

Effect of magnetic field on thermos: Viscoelastic cylinder subjected to a constant thermal shock



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ARTICLE INFO

Article history:

Received 10 May 2019

Received in revised form

5 August 2019

Accepted 12 November 2019

Keywords:

Thermal stress

Generalized magneto-thermo-viscoelastic

Solid cylinder

Thermal shock

Laplace transform technique

Numerical computations

ABSTRACT

In this paper, we will discuss the problem of distribution of thermal stresses and temperature in a generalized Magneto-Thermo-Viscoelastic Solid Cylinder of radius L . The surface of the cylinder is assumed to be free traction and subjected to a constant thermal shock. The Laplace transform technique is used to solve the problem. A solution to the problem in the physical domain is obtained by using a numerical method of MATLAB Programmer and the expression for the temperature, strain, and stress are obtained. Numerical computations are carried out for a particular material for illustrating the results. Finally, the results obtained are presented graphically to show the effect of time on the field variables. And to show a comparison between Lord-Shulman and Coupled theory.

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1. Introduction

Interactions between strain and electromagnetic fields are largely being undertaken due to its various applications in many branches of science and technology. Development of magneto elasticity also induces us to study various problems of geophysics, seismology, and related topics. Without going into the details of the research work published so far in the fields of magneto elasticity, magneto-thermo-elasticity, magneto-thermo viscoelasticity we mention some recent papers. Acharya and Sengupta (1978) studied the magneto-thermoelastic surface waves in initially stressed conducting media. Chaudhary et al. (2004) investigated the reflection/Transmission of plane SH wave through a self-reinforced elastic layer between two half-spaces. Plane SH-wave response from elastic slab interposed between two different self-reinforced elastic solids studied by Chaudhary et al. (2006). Discussed the surface waves in magneto-elastic initially stressed conducting media been illustrated by De and Sengupta (1972). Othman and Song (2006) studied the effect of rotation on the reflection of magneto-thermoelastic waves under thermoelasticity without

energy dissipation. Magneto-thermo elastic wave propagation at the interface between two micropolar viscoelastic media discussed by Song et al. (2006). Tianhu et al. (2004) studied a two-dimensional generalized thermal shock problem for a half-space in electromagneto-thermoelasticity. Verma et al. (1988) investigated the magneto-elastic transverse surface waves in self-reinforced elastic solids. Effect of the rotation on an infinite generalized magneto-thermoelastic diffusion body with a spherical cavity studied by Abd-Alla et al. (2011). Bayones (2012) studied the influence of diffusion on the generalized magneto-thermo-viscoelastic problem of homogenous isotropic material. Viscoelastic materials are those for which the relationship between stress and strain depends on time. All materials exhibit some viscoelastic response. In common metals such as steel, aluminum, copper etc. Abd-Alla and Abo-Dahab (2009) investigated the time-harmonic sources in a generalized magneto-thermo-viscoelastic continuum with and without energy dissipation. Roychoudhuri and Banerjee (1998) investigated the magneto-thermoelastic interactions in an infinite viscoelastic cylinder of temperature rate dependent material subjected to a periodic loading. Spherically symmetric thermo-viscoelastic waves in a viscoelastic medium with a spherical cavity discussed by Banerjee and Roychoudhuri (1995). The problem of magneto-thermo-viscoelastic interactions in an unbounded body with a spherical cavity subjected to periodic loading is discussed by Abd-Alla et al. (2004).

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<https://doi.org/10.21833/ijaas.2020.01.012>

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Linear viscoelasticity has been an important area of research since the period of Maxwell, Boltzmann, Voigt, and Kelvin. Valuable information regarding linear viscoelasticity theory may be obtained from Gross (1968). Much research like Biot (1955), Gurtin and Sternberg (1962), Iliushin and Pobedria (1970) and Tanner (1988) have contributed notably to Thermo viscoelasticity.

The Kelvin-Voigt model is one of the macroscopic mechanical models often used to describe the viscoelastic behavior of the material. This model represents the delayed elastic response subjected to stress when the deformation is time dependent but recoverable. The dynamic interaction of thermal and mechanical fields in solids has great practical applications in modern aeronautics nuclear reactors, and high energy particle accelerators, for example.

