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Lagrangian-Taylor differential transformation dynamics analysis of self-balancing inverted pendulum robot

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ABSTRACT

Robots are fast becoming a fixture in our lives. Kinematics and dynamics of self-balancing inverted pendulum robot modelled as an inverted are derived in this paper using Lagrange energy method. The derived equation of motion of the inverted pendulum robot was analyzed via Taylor differential transformation. Maple Computer software was used for the plotting of graphs for the result obtained. The results show that the position and motion of the inverted pendulum robot have a significant effect on achieving its self-balance.

1. Introduction

Robotics is an interdisciplinary branch of engineering and science that includes mechanical engineering, electrical engineering, computer science, and others. Robotics deals with the design, construction, operation, and use of robots, as well as computer systems for their control, sensory feedback, and information processing. These technologies are used to develop machines that can substitute for humans. Robots can be used in any situation and for any purpose, but today many are used in dangerous environments (including bomb detection and de-activation), manufacturing processes, or where humans cannot survive. Robots can take on any form but some are made to resemble humans in appearance. This is said to help in the acceptance of a robot in certain replicative behaviors usually performed by people. Such robots attempt to replicate walking, lifting, speech, cognition, and basically anything a human can do. Many of today's robots are inspired by nature, contributing to the field of bio-inspired robotics. For years now, robots have worked tirelessly in the shadows to increase or enhance the productivity of humans. The capabilities of robots have evolved well beyond the banality of those grainy industrial films. Today's industrial robots have incredible dexterity to match their brute strength, and can actually learn on the job. And then there's an entirely new breed of robots—some in humanoid form, and others that take highly practical forms all their own—that can walk, talk, save lives, and perform critical jobs in extreme environments, or simply take care of mundane tasks at home while we're out enjoying our lives (Yazdani et al., 2016; Chen and Wu, 1996).

The inverted pendulum is a classic automation problem that has numerous theoretical approaches as well as a multitude of practical applications (Hassan, 2008; Matesica et al., 2016). It can used to model the motion of parts or whole robot.

An inverted pendulum robot is a dynamical system. It is a classic automation problem that has numerous theoretical approaches as well as a multitude of practical applications. Its focal point of mass is over its turn point (Agarana and Agboola, 2015; Agarana and Iyase, 2015). While a typical pendulum is steady when hanging downwards, an inverted pendulum is inalienably shaky, and must be effectively adjusted so as to stay upright; this should be possible either by applying a torque at the turn point, by moving the rotate point on a level plane as a major aspect of an input framework, changing the rate of revolution of a mass mounted on the pendulum on a hub parallel to the turn hub and subsequently creating a net torque on the pendulum, or by swaying the rotate point vertically (Agarana and Iyase, 2015; Agarana and Bishop, 2015; Agarana and Emetere, 2016). The dynamical system is modelled as Lagrange's equation. It is a linear second-order non- homogenous partial differential equation. This equation was transformed to series using Taylor Differential Transformation method (TDTM). Application of Differential transformation

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and related methods has proved over the years to be very efficient in solving differential equations. They are often referred to as semi – analytical methods.

2. Problem formulations

2.1. Equation of motion

Using Lagrange's equations, which employ a single scalar function rather than vector components, to derive the equations modelling an inverted pendulum we take partial derivatives. In classical mechanics, the natural form of the Lagrangian is defined as (Chen and Wu, 1996):

$$L = E_k - E_p \quad (1)$$

where, $E_k$and E_p are kinetic energy and potential energy respectively. $E_p$is defined by its mass m, and the gravitational constant g:

$$E_p = mgh \quad (2)$$

the kinetic energy E_k of a point object is defined by its mass m and velocity v:

$$E_k = \frac{1}{2}mv^2. \quad (3)$$

Equation of motion can be directly derived by substitution using Euler Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \theta}\right) = \frac{\partial L}{\partial \theta} \quad (4)$$

where θ is the angle the pendulum makes with the upward vertical.

