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Applications of He's semi-inverse variational method and ITEM to the nonlinear long-short wave interaction system

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ABSTRACT

This work deals with exact soliton solutions of the nonlinear long-short wave interaction system, utilizing two analytical methods. The system of coupled long-short wave interaction equations is studied by two analytical methods, namely, the generalized tan $(\phi/2)$-expansion method and He's semi-inverse variational method, based upon the integration tools. Moreover, in this paper, we generalize two aforementioned methods which give new soliton wave solutions. Abundant exact traveling wave solutions including solitons, kink, periodic and rational solutions have been found. These solutions might play an important role in engineering and physics fields. By using these methods, exact solutions including the hyperbolic function solution, traveling wave solution, soliton solution, rational function solution, and periodic wave solution of this equation have been obtained. In addition, by using Matlab, some graphical simulations were done to see the behavior of these solutions.

1. Introduction

In this paper, we consider the nonlinear long-short wave interaction system (Bekir et al., 2013) as follows:

$$
\begin{aligned}
&iu_t + u_{xx} - uv = 0,\\
&v_t + v_x + (|u|^2)_x = 0.
\end{aligned}
\tag{1.1}
$$

The nonlinear long-short wave interaction systems with considering a general theory for interactions between short and long waves first introduced by Benney (1977). Describes of the nonlinear resonance interaction of multiple short waves with a long wave in two spatial dimension by considering a general multi-component (2 + 1)-dimensional long-wave-shortwave resonance interaction system with arbitrary nonlinearity coefficients have been investigated by Sakkaravarthi et al. (2014) by applying the Hirota (1985) bilinearization method. The entangled mapping approach based on the general reduction theory was investigated by Dai and Liu (2012), in which they have derived new type of variable separation solution for the (2 + 1)-dimensional long wave short wave interaction model. By utilizing the first integral method obtained one-soliton solutions and also by help aforesaid method is used to construct exact solutions of the nonlinear long-short wave resonance equations (Jafari et al., 2015). Apart from this, study on the long-short-wave interaction system by utilizing (G'/G)-expansion method was also carried out in Bekir et al. (2013). Triki et al. (2015) studied the long-wave short-wave interaction equation by help the simplest equation approach also obtained soliton solutions as well as other solutions such as singular periodic solutions and plane waves. Later on, the nonlinear long-short wave interaction system was studied by investigating the transverse linear instability of one-dimensional solitary wave solution (Erbay and Erbay, 2012). Dias et al. (2010), proof of the global existence and uniqueness of the solution of the Cauchy problem and also proof of the convergence of the whole sequence of solutions have been studied. Finally, by applying the new modified exp $(-\Omega\ (\xi))$-expansion method sets of solutions including, hyperbolic, complex, and dark soliton solutions have obtained in Baskonus et al. (2017).

It has been discovered that many models in mathematics and physics are described by nonlinear Partial differential equations. Indeed modeling

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physical problems using partial differential equations with the exact parameters is not always easy but also impossible in the real problems. For this purpose, one way is using integration methods for finding the exact solutions. One of the most recent approaches is using numerical methods including the multiresolution analysis (Seyedi et al., 2015), the multi-scale analysis (Seyedi et al., 2018), semi-analytical methods (Dehghan and Manafian, 2009; Dehghan et al., 2010; Rashidi et al., 2013) or analytical methods (Manafian, 2015; 2016; 2018; Foroutan et al., 2018; Sendi et al., 2019; Dehghan et al., 2011a; 2011b; Manafian and Lakestani, 2015a; 2015b; 2015c; Biswas, 2009; Bekir and Aksoy, 2012; Manafian and Lakestani, 2016a; 2016b; 2016c; Manafian et al., 2016a; 2016b; Aghdaei and Manafian, 2016). Also, of applied methods for solving nonlinear partial differential equation is He's semi-inverse variational principle, introduced by He (2006). For further information see references Kohl et al. (2009), Zhang (2007), Biswas et al. (2012a, 2012b), Sassaman et al. (2010). So instead of using current models of partial differential equations, we can transfer PDEs to ordinary differential equations. Hence there occurs a need to use solitary wave variable that would appropriately transforms PDEs to ODEs and solve them. In recent decade, exact solutions of nonlinear differential equations have been attracted attention from all over the world. Therefore, some newly published papers can be pointed to new exact solutions in new works in which given in Refs. (Cattani et al., 2018a; 2018b; Sulaiman et al., 2018; Baskonus et al., 2018b; Ciancio et al., 2018; Baskonus, 2016; 2017; Baskonus and Cattani, 2018).