Biot (1956) formulated the coupled thermo-elasticity theory to eliminate the paradox inherent in the classical uncoupled theory that elastic deformation has no effect on the temperature. The field equations for both the theories are of a mixed parabolic-hyperbolic type, which predicts infinite speeds for thermo-elastic signals, contrary to physical observations.

Hetnarski and Ignaczak (1999) examine five generalizations to the coupled theory of thermo-elasticity. The first generalization is due to Lord and Shulman who formulated the generalized thermo-elasticity theory involving one thermal relaxation time. This theory is referred to as L-S theory or extended thermo-elasticity in which the Maxwell Cattaneo law replaces the Fourier law of heat conduction by introducing a single parameter that acts as relaxation time.

The second generalization to the coupled thermo-elasticity theory is due to Green and Lindsay (1972), called G-L theory or the temperature-rate dependent theory, which involves two relaxation times.

The third generalization to the coupled thermo-elasticity theory is known as low-temperature thermo-elasticity introduced by Hetnarski and Ignaczak (1996), called H-I theory. This model is characterized by a system of non-linear field equations. Low-temperature non-linear models of heat conduction that predict wave-like thermal signals and which are supposed and studied in some works by Kosiński (1989).

The fourth generalization to the coupled theory is concerned with the thermo-elasticity theory without energy dissipation introduced by Green and Naghdi (1991), referred to as G-N theory of type II in which the classical Fourier law is replaced by heat flux rate-temperature gradient relation. The heat transport equation does not involve a temperature-rate term and as such this model admits undamped thermo-elastic wave in thermo-elastic material. In the context of the linearized version of this theory, a theorem on the uniqueness of the solution has been established by Chandrasekharaiah (1986).

The fifth generalization to the thermo-elasticity theory is known as the dual phase lag thermo-

elasticity developed by Tzou (1995) and Chandrasekharaiah (1998). Tzou (1995) considered micro-structural effects into the delayed response in time in the macroscopic formulation by taking into account that the increase of the lattice temperature is delayed to phonon-electron interactions on the macroscopic level. A macroscopic lagging (or delayed) response between the temperature gradient and the heat flux vector seems to be a possible outcome due to such progressive interactions.

Tzou (1995) introduced two-phase lags to both the heat flux vector and the temperature gradient and considered a constitutive equation to describe the lagging behavior in the heat conduction in solids. Here the classical Fourier law is replaced by an approximation to a modification of the law with two different translations for the heat flux vector and the temperature gradient.

Mukhopadhyay (2000) explored the effect of thermal relaxation time parameters on Thermo viscoelastic interactions in an infinite body with a spherical cavity subjected to periodic loadings on the boundary of the cavity. Ezzat et al. (2001) applied the state-space formulation to generalized Thermo viscoelasticity with thermal relaxation.

The theory of electro-magneto-thermo-viscoelasticity has aroused much interest in many industrial applications, particularly in a nuclear device. Where there exists a primary magnetic field. Various investigations have been carried out by considering the interaction between magnetic, thermal and strain fields. Analyses of such problems also influence various applications in biomedical engineering as well as in different geometric studies. Ezzat et al. (2009) have studied two temperature theories in generalized magneto-thermo-viscoelasticity.

Many Applications of state space approach developed for different type of problems in thermo-elasticity (Youssef and El-Bary, 2014; 2018; Youssef et al., 2014; 2017; Ezzat and El-Bary, 2009; 2012, 2014; 2015a; 2015b; 2016a; 2016b; 2017a; 2017b; 2017c; 2017d; 2018; Ezzat et al., 2010; 2015a; 2015b; 2017; Ismail et al., 2017; Khamis et al., 2017; El-Karamany et al., 2018) some of them consider one temperature and other discussed two temperature.

