\lambda})\mu(x)}\right)\right) = \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)} \forall \gamma \in [0,1]$$

and

$$\lambda \geq 0 \tag{4}$$

So, for any $\lambda \geq 0$, $\gamma \in [0,1]$, the IFS, $'I'$ can be written as;

$$I = \langle\left(x, \mu(x), \emptyset(\mu(x))\right) : x \in X\rangle$$

or

$$I = \langle\left(x, \mu(x), \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}\right) : x \in X\rangle \tag{5}$$

The newly generated intuitionistic fuzzy set as defined in (5) will have the following characteristics;

3.1. (α, β) −cut for generated intuitionistic fuzzy set

For generated intuitionistic fuzzy set I, the (α, β) −cut is a crisp set as follows;

$$I_{(\alpha,\beta)} = \langle x \in X: \mu(x) \geq \alpha, \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)} \leq \beta\rangle, \quad \forall \alpha, \beta \in [0,1].$$

3.2. Support of generated intuitionistic fuzzy set

For generated intuitionistic fuzzy set I, the support of I is a crisp set as follows;

$$\text{Supp(I)} = \{x \in X: \mu(x) > 0, \ \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)} > 0\}.$$

3.3. Core of generated intuitionistic fuzzy set

For generated intuitionistic fuzzy set I, the core of I is a crisp set as follows;

$$\text{Core(I)} = \{x \in X: \mu(x) = 1, \ \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)} = 0\}.$$

3.4. Arithmetic operations on generated intuitionistic fuzzy set

Let $I_1 = \langle\left(x, \mu(x), \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}\right) : x \in X\rangle$ and $I_2 = \langle\left(x, \mu'(x), \frac{1-\mu'(x)}{1-\gamma(1-e^{\lambda})\mu'(x)}\right) : x \in X\rangle$ be two intuitionistic fuzzy sets, then:

a) $I_1 \cap I_2 = \langle\left(x, min(\mu(x), \mu'(x)), max(\frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}, \frac{1-\mu'(x)}{1-\gamma(1-e^{\lambda})\mu'(x)}\right) : x \in X\rangle$.

b) $I_1 \cup I_2 = \langle\left(x, max(\mu(x), \mu'(x)), min(\frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}, \frac{1-\mu'(x)}{1-\gamma(1-e^{\lambda})\mu'(x)}\right) : x \in X\rangle$.

c) $I_1 + I_2 = \langle\left(x, \mu(x) + \mu'(x) - \mu(x).\mu'(x), \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}.\frac{1-\mu'(x)}{1-\gamma(1-e^{\lambda})\mu'(x)}\right) : x \in X\rangle$.

d) $I_1 * I_2 = \langle\left(x, \mu(x).\mu'(x), \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)} + \frac{1-\mu'(x)}{1-\gamma(1-e^{\lambda})\mu'(x)} - \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}.\frac{1-\mu'(x)}{1-\gamma(1-e^{\lambda})\mu'(x)}\right) : x \in X\rangle$

e) $I * I = \langle\left(x, (\mu(x))^2, 2\frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)} - \left(\frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}\right)^2\right) : x \in X\rangle$

$=\langle\left(\mathrm{x}, (\mu(\mathrm{x}))^2, 1-\left(1-\frac{1-\mu(\mathrm{x})}{1-\gamma(1-e^{\lambda})\mu(\mathrm{x})}\right)^2\right): \mathrm{x} \in \mathrm{X}\rangle.$

So, for any positive integer m,

f) $I^m = \langle\left(x, (\mu(x))^{\mathrm{m}}, 1-\left(1-\frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}\right)^{\mathrm{m}}\right): x \in \mathrm{X}\rangle.$

3.5. Convexity of generated intuitionistic fuzzy set

Let $\emptyset(\mu(x)) = \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}$ be generalized intuitionistic fuzzy generator, then generalized IFS.

$I = \langle(x, \mu(x), \emptyset(\mu(x))): x \in X\rangle$ is convex if;

a) $\mu(tx+(1-ty)) \geq t\mu(x) + (1-t)\mu(y)$,
b) $\frac{1-\mu(tx+(1-t)y)}{1-\gamma(1-e^{\lambda})\mu(tx+(1-t)y)} \leq \mathrm{t}\frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)} + (1-\mathrm{t})\frac{1-\mu(y)}{1-\gamma(1-e^{\lambda})\mu(y)}$

$\forall x, y \in X$ and $t \in (0, 1)$.