In this paper, a novel and high accuracy method based on the classical Galerkin method proposed by Seyedi et al. (2018). They used Alpert Wavelet basis in the spectral methods and could solve the nano-fluid problems with high accuracy. Using the integration methods, we construct two analytical methods for Eq. 1.1, give corresponding algebraic equations, and show the efficiency of these schemes by the applied equation. Compared with some existed results, these methods are especially well designed for the solution of PDEs as particular the nonlinear long-short wave interaction system. The aim of this paper is to obtain analytical solutions of the aforementioned equation, and to determine the accuracy of these methods in solving this equation. The rest of the Paper is organized as follows: In Section 2, we present the He's semi-inverse variational principle method and the improved tan $(\phi/2)$-expansion method. In Section 3, we use transformations for converting the nonlinear longshort wave interaction system to an ODE form. In Section 4, by help of methods applied in section 2 we drive new soliton wave solutions for the nonlinear long-short wave interaction system. Moreover, in Section 5, we give the simulation and discussion of the solutions with depicting figures. Also conclusion is given in Section 6.

2. Methodology

2.1. The He's semi-inverse variational principle method

We describe the He's semi-inverse variational principle method for the given partial differential equation. First we give a description of this method, by noting the following steps:

Step 1: We suppose that given nonlinear partial differential equation for u(x, t) to be in the form

$$N(u, u_x, u_t, u_{xx}, u_{tt}, \dots) = 0, \tag{2.1}$$

which can be converted to an ODE

$$Q(u, ku', wu', k^2u'', w^2u'', \dots) \tag{2.2}$$

by the transformation $\xi = kx + wt$, as wave variable. Also, μ is constant to be determined later.

Step 2: According to He's semi-inverse method, we construct the following trial-functional

$$J(U) = \int L d\xi \tag{2.3}$$

where L is an unknown function of U and its derivatives.

Step 3: By the Ritz method, we can obtain different forms of solitary wave solutions, such as

$$U(\xi) = A sech(B\xi), \quad U(\xi) = A csch(B\xi), \quad U(\xi) = A tanh(B\xi), \quad U(\xi) = A coth(B\xi), \tag{2.4}$$

and so on. For example in this paper, we search a soliton solution in the form

$$U(\xi) = A sech(B\xi), \tag{2.5}$$
$$U(\xi) = A tanh(B\xi), \tag{2.6}$$

where A and B are constants to be further determined. Substituting Eqs. 2.5 or 2.6 into Eq. 2.3 and making J stationary with respect to A and B results in

$$\frac{\partial J}{\partial A} = 0, \tag{2.7}$$
$$\frac{\partial J}{\partial B} = 0. \tag{2.8}$$

Solving Eqs. (2.7) and (2.8), we obtain A and B. Hence the soliton solutions (2.5) or (2.6) are well determined.

2.2. Description of the ITEM

The ITEM is well-known analytical method which was improved and developed by Sendi et al. (2019).

Step 1: We suppose that given nonlinear partial differential equation for u(x, t) to be in the form

$$N(u, u_x, u_t, u_{xx}, u_{tt}, \dots) = 0, \tag{2.9}$$

which can be converted to an ODE

$$Q(u, ku', wu', k^2u'', w^2u'', \dots) \tag{2.10}$$

by the transformation $\xi = kx + wt$ is the wave variable. Also, μ is constant to be determined later.

Step 2: Suppose the traveling wave solution of Eq. 2.10 can be expressed as follows:

$$u(\xi) = S(\phi) = \sum_{k=-m}^{m} A_k [p + \tan(\phi/2)]^k \tag{2.11}$$

where $Ak(0 \le k \le m)$ and $A-k = Bk(1 \le k \le m)$ are constants to be determined, such that $Am \neq 0, Bm \neq 0$ and $\phi = \phi(\xi)$ satisfies the following ordinary differential equation:

$$\phi'(\xi) = a\sin(\phi(\xi)) + b\cos(\phi(\xi)) + c. \tag{2.12}$$

We will consider the following special solutions of Eq. 2.12:

Family 1: When $\Delta = a^2 + b^2 - c^2 < 0$ and $b - c \neq 0$, then $\phi(\xi) = 2\tan^{-1}\left[\frac{a}{b-c} - \frac{\sqrt{-\Delta}}{b-c}\tan\left(\frac{\sqrt{-\Delta}}{2}\bar{\xi}\right)\right]$.
Family 2: When $\Delta = a^2 + b^2 - c^2 > 0$ and $b - c \neq 0$, then $\phi(\xi) = 2\tan^{-1}\left[\frac{a}{b-c} + \frac{\sqrt{\Delta}}{b-c}\tanh\left(\frac{\sqrt{\Delta}}{2}\bar{\xi}\right)\right]$.
Family 3: When $\Delta = a^2 + b^2 - c^2 > 0$, $b \neq 0$ and $c = 0$, then $\phi(\xi) = 2\tan^{-1}\left[\frac{a}{b} + \frac{\sqrt{b^2+a^2}}{b}\tanh\left(\frac{\sqrt{b^2+a^2}}{2}\bar{\xi}\right)\right]$.
Family 4: When $\Delta = a^2 + b^2 - c^2 < 0$, $c \neq 0$ and $b = 0$, then $\phi(\xi) = 2\tan^{-1}\left[-\frac{a}{c} + \frac{\sqrt{c^2-a^2}}{c}\tan\left(\frac{\sqrt{c^2-a^2}}{2}\bar{\xi}\right)\right]$.
Family 5: When $\Delta = a^2 + b^2 - c^2 > 0$, $b - c \neq 0$ and $a = 0$, then $\phi(\xi) = 2\tan^{-1}\left[\sqrt{\frac{b+c}{b-c}}\tanh\left(\frac{\sqrt{b^2-c^2}}{2}\bar{\xi}\right)\right]$.
Family 6: When $a = 0$ and $c = 0$, then $\phi(\xi) = \tan^{-1}\left[\frac{e^{2b\bar{\xi}}-1}{e^{2b\bar{\xi}}+1}, \frac{2e^{b\bar{\xi}}}{e^{2b\bar{\xi}}+1}\right]$.
Family 7: When $b = 0$ and $c = 0$, then $\phi(\xi) = \tan^{-1}\left[\frac{2e^{a\bar{\xi}}}{e^{2a\bar{\xi}}+1}, \frac{e^{2a\bar{\xi}}-1}{e^{2a\bar{\xi}}+1}\right]$.
Family 8: When $a^2 + b^2 = c^2$, then $\phi(\xi) = 2\tan^{-1}\left[\frac{a\bar{\xi}+2}{(b-c)\bar{\xi}}\right]$.
Family 9: When $a = b = c = ka$, then $\phi(\xi) = 2\tan^{-1}\left[e^{ka\bar{\xi}} - 1\right]$.
Family 10: When $a = c = ka$ and $b = -ka$, then $\phi(\xi) = -2\tan^{-1}\left[\frac{e^{ka\bar{\xi}}}{-1+e^{ka\bar{\xi}}}\right]$.
Family 11: When $c = a$, then $\phi(\xi) = -2\tan^{-1}\left[\frac{(a+b)\,e^{b\bar{\xi}}-1}{(a-b)\,e^{b\bar{\xi}}-1}\right]$.
Family 12: When $a = c$, then $\phi(\xi) = 2\tan^{-1}\left[\frac{(b+c)\,e^{b\bar{\xi}}+1}{(b-c)\,e^{b\bar{\xi}}-1}\right]$.
Family 13: When $c = -a$, then $\phi(\xi) = 2\tan^{-1}\left[\frac{e^{b\bar{\xi}}+b-a}{e^{b\bar{\xi}}-b-a}\right]$.
Family 14: When $b = -c$, then $\phi(\xi) = 2\tan^{-1}\left[\frac{ae^{a\bar{\xi}}}{1-ce^{a\bar{\xi}}}\right]$.
Family 15: When $b = 0$, and $a = c$, then $\phi(\xi) = -2\tan^{-1}\left[\frac{c\bar{\xi}+2}{c\bar{\xi}}\right]$.
Family 16: When $a = 0$, and $b = c$, then $\phi(\xi) = 2\tan^{-1}\left[c\bar{\xi}\right]$.
Family 17: When $a = 0$, and $b = -c$, then $\phi(\xi) = -2\tan^{-1}\left[\frac{1}{c\bar{\xi}}\right]$.
Family 18: When $a = 0$, and $b = 0$, then $\phi(\xi) = c\xi + C$.
Family 19: When $b = c$, then $\phi(\xi) = 2\tan^{-1}\left[\frac{e^{a\bar{\xi}}-c}{a}\right]$.

where $\bar{\xi} = \xi + C$, $p, A0, Ak, Bk (k = 1, 2, \dots, m)$, a, b and c are constants to be determined later.

Step 3: Determine m. This, usually, can be accomplished by balancing the linear term(s) of highest order with the highest-order nonlinear term(s) in Eq. 2.10. But, the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in Eq. 2.10. If $m = q/p$ (where $m = q/p$ be a fraction in the lowest terms), we let

$$u(\xi) = v^{q/p}(\xi), \tag{2.13}$$

then substitute Eq. 2.13 into Eq. 2.10 and then determine the value of m in new Eq. 2.10. If m be a negative integer, we let

$$u(\xi) = v^m(\xi), \tag{2.14}$$

then substitute Eq. 2.14 into Eq. 2.10. Then we determine the new value of m in obtained equation.

Step 4: Substituting (2.11) into Eq. 2.10 with the value of m obtained in Step 2. Collecting the coefficients of $\tan(\phi/2)k$, $\cot(\phi/2)k (k = 0, 1, 2, \dots)$, then setting each coefficient to zero, we can get a set of over-determined equations for $A0, Ak, Bk (k = 1, 2, \dots, m)$ a, b, c and p with the aid of symbolic computation Maple.

Step 5: Solving the algebraic equations in Step 3, then substituting $A0, A1, B1, \dots, Am, Bm, \mu, p$ in (2.11).