2. Basic governing equations

The governing equations in the context of the theory of generalized thermo viscoelasticity with magnetic field for the isotropic and homogeneous elastic medium are considered as:

- The equation of motion:

$$\sigma_{ij,i} + F_i = \rho \ddot{u}_i \quad (1)$$

where,

$$F = \mu_o J \times H$$

• Heat conduction equation:

$$K\theta_{,ii} = \left(\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right) \left(\rho C_E \theta + \gamma_e T_o \left(1 + \gamma_o \frac{\partial}{\partial t}\right) e\right) \quad (2)$$

• Constitutive relations:

$$\sigma_{ij} = 2 \left(\mu_e + \mu_v \frac{\partial}{\partial t}\right) e_{ij} + \left(\lambda_e + \mu_v \frac{\partial}{\partial t}\right) e \delta_{ij} - \gamma \theta \delta_{ij} \quad (3)$$

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}). \quad (4)$$

We take the linearized Maxwell's equations governing the electromagnetic field for a perfectly conducting medium as:

$$\text{curl } h = J + \epsilon_o \frac{\partial E}{\partial t}, \quad (5)$$

$$\text{curl } E = -\mu_o \frac{\partial H}{\partial t}, \quad (6)$$

$$E = -\mu_o \left(\frac{\partial u}{\partial t} \times H\right), \quad (7)$$

$$\text{div } h = 0. \quad (8)$$

We derive the analytical formulation of the problem in cylindrical coordinates (r, θ, z) with the z -axis coinciding with the axis of the cylinder. We consider the strains axis symmetrical about the z -axis. We have only the radial displacement $u_r = u(r, t)$, the circumferential displacement $u_\theta = 0$ and the longitudinal displacement $u_z = 0$.

we consider a perfectly conducting isotropic homogeneous generalized thermos viscoelastic cylinder subjected to a constant magnetic field $H(0, H_o, 0)$ which produce an induced magnetic field $h(0, h_o, 0)$ and induced electric field $E(0, 0, E)$. We assume one dimensional motion for which all the field quantities are functions of r and t . The displacement components take the form,

$$u_r = u(r, t), u_\theta = u_z = 0. \quad (9)$$

The strain components become,

$$e = \frac{\partial u}{\partial r} + \frac{u}{r} \quad (10)$$

The components of magnetic field vectors are:

$$H_r = 0, H_\theta = H_o, H_z = 0. \quad (11)$$

The electric intensity vector E is parallel to the current density vector J . Hence components of E and J are given by

$$E_r = E_z = 0, E_\theta = E \quad (12)$$

$$J_r = J_\theta = 0, J_\theta = J. \quad (13)$$

Now, the Maxwell's Eqs. 6 to 9 provide the following results:

$$E = \mu_o H_o \frac{\partial u}{\partial t}, \quad (14)$$

$$h = -H_o \left(\frac{\partial u}{\partial r} + \frac{u}{r}\right), \quad (15)$$

$$J = -\frac{\partial h}{\partial r}. \quad (16)$$

Using Eqs. 11, 14, and 16 into the relation,

$$F = \mu_o J \times H$$

We obtain,

$$F_r = \mu_o H_o^2 \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2}\right). \quad (17)$$

From Eq. 3 we obtain the components of the stress tensor as:

$$\sigma_{rr} = \left(\lambda_e + 2\mu_e\right) + \left(\lambda_v + 2\mu_v\right) \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) + \left(\lambda_e + \lambda_v \frac{\partial}{\partial t}\right) \frac{u}{r} - \gamma \theta, \quad (18)$$

$$\sigma_{\phi\phi} = \left(\lambda_e + \lambda_v \frac{\partial}{\partial t}\right) \left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) + 2 \left(\mu_e + \mu_v \frac{\partial}{\partial t}\right) \frac{u}{r} - \gamma \theta, \quad (19)$$

$$\sigma_{zz} = \left(\lambda_e + \lambda_v \frac{\partial}{\partial t}\right) \left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) - \gamma \theta, \quad (20)$$

$$\sigma_{r\phi} = \sigma_{r\psi} = \sigma_{\psi\phi} = 0. \quad (21)$$

Also, we arrived at:

$$\rho \frac{\partial^2 u}{\partial t^2} = \left(\lambda_e + 2\mu_e\right) + \left(\lambda_v + 2\mu_v\right) \frac{\partial}{\partial t} + \mu_o H_o^2 \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2}\right) - \gamma \frac{\partial \theta}{\partial r} \quad (22)$$