2.2. Inverted pendulum dynamic problem formulation

From the Fig. 1 we have:

$$x = l\, sin\theta \quad (5)$$
$$y = l\, cos\theta \quad (6)$$

this implies

$$\frac{dx}{dt} = l\, cos(\theta)\frac{d\theta}{dt} \quad (7)$$

and

$$\frac{dy}{dt} = -l\, sin(\theta)\frac{d\theta}{dt} \quad (8)$$

application of the Lagrangian gives:

$$L = \frac{1}{2}ml^2\left(\frac{d\theta}{dt}\right)^2 - mgl\, cos\theta \quad (9)$$

which implies

$$\frac{\partial L}{d\theta} = mgl\, sin\theta \quad (10)$$
$$\frac{\partial L}{\partial\left(\frac{d\theta}{dt}\right)} = ml^2\frac{d\theta}{dt} \quad (11)$$
$$\frac{d}{dt}\left(\frac{\partial L}{\partial\left(\frac{d\theta}{dt}\right)}\right) = ml^2\frac{d^2y}{dt^2} \quad (12)$$
$$ml^2\frac{d^2\vartheta}{dx^2}mgl\, sin\theta \quad (13)$$

which leads to

$$\frac{d^2\theta}{dt^2} = \frac{g}{l}\, sin\theta \quad (14)$$

with the oscillator, the above equation becomes:

$$\frac{d^2\theta}{dt^2} = \frac{1}{l}[g - A\omega^2 sin(\omega t)]sin\theta \quad (15)$$

where ωt is the phase the driving force term.

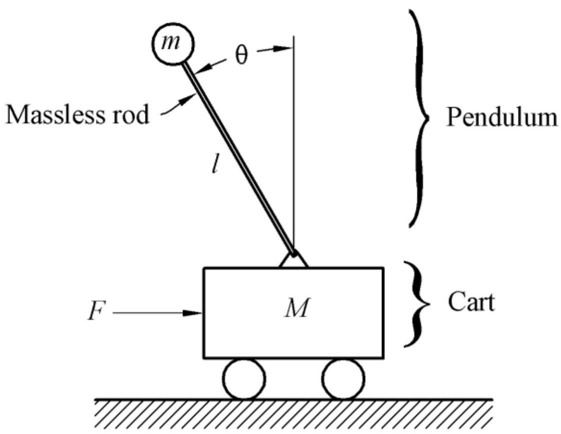

Fig. 1: Inverted pendulum robot on a cart

2.3. Taylor differential transformation

Given the k^{th} derivative of the function $\theta(t)$ with respect to time t as

$$\frac{\partial^k\theta(t)}{\partial t^k} = \varphi(t,k), \quad (16)$$

the differential transformation of the function $\theta(t)$ at $t = t_1$, is defined as (Agarana and Emetere, 2016):

$$\theta(t) = \varphi(t,k) = \left[\frac{\partial^k\theta(t)}{\partial t^k}\right]_{t=t_1}. \quad (17)$$

From the Taylor theorem, it is assumed that the function $\theta(t)$ can be expanded in the form of Taylor series as follows (Hassan, 2008):

$$\theta(t) = \sum\frac{(t-t_1)}{k!}\theta(k). \quad (18)$$

For generalization, following Chen and Wu (1996), if

$$\theta(k) = M(k)\left[\frac{\partial^k q(t)\theta(t)}{\partial t^k}\right]_{t=t_0} \quad (19)$$

then

$$\theta(t) = \frac{1}{q(t)}\sum_{k=0}^{\infty}\frac{(t-t_0)^k}{k!}\frac{\theta(k)}{M(k)} \quad (20)$$

where $M(k) \neq 0$ is called the proportional coefficient of the differential transformation, and $q(t) \neq 0$ is called the transformation kernel of the given function $\theta(t)$.

3. Analysis

Based on the aforementioned equations we can write:

$$\frac{d^2\theta}{dt^2} = \frac{1}{l}[g - A\omega^2 sin(\omega t)]sin\theta \quad (21)$$
$$l\frac{d^2\theta}{dt^2} - gsin\theta + A\omega^2 sin(\omega t)sin\theta = 0 \quad (22)$$

with the initial conditions

$$\theta_{t=0} = \theta_0 \quad (23)$$
$$\left[\frac{d\theta}{dt}\right]_{t=0} = \theta_0 = v_0 \quad (24)$$

where, l, g, t, A, ω, θ_0, v_0 denote length, acceleration due to gravity, time, initial displacement and velocity. Following Chen and Wu (1996) introducing the following dimensionless parameters and variables, C, K, X and r as defined.

$$C = \frac{c\theta_0}{mv_0}, K = \frac{k\theta_0^2}{mv_0^2}, X = \frac{\theta}{\theta_0}, r = \frac{v_0 t}{\theta_0} \quad (25)$$

with the following assumptions; $sin\theta = \theta, l = 1, B = A\omega^2, wt = \theta$ we have

$$\ddot{\theta} - g\theta + B\theta^2 = 0 \quad (26)$$
$$\theta_0 = 1 \quad (27)$$
$$\dot{\theta}_0 = 1 \quad (28)$$

$$\text{Let } \theta_0 = \theta 1 = \theta^2 = \theta.\theta \quad (29)$$

$$\text{and } \theta 2 = \theta^3 = \theta 1.\theta \quad (30)$$

but,

$$\theta 1(k) = \theta(k).\theta(k) = \sum_{l=0}^{k} \theta(l)\theta(k-l) \quad (31)$$
$$\theta 2(k) = \theta 1(k).\theta(k) = \sum_{l=0}^{k} \theta 1(l)\theta(k-l). \quad (32)$$