3. The LSWI systems

In this paper, we consider the nonlinear long-short wave interaction systems (Baskonus et al., 2017; 2018a) in the form

$$\begin{aligned} &iu_t + u_{xx} - uv = 0, \\ &v_t + v_x + (|u|^2)_x = 0. \end{aligned} \tag{3.1}$$

Combine the real variables x and t by a compound variable ξ

$$\begin{aligned} &u(x,t) = \exp(i\eta)\,U(\xi), \qquad \eta = \alpha x + \beta t, \\ &v(x,t) = V(\xi), \qquad \xi = kx + wt, \end{aligned} \tag{3.2}$$

If we take the necessary derivations of Eq. 3.2 for Eq. 3.1, then we get the following nonlinear ODEs,

$$(w + 2\alpha k)iU' - (\alpha^2 + \beta)U + k^2U'' - UV = 0, \tag{3.3}$$
$$(k + w)V' + k(U^2)' = 0. \tag{3.4}$$

Consider the complex part of Eq. 3.3 to zero, will obtain

$$w = -2\alpha k. \tag{3.5}$$

By integrating Eq. 3.4 and considering Eq. 3.5, we get to

$$V = \frac{-k}{k+w}U^2 = \frac{-k}{k-2\alpha k}U^2 = \frac{-1}{1-2\alpha}U^2. \tag{3.6}$$

Now, when we substitute Eqs. 3.5 and 3.6 into Eq. 3.3, we obtain the NODE as

$$k^2U'' - (\alpha^2+\beta)U + \frac{1}{1-2\alpha}U^3 = 0, \tag{3.7}$$

4. Test problems

4.1. Applying section 2.1 for the LSWI systems

By He's semi-inverse principle (He, 2006; Kohl et al., 2009; Zhang, 2007), we can obtain the following variational formulation by using of the Eq. 3.7

$$J = \int_0^\infty \left[\frac{1}{2}k^2(U')^2 - \frac{1}{2}(\alpha^2+\beta)U^2 + \frac{1}{4(1-2\alpha)}U^4\right]d\xi. \tag{4.1}$$

By a Ritz-like method, we search a soliton solution in the form

$$U(\xi) = A\,\mathrm{sech}(B\xi), \tag{4.2}$$

where A and B are unknown constants to be further determined. Substituting Eq. 4.2 into Eq. 4.1, we have

$$J = \int_0^\infty \left[\frac{1}{2}k^2A^2B^2sech^2(B\xi)tanh^2(B\xi) - \frac{1}{2}(\alpha^2+\beta)A^2sech^2(B\xi) + \frac{1}{4(1-2\alpha)}A^4sech^4(B\xi)\right]d\xi. \tag{4.3}$$

Making J stationary with A and B yields

$$J = \frac{1}{6}k^2A^2B - \frac{1}{2B}(\alpha^2+\beta)A^2 + \frac{1}{6(1-2\alpha)}A^4.$$

$$\frac{\partial J(A,B)}{\partial A} = \frac{1}{3}k^2AB - \frac{1}{B}(\alpha^2+\beta)A + \frac{2}{3(1-2\alpha)}A^3 = 0, \tag{4.4}$$

$$\frac{\partial J(A,B)}{\partial B} = \frac{1}{6}k^2A^2 + \frac{1}{2B^2}(\alpha^2+\beta)A^2 - \frac{1}{6(1-2\alpha)B^2}A^4 = 0. \tag{4.5}$$

Solving Eqs. 4.4 and 4.5, we obtain

$$A = \pm\sqrt{2(1-2\alpha)(\alpha^2+\beta)}, \quad B = \pm\frac{1}{k}\sqrt{-(\beta+\alpha^2)}. \tag{4.6}$$

By utilizing the transformations (3.2) and (3.6), we will have

$$u(x,t) = \pm\sqrt{2(1-2\alpha)(\alpha^2+\beta)}\ \mathrm{sech}\left(\pm\frac{1}{k}\sqrt{-(\beta+\alpha^2)}(kx-2\alpha kt)\right)e^{i(\alpha x+\beta t)}, \tag{4.7}$$

$$v(x,t) = 2(\alpha^2+\beta)\ sech^2\left(\pm\frac{1}{k}\sqrt{-(\beta+\alpha^2)}(kx-2\alpha kt)\right). \tag{4.8}$$

Also, we search a soliton solution in the form

$$U(\xi) = A\tanh(B\xi), \tag{4.9}$$

where A and B are unknown constants to be further determined. Substituting Eq. 4.9 into Eq. 4.1, we have

$$J = \int_0^\infty \left[\frac{1}{2}k^2A^2B^2sech^4(B\xi) - \frac{1}{2}(\alpha^2+\beta)A^2tanh^2(B\xi) + \frac{1}{4(1-2\alpha)}A^4tanh^4(B\xi)\right]d\xi \tag{4.10}$$
$$= \frac{1}{3}k^2A^2B + \frac{1}{2B}(\alpha^2+\beta)A^2 - \frac{1}{3(1-2\alpha)B}A^4.$$