Now, we shall use the following non dimensional variables,

$$\dot{r} = C_1 \eta r, \dot{u} = C_1 \eta u, \dot{t} = C_1^2 \eta t, \sigma'_{ij} = \frac{\sigma_{ij}}{\mu_e}, \theta' = \frac{\theta}{T_o}, h' = \frac{h}{H_o}, \dot{E} = \frac{E}{\mu_o H_o C_1}, \dot{J} = \frac{J}{\eta H_o C_1}.$$

Eqs. 14 to 16 and 18 to 22 take the following form (dropping the primes for convenience).

$$J = \frac{\partial e}{\partial r} \quad (23)$$

$$h = -\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) \quad (24)$$

$$E = \frac{\partial u}{\partial t} \quad (25)$$

$$\sigma_{rr} = \left(1 + a \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial r} + \left(b_1 + b_2 \frac{\partial}{\partial t}\right) \frac{u}{r} - \theta, \quad (26)$$

$$\sigma_{\phi\phi} = \left(1 + a \frac{\partial}{\partial t}\right) \frac{u}{r} + \left(b_1 + b_2 \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial r} - \theta, \quad (27)$$

$$\sigma_{zz} = \left(b_1 + b_2 \frac{\partial}{\partial t}\right) \left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) - \theta \quad (28)$$

$$\sigma_{r\phi} = \sigma_{r\psi} = \sigma_{\phi\psi}, \quad (29)$$

$$\frac{\partial^2 u}{\partial t^2} = \left(1 + R_H + a \frac{\partial}{\partial t}\right) \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2}\right) - \frac{\partial e}{\partial r} \quad (30)$$

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} = \left(1 + \tau_o \frac{\partial}{\partial t}\right) \frac{\partial \theta}{\partial t} + \epsilon \left(1 + \tau_o \frac{\partial^2}{\partial t^2}\right) \left(\frac{\partial u}{\partial r} + \frac{u}{r}\right), \quad (31)$$

where,

$$C_1^2 = \frac{\lambda_e + 2\mu_e}{\rho}, C_2^2 = \frac{\lambda_v + 2\mu_v}{\rho}, a = \frac{C_2^2}{C_1^2}, b_1 = \frac{\lambda_e}{\rho C_1^2}, b_2 = \frac{\lambda_v}{\rho C_2^2}, \epsilon = \frac{\gamma^2 \theta_o}{\rho^2 C_E C_1^2}, R_H = \frac{\mu_o H_o^2}{\lambda_e + 2\mu_e}.$$

3. Laplace transform domain

Taking the Laplace transform of Eqs. 23 to 31 by using homogeneous initial conditions, defined and denoted as:

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t) dt, \quad s > 0$$

We obtain,

$$\bar{J} = \frac{\partial \bar{e}}{\partial r} \quad (32)$$

$$\bar{h} = -\left(\frac{\partial \bar{u}}{\partial r} + \frac{\bar{u}}{r}\right) \quad (33)$$

$$\begin{aligned} \bar{E} &= s\bar{u} & (34) \\ \frac{d^3\theta}{dr^3} + \frac{1}{r} \frac{d^2\theta}{dr^2} - \frac{1}{r^2} \frac{d\bar{\theta}}{dr} &= C_{21}\bar{u} + C_{22} \frac{d\bar{\theta}}{dr} & (35) \\ \frac{d^2\bar{u}}{dr^2} + \frac{1}{r} \frac{d\bar{u}}{dr} - \frac{\bar{u}}{r^2} &= C_{11}\bar{u} + C_{12} \frac{d\bar{\theta}}{dr} & (36) \\ \sigma_{rr} &= (1 + as) \frac{d\bar{u}}{dr} + (b_1 + b_2s) \frac{\bar{u}}{r} - \theta & (37) \\ \sigma_{\phi\phi} &= (1 + as) \frac{\bar{u}}{r} + (b_1 + b_2s) \frac{d\bar{u}}{dr} - \theta & (38) \\ \sigma_{zz} &= (b_1 + b_2s) \left(\frac{d\bar{u}}{dr} + \frac{\bar{u}}{r} \right) - \theta & (39) \end{aligned}$$

where,

$$C_{11} = \frac{s^2}{1+as+R_H}, C_{12} = \frac{C_{11}}{s^2}, C_{21} = \varepsilon s(1 + \tau_0 s)C_{11}, C_{22} = s(1 + \tau_0 s)(1 + \varepsilon C_{12}).$$