Taking the Taylor differential transformation with respect to dimensionless time r, the above equations becomes (Hassan, 2008):

$$\frac{(k+2)(k+1)}{H^2}\theta(k+2) - g\theta(k) + B\theta 2(k) = 0 \quad (33)$$
$$\theta(0) = \theta_0 = 1 \quad (34)$$
$$\theta(1) = H\dot{\theta}_0 = H. \quad (35)$$

For $k = 0$ and 1 the following equations are obtained

$$\theta 1(0) = \theta(0)\theta(0) = 1 \quad (36)$$
$$\theta 2(0) = \theta 1(0)\theta 1(0) = 1 \quad (37)$$
$$\theta 1(1) = \theta(0)\theta(1) + \theta(1)\theta(0) = H + H = 2H \quad (38)$$
$$\theta 2(1) = \theta 1(0)\theta(1) + \theta(1)\theta(0) = H + 2H = 3H \quad (39)$$

also, for $k = 1$,

$$\frac{2}{H^2}\theta(2) - g\theta(0) + B\theta 2(0) = 0 \quad (40)$$

$$\theta(2) = -\frac{H^2}{2}[g + B] = \frac{A_0 H^2}{2!} \quad (41)$$

where $A_0 = -(g + B)$

for $k = 2$,

$$\theta 1(2) = \theta(0)\theta(2) + \theta(1)\theta(1) + \theta(2)\theta(0) = (A_0 + 1)H^2 \quad (42)$$

$$\theta 2(2) = \theta 1(0)\theta(2) + \theta 1(1)\theta(1) + \theta 1(2)\theta(0) = \left(\frac{3}{2}A_0 + 3\right)H^2 \quad (43)$$

for $k = 3$,

$$\frac{6}{H^2}\theta(3) - g\theta(1) + B\theta^2(1) = 0 \quad (44)$$

By substitution,

$$\theta(3) = \frac{H^2}{6}[g\theta(1) - B\theta 2(1)] = \frac{H^2}{6}[g\theta(1) - B\theta 2(1)] = \frac{H^2}{6}[g(H) - B(3H)] = \frac{H^3}{6}(g - 3B) = \frac{J_0 H^3}{6} \quad (45)$$

where $J_0 = g - 3B$

for $k = 3$, the following can be obtained

$$\theta 1(3) = \theta(0)\theta(2) + \theta(2)\theta(1) + \theta(3)\theta(0) = \left(A_0 + \frac{J_0}{3}\right)H^3 \quad (46)$$
$$\theta 2(3) = \theta 1(0)\theta(32) + \theta 1(1)\theta(2) + \theta 1(2)\theta(1) + \theta 1(3)\theta(0) = \left(3A_0 + \frac{J_0}{3} + 1\right)H^3 \quad (47)$$

for $k = 2$,

$$\frac{12}{H^2}\theta(4) - g\theta(2) + B\theta 2(2) = 0 \quad (48)$$

By substitution again,

$$\theta(4) = \frac{H^2}{12}[g\theta(2) - B\theta 2(2)] = \frac{H^2}{12}\left[g\left(\frac{A_0 H^2}{2}\right) - B\left(\frac{3}{2}A_0 + 3\right)H^2\right] = \frac{H^4}{24}[gA_0 - 3BA_0 + 6B] \quad (49)$$
$$\theta(4) = \frac{y_0 H^4}{24} \quad (50)$$

where

$$y_0 = gA_0 - 3A_0 B + 6B \quad (51)$$

4. Results

The dimensionless displacement, velocity and acceleration, $\theta(t)$, $\dot{\theta}(t)$, $\ddot{\theta}(t)$, respectively, can be written, from the analysis as follows:

$$\theta(r) = \sum_{k=0}^{\infty} \theta(k)\left(\frac{r}{H}\right)^k = \theta(0) + \theta(1)\frac{r}{H} + \theta(2)\left(\frac{r}{H}\right)^2 + \theta(3)\left(\frac{r}{H}\right)^3 + \theta(4)\left(\frac{r}{H}\right)^4 + \cdots + \theta(n)\left(\frac{r}{H}\right)^n \dot{\theta}(r) = \frac{d\theta}{dr} = 1 + A_0 r + \frac{J_0}{2!}r^2 + \frac{J_0}{3!}r^3 + \cdots + \frac{\theta_0^n}{(n-1)!}r^{n-1}$$
$$\ddot{\theta}(r) = \frac{d^2\theta}{dr^2}A_0 + J_0 r + \frac{J_0}{2!}r^2 + \cdots + + \frac{\theta_0^n}{(n-2)!}r^{n-1} = 1 + H\frac{r}{H} + \frac{A_0 H^2}{2!}\left(\frac{r}{H}\right)^2 + \frac{J_0 H^3}{3!}\left(\frac{r}{H}\right)^3 + \frac{J_0 H^4}{4!}\left(\frac{r}{H}\right)^4 + \cdots + \frac{\theta_0^{(n)} H^n}{n!}\left(\frac{r}{H}\right)^n = 1 + r + \frac{A_0}{2!}r^2 + \frac{J_0}{3!}r^3 + \frac{J_0}{4!}r^4 + \cdots + \frac{\theta_0^{(n)}}{n!}r^n$$

where; $\theta_0^{(n)} = \left[\frac{d^n\theta}{dr^n}\right]_{r\neq 0}$ denotes the nth derivative of the dimensionless displacement $\theta(r)$, with respect to dimensionless time r at $r \neq 0$.

5. Conclusion

This study set out to analyze the dynamics of inverted pendulum robot. The equation of motion of the system was derived using the Lagrange energy method. The second order ordinary differential equation of motion was transformed to its algebraic form using the Taylor differential transformation technique. From the analysis and evaluation results obtained are represented graphically in Figs. 2-4, using computer software – Maple. The displacement, velocity and acceleration of the inverted pendulum robot are functions of time r. At a particular time r, when the system is perturbed, its measure of displacement is higher than its velocity, while the velocity is higher than the acceleration. The Taylor differential transformation technique has proven to be a good and easy method of analyzing the dynamic behaviour of inverted pendulum system.

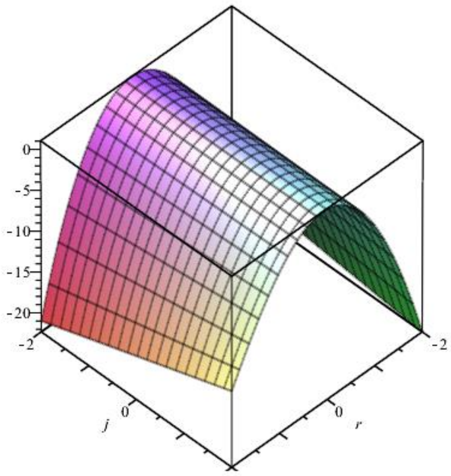

Fig. 2: Displacement of inverted pendulum robot at various time

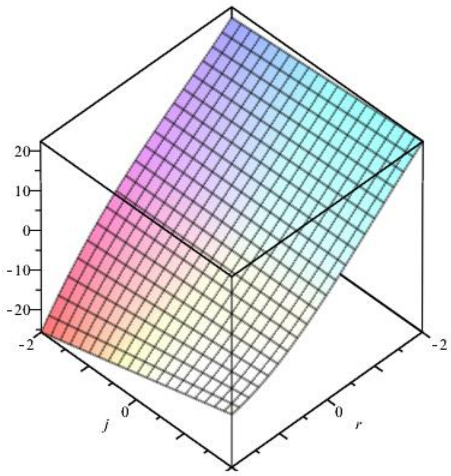

Fig. 3: Velocity of inverted pendulum robot at various time

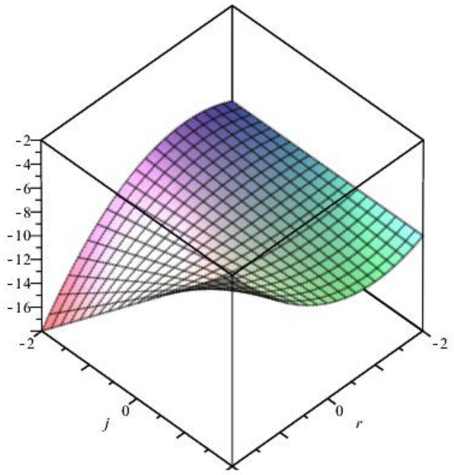

Fig. 4: Acceleration of inverted pendulum robot at various times

Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

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