Making J stationary with A and B yields

$$\frac{\partial J(A,B)}{\partial A} = \frac{2}{3}k^2AB - \frac{1}{B}(\alpha^2+\beta)A - \frac{4}{3(1-2\alpha)B}A^3 = 0, \tag{4.11}$$

$$\frac{\partial J(A,B)}{\partial B} = \frac{1}{3}k^2A^2 - \frac{1}{2B^2}(\alpha^2+\beta)A^2 + \frac{1}{3(1-2\alpha)B^2}A^4 = 0. \tag{4.12}$$

Solving Eqs. 4.11 and 4.12, we obtain

$$A = \pm\sqrt{(1-2\alpha)(\alpha^2+\beta)}, \quad B = \pm\frac{1}{k}\sqrt{\frac{\beta+\alpha^2}{2}} \tag{4.13}$$

$$u(x,t) = \pm\sqrt{(1-2\alpha)(\alpha^2+\beta)}\ \tanh\left(\pm\frac{1}{k}\sqrt{\frac{\beta+\alpha^2}{2}}(kx-2\alpha kt)\right)e^{i(\alpha x+\beta t)}, \tag{4.14}$$

By utilizing the transformations (3.2) and (3.6), we will have

$$v(x,t) = (\alpha^2+\beta)\ tanh^2\left(\pm\frac{1}{k}\sqrt{\frac{\beta+\alpha^2}{2}}(kx-2\alpha kt)\right). \tag{4.15}$$

Likewise, we search another soliton solution in the form

$$U(\xi) = A\,\mathrm{csch}(B\xi), \tag{4.16}$$

where A and B are unknown constants to be further determined. Substituting Eq. 4.16 into Eq. 4.1, we have

$$J = \int_0^\infty \left[\frac{1}{2}k^2A^2B^2csch^2(B\xi)\ coth^2(B\xi) - \frac{1}{2}(\alpha^2+\beta)A^2csch^2(B\xi) + \frac{1}{4(1-2\alpha)}A^4csch^4(B\xi)\right]d\xi \tag{4.17}$$
$$= -\frac{1}{6}k^2A^2B + \frac{1}{2B}(\alpha^2+\beta)A^2 + \frac{1}{6(1-2\alpha)B}A^4.$$

Making J stationary with A and B yields

$$\frac{\partial J(A,B)}{\partial A} = -\frac{1}{3}k^2AB + \frac{1}{B}(\alpha^2+\beta)A + \frac{2}{3(1-2\alpha)B}A^3 = 0, \tag{4.18}$$

$$\frac{\partial J(A,B)}{\partial B} = -\frac{1}{6}k^2A^2 - \frac{1}{2B^2}(\alpha^2+\beta)A^2 - \frac{1}{6(1-2\alpha)B^2}A^4 = 0. \tag{4.19}$$

Solving Eqs. 4.18 and 4.19, we obtain

$$A = \pm\sqrt{2(1-2\alpha)(\alpha^2+\beta)}, \quad B = \pm\frac{1}{k}\sqrt{-(\beta+\alpha^2)}. \tag{4.20}$$

By using the transformations (3.2) and (3.6), we will have

$$u(x,t) = \pm\sqrt{-2(1-2\alpha)(\alpha^2+\beta)}\ csch\left(\pm\frac{1}{k}\sqrt{-(\beta+\alpha^2)}(kx-2\alpha kt)\right)e^{i(\alpha x+\beta t)}, \tag{4.21}$$

$$v(x,t) = -2(\alpha^2+\beta)\ csch^2\left(\pm\frac{1}{k}\sqrt{-(\beta+\alpha^2)}(kx-2\alpha kt)\right). \tag{4.22}$$

As last example, we search a soliton solution in the form

$$U(\xi) = A\coth(B\xi), \tag{4.23}$$

where A and B are unknown constants to be further determined. Substituting Eq. 4.23 into Eq. 4.1, we have

$$J = \int_0^\infty \left[\frac{1}{2}k^2A^2B^2 csch^4(B\xi) - \frac{1}{2}(\alpha^2+\beta)A^2 coth^2(B\xi) + \frac{1}{4(1-2\alpha)}A^4 coth^4(B\xi)\right] d\xi \tag{4.24}$$
$$= \frac{1}{3}k^2A^2B + \frac{1}{2B}(\alpha^2+\beta)A^2 + \frac{1}{3(1-2\alpha)B}A^4.$$

Making J stationary with A and B yields

$$\frac{\partial J(A,B)}{\partial A} = \frac{2}{3}k^2AB + \frac{1}{B}(\alpha^2+\beta)A - \frac{4}{3(1-2\alpha)B}A^3 = 0, \tag{4.25}$$
$$\frac{\partial J(A,B)}{\partial B} = \frac{1}{3}k^2A^2 - \frac{1}{2B^2}(\alpha^2+\beta)A^2 + \frac{1}{3(1-2\alpha)B^2}A^4 = 0. \tag{4.26}$$