4. Solution of the problem

Now to solve Eqs. 35 and 36 put $\phi = \frac{d\bar{\theta}}{dr}$ and $\bar{u} = \frac{dv}{dr}$ we get:

$$\nabla^4 v - (C_{11} + C_{22})\nabla^2 v + (C_{11}C_{22} - C_{21}C_{12})v = 0 \quad (40)$$

Eq. 40 can be written as:

$$(\nabla^2 - K_1^2)(\nabla^2 - K_2^2)v = 0 \quad (41)$$

where K_1^2 and K_2^2 are the roots of the characteristic equation,

$$K^4 - (C_{11} + C_{22})K^2 + (C_{11}C_{22} - C_{21}C_{12}) = 0 \quad (42)$$

The solution of Eq. 41 takes the form,

$$v = v_1 + v_2$$

where, v_1 satisfy the following equation,

$$(\nabla^2 - K_1^2)v_1 = 0 \quad (43)$$

the solution of Eq. 43 takes the form:

$$v_1 = A_1 I_0(K_1 r) + B_1 I_1(K_1 r) \quad (44)$$

where, $I_0(K_1 r)$ is the Modified Bessel function of the first kind of order zero; $I_1(K_1 r)$ is the Modified Bessel function of the second kind of order Zer; A_1, B_1 are parameters depending on s only to be determined from the boundary conditions. For boundedness, we take $B_1 = 0$ then,

$$v_1 = A_1 I_0(K_1 r) \quad (45)$$

by the same way we get:

$$v_2 = A_2 I_0(K_2 r) \quad (46)$$

then,

$$v = A_1 I_0(K_1 r) + A_2 I_0(K_2 r). \quad (47)$$

Finally, we get:

$$\bar{u} = A_1 K_1 I_1(K_1 r) + A_2 K_2 I_1(K_2 r). \quad (48)$$

In a similar manner we get:

$$\bar{\theta} = B_1 I_0(K_0 r) + B_2 I_0(K_2 r). \quad (49)$$

These complete the solution to the problem.

5. Numerical inversion of Laplace transforms

In order to invert the Laplace transforms in the above equations we shall use a numerical technique based on Fourier expansions of functions. Let $\bar{g}(s)$ be the Laplace transform of a given function $g(t)$. The inversion formula of Laplace transforms states that:

$$g(t) = \frac{1}{2\pi i} \int_{d-i\infty}^{d+i\infty} e^{st} \bar{g}(s) ds$$

where d is an arbitrary positive constant greater than all the real parts of the singularities of $\bar{g}(s)$. Takings $d = d + iy$, we get:

$$g(t) = \frac{e^{dt}}{2\pi} \int_{-\infty}^{\infty} e^{iky} \bar{g}(d + iy) dy$$

This integral can be approximated by,

$$g(t) = \frac{e^{dt}}{2\pi} \sum_{k=-\infty}^{\infty} e^{ikt\Delta y} \bar{g}(d + ik\Delta y) \Delta y,$$

taking $\Delta y = \frac{\pi}{t_1}$ we obtain:

$$g(t) = \frac{e^{dt}}{t_1} \left(\frac{1}{2} \bar{g}(d) + Re \left(\sum_{k=1}^{\infty} e^{ikt\pi/t_1} \bar{g}(d + ik\pi/t_1) \right) \right).$$