4. Some basic results based on constructed intuitionistic fuzzy generator and generated intuitionistic fuzzy set

For the generated intuitionistic fuzzy set and intuitionistic fuzzy generator, we will prove the following results with their mathematical arguments.

Result 1: Generated intuitionistic fuzzy set,

$I = \langle\left(x, \mu(x), \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}\right): x \in X\rangle$

is convex if;

a) $\mu(tx+(1-ty)) \geq min\{\mu(x), \mu(y)\}$,
b) $\frac{1-\mu(tx+(1-t)y)}{1-\gamma(1-e^{\lambda})\mu(tx+(1-t)y)} \leq max\left\{\frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}, \frac{1-\mu(y)}{1-\gamma(1-e^{\lambda})\mu(y)}\right\}$

$\forall x, y \in X$ and $t \in (0, 1)$.

Proof: Let us assume that intuitionistic fuzzy set $'I'$ is convex and let $\alpha = \mu(x) \leq \mu(y)$, and $\beta = \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)} \geq \frac{1-\mu(y)}{1-\gamma(1-e^{\lambda})\mu(y)} \Rightarrow x, y \in I_{(\alpha,\beta)}$, where $I_{(\alpha,\beta)}$ is the (α,β)-cut of I. So, $tx+(1-ty) \in I_{(\alpha,\beta)}$ for any $t \in (0, 1)$. Therefore,

$$\mu(tx+(1-ty)) \geq \alpha \Rightarrow \mu(tx+(1-ty)) \geq min\{\mu(x), \mu(y)\}$$

and,

$$\frac{1-\mu(tx+(1-t)y)}{1-\gamma(1-e^{\lambda})\mu(tx+(1-t)y)} \leq \beta$$
$$\Rightarrow \frac{1-\mu(tx+(1-t)y)}{1-\gamma(1-e^{\lambda})\mu(tx+(1-t)y)} \leq max\left\{\frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}, \frac{1-\mu(y)}{1-\gamma(1-e^{\lambda})\mu(y)}\right\}$$

Conversely, let the given conditions a and b holds, then we need to show that $'I'$ is convex,

For any

$x, y \in I_{(\alpha,\beta)}$
$\mu(x) \geq \alpha, \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)} \leq \beta$

and,

$\mu(y) \geq \alpha, \frac{1-\mu(y)}{1-\gamma(1-e^{\lambda})\mu(y)} \leq \beta$

and for any $t \in (0, 1)$

$$\mu(tx+(1-ty)) \geq min(\mu(x), \mu(y))$$
$$\Rightarrow \mu(tx+(1-ty)) \geq \alpha$$

and,

$$\frac{1-\mu(tx+(1-t)y)}{1-\gamma(1-e^{\lambda})\mu(tx+(1-t)y)} \leq max\{\frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}, \frac{1-\mu(y)}{1-\gamma(1-e^{\lambda})\mu(y)}\}$$
$$\Rightarrow \frac{1-\mu(tx+(1-t)y)}{1-\gamma(1-e^{\lambda})\mu(tx+(1-t)y)} \leq \beta$$
$$\Rightarrow tx+(1-ty) \in I_{(\alpha,\beta)},$$

this implies that $'I'$ is convex.

Result 2: Show that the generalized intuitionistic fuzzy generator $\emptyset(\mu(x)) = \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)} \forall \gamma \in [0, 1]$ and $\lambda \geq 0$, defined on a Universal set $'X'$ is involutive.