Solving Eqs. 4.25 and 4.26, we obtain

$$A = \pm\sqrt{(1-2\alpha)(\alpha^2+\beta)}, \qquad B = \pm\frac{1}{k}\sqrt{\frac{\beta+\alpha^2}{2}}. \tag{4.27}$$

By utilizing the transformations (3.2) and (3.6), we will have

$$u(x,t) = \pm\sqrt{(1-2\alpha)(\alpha^2+\beta)}\ coth\left(\pm\frac{1}{k}\sqrt{\frac{\beta+\alpha^2}{2}}(kx-2\alpha kt)\right) e^{i(\alpha x+\beta t)}, \tag{4.28}$$
$$v(x,t) = (\alpha^2+\beta)\ coth^2\left(\pm\frac{1}{k}\sqrt{\frac{\beta+\alpha^2}{2}}(kx-2\alpha kt)\right). \tag{4.29}$$

4.2. Applying section 2.2 for the LSWI systems

By considering Eq. 3.7, and balancing the terms U'' and $U3$ by using homogenous principle, we get

$$m+2 = 3m, \Rightarrow \quad m = 1. \tag{4.30}$$

To get a closed form solution, we use the transformation as

$$U(\xi) = A_0 + A_1[p+\tan(\phi/2)] + B_1[p+\tan(\phi/2)]^{-1}. \tag{4.31}$$

By substituting (4.31) into Eq. 3.7 and collecting all terms with the same order of tan $(\Phi(\xi)/2)$ together, the left hand side of (4.31) are converted into polynomial in tan $(\Phi(\xi)/2)$. Setting each coefficient of each polynomial to zero, we derive a set of algebraic equations for a, b, c, μ, α, β, k, w, $A0$, $A1$, and $B1$. Solving the obtained algebraic equations, we have the following sets of coefficients for the solutions of (3.1) as given below:

Case 1:

$$p = -\frac{a}{b-c},\ b=b,\ c=c,\ \Delta = a^2+b^2-c^2,\ k=k,\ \Omega = (b-c)p^2+b+c,\ \alpha = \frac{1}{2}+\frac{{B_1}^2}{k^2\Omega^2}, \tag{4.32}$$
$$\beta = -\frac{1}{2}(b-c+1) - \frac{{B_1}^2}{k^2\Omega^2},\quad A_0 = 0,\quad A_1 = 0,\quad B_1 = B_1$$

By using of transformations of (3.1) and (4.32), we can obtain the following complex dark solutions for Eq. 3.1 as

Case 1.1: Family 1

$$u_1(x,t) = \frac{-B_1}{\sqrt{-\Delta}}\cot\left(\frac{\sqrt{-\Delta}}{2}\xi(x,t)\right) e^{i\left[\left(\frac{1}{2}+\frac{{B_1}^2}{k^2\Omega^2}\right)x-\left(\frac{1}{2}(b-c+1)+\frac{{B_1}^2}{k^2\Omega^2}\right)t\right]}, \tag{4.33}$$
$$v_1(x,t) = -\frac{k^2\Omega^2(b-c)^2}{2\Delta}\cot^2\left(\frac{\sqrt{-\Delta}}{2}\xi(x,t)\right),\quad \xi(x,t) = kx - k\left(1+\frac{2{B_1}^2}{k^2\Omega^2}\right)t.$$

Case 1.2: Family 2

$$u_2(x,t) = \frac{B_1(b-c)}{\sqrt{\Delta}}\coth\left(\frac{\sqrt{\Delta}}{2}\xi(x,t)\right) e^{i\left[\left(\frac{1}{2}+\frac{{B_1}^2}{k^2\Omega^2}\right)x-\left(\frac{1}{2}(b-c+1)+\frac{{B_1}^2}{k^2\Omega^2}\right)t\right]}, \tag{4.34}$$
$$v_2(x,t) = -\frac{k^2\Omega^2(b-c)^2}{2\Delta}\coth^2\left(\frac{\sqrt{\Delta}}{2}\xi(x,t)\right),\quad \xi(x,t) = kx - k\left(1+\frac{2{B_1}^2}{k^2\Omega^2}\right)t.$$

Case 1.3: Family 6

$$u_3(x,t) = B_1\cot\left(\frac{1}{2}\arctan\left[\frac{e^{2b\xi(x,t)}-1}{e^{2b\xi(x,t)}+1}, \frac{2e^{b\xi(x,t)}}{e^{2b\xi(x,t)}+1}\right]\right) e^{i\left[\left(\frac{1}{2}+\frac{{B_1}^2}{k^2b^2}\right)x-\left(\frac{1}{2}(b+1)+\frac{{B_1}^2}{k^2b^2}\right)t\right]} \tag{4.35}$$
$$v_3(x,t) = \frac{k^2b^2}{2}\cot^2\left(\frac{1}{2}\arctan\left[\frac{e^{2b\xi(x,t)}-1}{e^{2b\xi(x,t)}+1}, \frac{2e^{b\xi(x,t)}}{e^{2b\xi(x,t)}+1}\right]\right),\quad \xi(x,t) = kx - k\left(1+\frac{2{B_1}^2}{k^2b^2}\right)t.$$