For numerical purposes, this is approximated by the function,

$$g_N(t) = \frac{e^{dt}}{t_1} \left(\frac{1}{2} \bar{g}(d) + Re \left(\sum_{k=1}^N e^{ikt\pi/t_1} \bar{g}(d + ik\pi/t_1) \right) \right), \quad (50)$$

where N is a sufficiently large integer chosen such that:

$$\frac{e^{dt}}{t_1} Re \left(e^{iN\pi t/t_1} \bar{g}(d + iN\pi/t_1) \right) < \eta,$$

where η is a reselected small positive number that corresponds to the degree of accuracy to be achieved Formula (49) is the numerical inversion formula valid for $0 \leq t \leq t_1$. In particular, we chooset = t_1 , getting:

$$g_N(t) = \frac{e^{dt}}{t_1} \left(\frac{1}{2} \bar{g}(d) + Re \left(\sum_{k=1}^N (-1)^k \bar{g}(d + ik\pi/t_1) \right) \right)$$

6. Numerical results and discussions

The copper material was chosen for purposes of numerical evaluations and constants of the problem were taken as follows:

$$K = 386 \text{ N/K } S, \alpha_r = 17.8(10)^{-5} K^{-1}, C_E = 383.1 \text{ m}^2/\text{K}, T_0 = 293\text{K}, \rho = 8954 \text{ kg/m}^3, \mu_e = 3.86(10)^{10} \text{ N/m}^2$$

$$\lambda_e = 7.76(10)^{10} \text{ N/m}^2, \tau_o = 0.001, \alpha_o = 6.8831(10)^{-13}, \alpha_1 = 6.8831(10)^{-13}, R = 1, \theta_1 = 1.$$

In order to study the effect of time and study the comparison between two theories on temperature, radial stress, displacement, and strain, we now present our results in the form of graphs (Figs. 1-8).

Fig. 1 is plotted to show the variation of temperature θ against r for a wide range of r , $1 \leq r \leq 3$ at small time $t=0.07$, for two theories (Coupled Theory) and (L-S Theory).

It is observed from this figure that the magnitude of the temperature is greater for (L-S Theory) than (Coupled Theory).

It can be noted that the speed of propagation of temperature is finite and coincide with the physical behavior of viscoelastic material.

Also, we can see from this figure that the boundary condition is satisfied.

Fig. 2 shows the variation of temperature θ against r for a wide range of r , $1 \leq r \leq 3$ for different values of time ($t=0.01, t=0.05$) and we have noticed that the time t has significant effects on temperature. The increasing of the value of t causes increasing of the value of temperature, and temperature θ vanishes more rapidly.

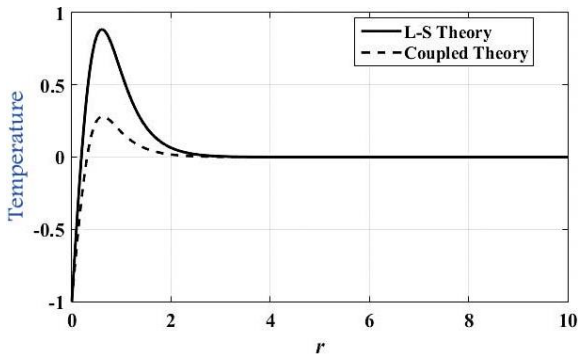


Fig. 1: Variation of temperature θ with distance r for two theories

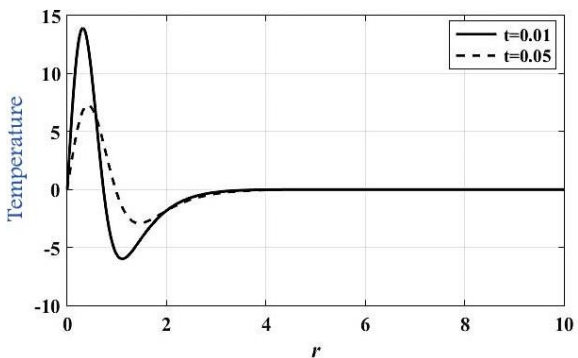


Fig. 2: Variation of temperature θ with distance r for different value of time t

Fig. 3 is plotted to show the variation of the radial stress σ_{rr} against r for a wide range of r , ($1 \leq r \leq 3$), at small time ($t=0.07$) for two theories (Coupled Theory) and (L-S Theory).