Proof: We have,

$$y = \emptyset(\mu(x)) = \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}$$

now,

$$\emptyset(y) = \frac{1-\mathrm{y}}{1-\gamma(1-e^{\lambda})\mathrm{y}} = \frac{1-\left(\frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}\right)}{1-\gamma(1-e^{\lambda})\left(\frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}\right)}$$
$$= \frac{\left(\frac{1-\gamma(1-e^{\lambda})\mu(x)-1+\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}\right)}{\left(\frac{1-\gamma(1-e^{\lambda})\mu(x)-\gamma(1-e^{\lambda})(1-\mu(x))}{1-\gamma(1-e^{\lambda})\mu(x)}\right)}$$
$$= \frac{\left(1-\gamma(1-e^{\lambda})\mu(x)-1+\mu(x)\right)}{\left(1-\gamma(1-e^{\lambda})\mu(x)-\gamma(1-e^{\lambda})(1-\mu(x))\right)} = \frac{\mu(x)\left(1-\gamma(1-e^{\lambda})\right)}{\left(1-\gamma(1-e^{\lambda})\right)}$$
$$\Rightarrow \emptyset(y) = \mu(x)$$
$$\Rightarrow \emptyset$$

is involutive.

Result 3: If an intuitionistic fuzzy generator $\emptyset(\mu(x)) = \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)} \forall \gamma \in [0, 1] and \lambda \geq 0$ defined on Universal set$'X'$, then find out the equilibrium point of $\emptyset$.

Proof: Let $\mu(x) \in [0, 1]$ is an equilibrium point of an intuitionistic fuzzy generator $\emptyset(\mu(x))$, then,

$$\emptyset(\mu(x)) = \mu(x) \Rightarrow \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)} = \mu(x)$$
$$\Rightarrow 1-\mu(x) = \mu(x) - \gamma(1-e^{\lambda})(\mu(x))^2$$

or

$$\gamma(1-e^{\lambda})(\mu(x))^2 - 2\mu(x) + 1 = 0$$

or

$$\mu(x) = \frac{1 \pm \sqrt{1-\gamma(1-e^{\lambda})}}{\gamma(1-e^{\lambda})} \gamma \neq 0 \ and \lambda > 0$$

But, $\mu(x) = \frac{1+\sqrt{1-\gamma(1-e^{\lambda})}}{\gamma(1-e^{\lambda})}$ will not satisfy the equilibrium point condition.

So, $\mu(x) = \frac{1-\sqrt{1-\gamma(1-e^{\lambda})}}{\gamma(1-e^{\lambda})} \gamma \neq 0 \ and \lambda > 0$, be the equilibrium point for generalized intuitionistic fuzzy generator $\emptyset$.

Result 4: Let,

$$\emptyset(\mu(x)) = \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x))}$$

be an intuitionistic fuzzy generator, then if $\emptyset(\mu(x))$ is monotonically decreasing, then there must exist a monotonically increasing function $'g'$ such that $g(\mu(x)) \leq \mu(x)$ and if $N'(\mu(x)) = 1-(\mu(x))$ be the standard negation of $(\mu(x))$ then $(g^{\circ}N')(\mu(x)) = \emptyset(\mu(x))$

Proof: Let g be any function such that,

$$g(\mu(x)) = \emptyset(1-\mu(x)),$$

Let $\emptyset$ is decreasing, then any $\mu(x_1)$ and $\mu(x_2)$ such that,

$$\mu(x_1) \leq \mu(x_2) \Rightarrow \emptyset(\mu(x_1)) \geq \emptyset(\mu(x_2)).$$

So, as $\emptyset(\mu(x))$ decreasing and hence $\emptyset(1-\mu(x)) = g(\mu(x))$ is increasing.

Now, we have $'g'$ as a monotonically increasing, further,

$$g(\mu(x)) = \frac{1-(1-\mu(x))}{1-\gamma(1-e^{\lambda})(1-\mu(x))} = \frac{\mu(x)}{1-\gamma(1-e^{\lambda})(1-\mu(x))} \leq \mu(x)$$

now,

$$(g^{\circ}N')(\mu(x)) = g(N'(\mu(x)) = g(1-(\mu(x))$$

as

$$N'(\mu(x)) = 1-(\mu(x))$$
$$= \emptyset(\mu(x)).$$

Result 5: Let $\emptyset(\mu(x)) = \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}$ be involution intuitionistic fuzzy generator then there exists a function $'g'$ such that,

$$g^{-1}(\mu(x)) + g(1-\mu(x)) = \emptyset(0), \mu(x) \in [0,1].$$

Proof: Let,

$$\emptyset(\mu(x)) = g(1-\mu(x))$$
$$\Rightarrow \frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)} = g(1-\mu(x))$$
$$\Rightarrow \emptyset\left(\frac{1-\mu(x)}{1-\gamma(1-e^{\lambda})\mu(x)}\right) = \emptyset(g(1-\mu(x)))$$
$$\Rightarrow \mu(x) = g(1-g(1-\mu(x))),$$