Case 1.4: Family 11

$$u_4(x,t) = -B_1\frac{be^{b\xi(x,t)}-(1-p)}{(3p-1)be^{b\xi(x,t)}-(1-p)^2} e^{i\left[\left(\frac{1}{2}+\frac{{B_1}^2}{k^2\Omega^2}\right)x-\left(\frac{1}{2}(b-a+1)+\frac{{B_1}^2}{k^2\Omega^2}\right)t\right]}, \tag{4.36}$$
$$v_4(x,t) = \frac{k^2\Omega^2}{2}\left(\frac{be^{b\xi(x,t)}-(1-p)}{(3p-1)be^{b\xi(x,t)}-(1-p)^2}\right)^2,\quad \xi(x,t) = kx - k\left(1+\frac{2{B_1}^2}{k^2\Omega^2}\right)t.$$

Case 1.5: Family 16

$$u_5(x,t) = B_1\left(\frac{1}{p+c\xi(x,t)}\right) e^{i\left[\left(\frac{1}{2}+\frac{{B_1}^2}{4k^2c^2}\right)x-\left(\frac{1}{2}+\frac{{B_1}^2}{4k^2c^2}\right)t\right]}, \tag{4.37}$$
$$v_5(x,t) = 2k^2c^2\left(\frac{1}{p+c\xi(x,t)}\right)^2,\quad \xi(x,t) = kx - k\left(1+\frac{{B_1}^2}{2k^2c^2}\right)t.$$

Case 2:

$$p=p,\quad a=a,\quad b=c,\quad c=c,\quad \Delta=a^2,\quad k=k,\quad \alpha = \frac{1}{2}+\frac{{A_0}^2}{k^2a^2}, \tag{4.38}$$
$$\beta = -\frac{1}{2}(1+k^2a^2) - \frac{{A_0}^2}{k^2a^2},\quad A_0 = A_0,\quad A_1 = 0,\quad B_1 = -\frac{2A_0(ap-c)}{a}.$$

By using of transformations of (3.1) and (4.38), we can obtain the following complex dark solutions for Eq. 3.1 as

Case 2.1: Family 7

$$u_6(x,t) = \left[A_0 - \frac{2A_0(ap-c)}{a}\left\{p + tan\left(\frac{1}{2}arctan\left[\frac{2e^{a\xi(x,t)}}{e^{2a\xi(x,t)}+1}, \frac{e^{a\xi(x,t)}-1}{e^{2a\xi(x,t)}+1}\right]\right)\right\}^{-1}\right] e^{i\left[\left(\frac{1}{2}+\frac{{B_1}^2}{k^2a^2}\right)x-\left(\frac{1}{2}(1+k^2a^2)+\frac{{B_1}^2}{k^2a^2}\right)t\right]}, \tag{4.39}$$

$v_6(x,t) = \frac{k^2a^2}{2A_0{}^2}\left[A_0 - \frac{2A_0(ap-c)}{a}\left\{p + \tan\left(\frac{1}{2}arctan\left[\frac{2e^{a\xi(x,t)}}{e^{2a\xi(x,t)}+1}, \frac{e^{a\xi(x,t)}-1}{e^{2a\xi(x,t)}+1}\right]\right)\right\}^{-1}\right]^2,$

Case 2.2: Family 19

$$u_7(x,t) = \left[A_0 - \frac{2A_0(ap-c)}{a}\left\{p + \frac{e^{a\xi(x,t)}-c}{a}\right\}^{-1}\right] e^{i\left[\left(\frac{1}{2}+\frac{B_1{}^2}{k^2a^2}\right)x - \left(\frac{1}{2}(1+k^2a^2)+\frac{B_1{}^2}{k^2a^2}\right)t\right]}, \quad (4.40)$$

$$v_7(x,t) = \frac{k^2a^2}{2A_0{}^2}\left[A_0 - \frac{2A_0(ap-c)}{a}\left\{p + \frac{e^{a\xi(x,t)}-c}{a}\right\}^{-1}\right]^2, \quad \xi(x,t) = kx - k\left(1+\frac{2A_0{}^2}{k^2a^2}\right)t.$$

Case 3:

$$p = p,\quad a = a,\quad b = b,\quad c = c,\quad \Delta = a^2+b^2-c^2,\quad k = k,\quad \alpha = \alpha,\quad \beta = -\alpha + \frac{1}{2}k^2(c^2-a^2), \quad (4.41)$$

$A_0 = \sqrt{\frac{2\alpha-1}{2}}(a+pb-pc)k,\quad A_1 = 0,\quad B_1 = -\sqrt{\frac{2\alpha-1}{2}}(2ap+p^2(b-c)-b-c)k.$

By using of transformations of (3.1) and (4.41), we can obtain the following complex dark solutions for Eq. 3.1 as