It is observed from this figure that the magnitude of the radial stress σ_{rr} is greater for (L-S Theory) than (Coupled Theory).

It can be noted that the speed of propagation of stress is finite and coincide with the physical behavior of viscoelastic material.

Fig. 4 shows the variation of the radial stress σ_{rr} against r for wide range of r , ($1 \leq r \leq 8$), for different values of time ($t=0.01, t=0.05$) and we have noticed that the time t has significant effects on stress, for wide range of ($r, 1 \leq r \leq 2.7$) the increasing of the value of t causes decreasing of the value of the radial stresses σ_{rr} and for wide range of $r, 2.7 \leq r \leq 8$ the increasing of the value of t causes increasing of the value of the radial stresses and radial stresses σ_{rr} vanishes more rapidly.

Fig. 5 is plotted to show the variation of strain e against r for a wide range of r , ($1 \leq r \leq 3$), at small time ($t=0.07$), for two theories (Coupled Theory) and (L-S Theory).

It is observed from this figure that the magnitude of the strain is greater for (L-S Theory) than (Coupled Theory). It can be noted that the speed of propagation of strain is finite and coincide with the physical behavior of viscoelastic material. Also, we can see from this figure that the boundary condition is satisfied.

Fig. 6 shows the variation of strain e against e for wide range of r , ($1 \leq r \leq 3$), at fractional parameter $\alpha=0.5$ and for different values of time ($t=0.1, t=0.5$) and we have noticed that the time t has significant effects on strain, for the wide range ($1 \leq r \leq 1.4$) the increasing of the value of t causes decreasing of the value of strain e , for the wide range ($1.4 \leq r \leq 3$) the increasing of the value of t causes increasing of the value of strain and strain e vanishes more rapidly.

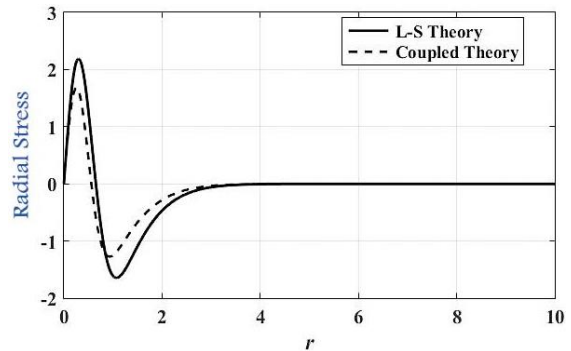


Fig. 3: Variation of radial stress σ_{rr} with distance r for two theories

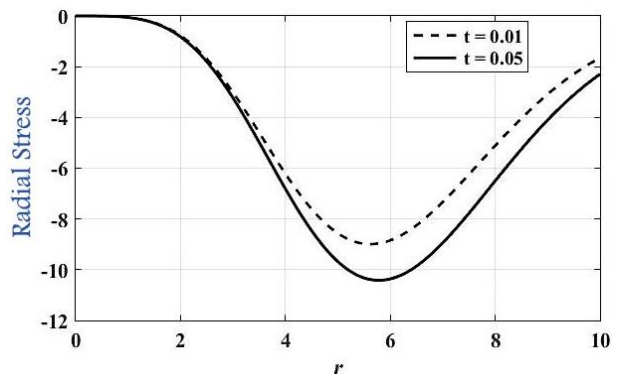


Fig. 4: Variation of radial stress σ_{rr} with distance r for different value of time t