as $'\emptyset'$ is involutive,

$$\Rightarrow g^{-1}(\mu(x)) + g(1-\mu(x)) = 1 \quad (6)$$

as $\emptyset(\mu(x)) = g(1-\mu(x))$, put $\mu(x) = 0$, we have $g(1) = \emptyset(0) = 1$, then from Eq. 6, we have
$\Rightarrow g^{-1}(\mu(x)) + g(1-\mu(x)) = \emptyset(0)$.

5. Applications of the proposed intuitionistic generator in medical diagnosis: Mathematical approach

With the increment in the variation and impreciseness of the information available to the physician, with the new generation of medical technologies, is quite a tedious job to classify the various sets of symptoms under a single platform. For a radio logistic in image processing is also a very tedious job to classify the symptoms correlating the disease. A unit symptom may also be the cause of serving severe diseases and the existence of several diseases. So, a technique or generator with more than one parameter is needed, so that the variation in the symptoms may be diagnosed. The newly constructed intuitionistic fuzzy generator may be useful in such conditions, where we may have a suitable range for the membership value to generate the intuitionistic fuzzy set. While we are making a medical diagnosis, imperfect, imprecise, and inaccurate information arise.

So, we provide a suitable way to handle the medical diagnostic process to reduce these kinds of uncertainties. The constructed intuitionistic fuzzy generator has a number of properties that make it appropriate for a particular disease or symptom in the medical field. For example, in case of cancer diagnosis in a patient, the opinion of two experts may be different, according to the first expert, the symptom of cancer is less than 50 percent and the other expert says that symptom in the patient is more than 50 percent, such type of uncertainties can easily handle by a fuzzy expert system based on our generated intuitionistic fuzzy set. In this present work, we have developed a non-membership with the help of membership by using a generator, and we tried to cover the more uncertainties present in the membership value by giving them a wide range. So, we will get more beneficial results in medical diagnosis. Proposed generated intuitionistic fuzzy set based fuzzy expert systems may have recognized to be useful in the medical diagnosis for the evaluation of qualitative and quantitative disease and symptoms in the patient, and we may give a suitable algorithm for the diagnosis.

We can also use the generalized intuitionistic generator in the medical image enhancement technique, as Chaira (2012) gave. Let us consider an image which is initially fuzzified as

$$\mu(g) = \frac{g-g_{min}}{g_{max}-g_{min}},$$

where g is the gray value of the image and g_{min}, g_{max} are the minimum and maximum gray values of the image, respectively. Then we can define an intuitionistic fuzzy membership function from intuitionistic fuzzy complement which is given as;

$$\mu'(\mathrm{g}) = 1 - \frac{1-\mu(g)}{1-\gamma(1-e^{\lambda})\mu(g)} = \frac{\left(1-\gamma(1-e^{\lambda})\right)\mu(g)}{1-\gamma(1-e^{\lambda})\mu(g)}$$

and the non-membership value is calculated as;

$$\emptyset(\mu(g)) = \frac{1-\mu'(g)}{1-\gamma(1-e^{\lambda})\mu'(g)},$$

the wider range of membership functions will allow us to increase the dynamic range of the image. So, our proposed generalized intuitionistic fuzzy generator will useful in enhancement technique to reduce the fuzziness and increase the image contrast.

6. Conclusion

On behalf of the newly generated intuitionistic fuzzy generator, in this paper, we have carried out the study of the general way of constructing the generated intuitionistic fuzzy set with the determination of the same properties. This study has made transparency in the conditions, which we have applied to form the intuitionistic fuzzy generator and applications in forming the generated intuitionistic fuzzy set. The advanced analysis of the newly constructed fuzzy generator has led us, in a natural way, for developing the generated intuitionistic fuzzy set, and the method allows us to prove the basic properties and some results by using the generated intuitionistic fuzzy set. Lastly, we presented the mathematical approach for the applications of the newly constructed fuzzy generator and generated an intuitionistic fuzzy set in medical diagnosis.

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Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

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