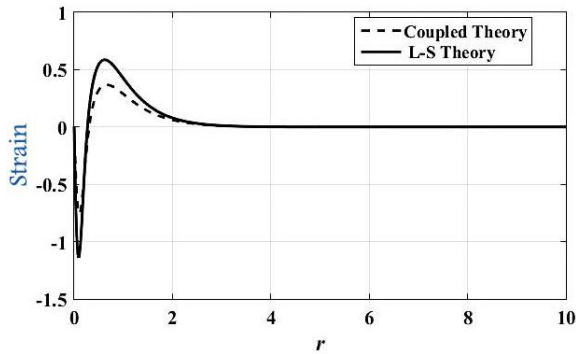


Fig. 5: Variation of strain e with distance r for two theories

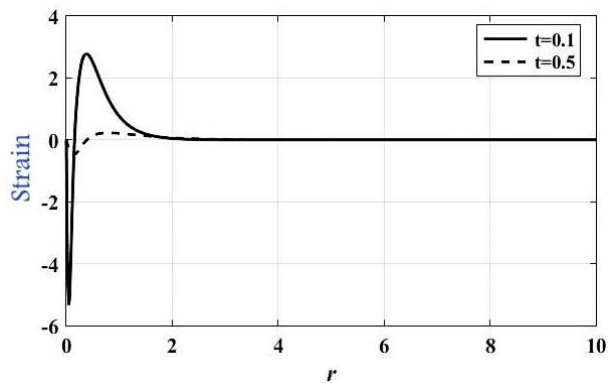


Fig. 6: Variation of strain e with distance r for different value of time t

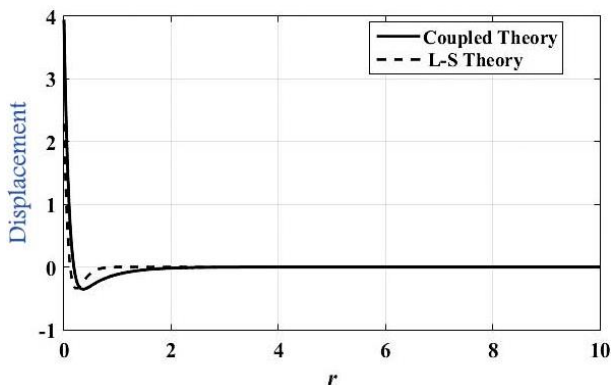


Fig. 7: Variation of displacement u with distance r for two theories

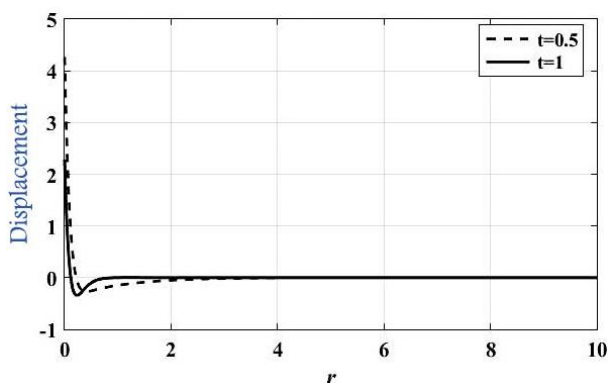


Fig. 8: Variation of displacement u with distance r for different value of time t

7. Conclusion

According to this work, many researchers in the field of generalized thermo-elasticity have applied

Lord-Shulman for thermo-elastic problem and very few of them can successfully be applied for magneto thermos Viscoelastic problem. In this paper, we deduce that the magnitude of all physical quantities is greater for (L-S Theory) than (Coupled Theory).

It can be noted that the speed of propagation of all physical quantities is finite and coincide with the physical behavior of Viscoelastic material.

List of symbols

λ_e, μ_e	Lame elastic constants
ρ	Density
C_E	Specific heat at constant strain
K	Thermal conductivity
α_t	Coefficient of linear thermal expansion
γ	$(3\lambda_e + 2\mu_e)\alpha_t$
λ_v, μ_v	Viscoelastic relaxation time
t	Time
q_i	Components of heat flux vector
σ_{ij}	Components of stress tensor
e_{ij}	Components of strain tensor
u_i	Components of displacement vector
T_o	Reference temperature
θ	Temperature increment
δ_{ij}	Kronicker delta
e	Cubical dilatation
L	Radius of the cylinder
μ_o	Magnetic permittivity
E	Electric displacement vector
J	Current density vector
H	Total magnetic intensity vector
h	Induced magnetic field vector
H_o	Initial uniform magnetic field
F_i	Components of Lorentz body force

Acknowledgment

The authors wish to acknowledge the approval and the support of this research study by Project NO. 7503-SCI-2017-1-8-F from the Deanship of Scientific Research in Northern Border University, Arar, KSA.

Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

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