Case 3.1: Family 1

$$u_8(x,t) = \left[\sqrt{\frac{2\alpha-1}{2}}(a+pb-pc)k - \sqrt{\frac{2\alpha-1}{2}}(2ap + p^2(b-c)-b-c)k \times \left\{p + \frac{a}{b-c} - \frac{\sqrt{-\Delta}}{b-c}\tan\left(\frac{\sqrt{-\Delta}}{2}\xi(x,t)\right)\right\}^{-1}\right] e^{i\left[\alpha x+\left(-\alpha+\frac{1}{2}k^2(c^2-a^2)\right)t\right]}, \quad (4.42)$$

$$v_8(x,t) = \frac{1}{2}\left[(a+pb-pc)k - (2ap+p^2(b-c)-b-c)k\left\{p + \frac{a}{b-c} - \frac{\sqrt{-\Delta}}{b-c}\tan\left(\frac{\sqrt{-\Delta}}{2}\xi(x,t)\right)\right\}^{-1}\right]^2,$$

where $\xi(x,t) = kx - 2k\alpha t$.

Case 3.2: Family 2

$$u_9(x,t) = \left[\sqrt{\frac{2\alpha-1}{2}}(a+pb-pc)k - \sqrt{\frac{2\alpha-1}{2}}(2ap + p^2(b-c)-b-c)k \times \left\{p + \frac{a}{b-c} + \frac{\sqrt{\Delta}}{b-c}\tanh\left(\frac{\sqrt{\Delta}}{2}\xi(x,t)\right)\right\}^{-1}\right] e^{i\left[\alpha x+\left(-\alpha+\frac{1}{2}k^2(c^2-a^2)\right)t\right]}, \quad (4.43)$$

$$v_9(x,t) = \frac{1}{2}\left[(a+pb-pc)k - (2ap+p^2(b-c)-b-c)k\left\{p + \frac{a}{b-c} + \frac{\sqrt{\Delta}}{b-c}\tanh\left(\frac{\sqrt{\Delta}}{2}\xi(x,t)\right)\right\}^{-1}\right]^2,$$

where $\xi(x,t) = kx - 2k\alpha t$.

Case 3.3: Family 6

$$u_{10}(x,t) = \sqrt{\frac{2\alpha-1}{2}}bk\left\{p - (p^2-1)\left\{p + \tan\left(\frac{1}{2}arctan\left[\frac{e^{2b\xi(x,t)}-1}{e^{2b\xi(x,t)}+1}, \frac{2e^{b\xi(x,t)}}{e^{2b\xi(x,t)}+1}\right]\right)\right\}^{-1}\right\} e^{i[\alpha x-\alpha t]}, \quad (4.44)$$

$$v_{10}(x,t) = \frac{1}{2}\left[pbk - (p^2-1)bk\left\{p + \tan\left(\frac{1}{2}arctan\left[\frac{e^{2b\xi(x,t)}-1}{e^{2b\xi(x,t)}+1}, \frac{2e^{b\xi(x,t)}}{e^{2b\xi(x,t)}+1}\right]\right)\right\}^{-1}\right]^2, \quad \xi(x,t) = kx - 2k\alpha t$$

Case 3.4: Family 11

$$u_{11}(x,t) = \sqrt{\frac{2\alpha-1}{2}}\left\{(a+pb-ap)k - (2ap+p^2(b-a)-b-a)k\left[p - \frac{(a+b)e^{b\xi(x,t)}-1}{(a-b)e^{b\xi(x,t)}-1}\right]^{-1}\right\} e^{i[\alpha x-\alpha t]}, \quad (4.45)$$

$$v_{11}(x,t) = \frac{1}{2}\left\{(a+pb-ap)k - (2ap+p^2(b-a)-b-a)k\left[p - \frac{(a+b)e^{b\xi(x,t)}-1}{(a-b)e^{b\xi(x,t)}-1}\right]^{-1}\right\}^2, \quad \xi(x,t) = kx - 2k\alpha t.$$

Case 3.5: Family 16

$$u_{12}(x,t) = \sqrt{\frac{2\alpha-1}{2}}\frac{2ck}{p+c\xi(x,t)} e^{i\left[\alpha x+\left(-\alpha+\frac{1}{2}k^2c^2\right)t\right]},\quad v_{12} = \frac{1}{2}\left\{\frac{2ck}{p+c\xi(x,t)}\right\}^2,\quad \xi(x,t) = kx - 2k\alpha t. \quad (4.46)$$

Case 4:

$$p = 0,\quad a = 0,\quad b = b,\quad c = c,\quad \Delta = a^2+b^2-c^2,\quad k = k,\quad \alpha = k^2(b^2-c^2)-\beta,\quad \beta = \beta, \quad (4.47)$$

$A_0 = 0,\quad A_1 = \sqrt{\frac{2k^2(b^2-c^2)-1-2\beta}{2}}(b-c)k,\quad B_1 = -\frac{1}{2}\frac{2k^2(b-c)(b+c)^2-(b+c)(2\beta+1)}{(b-c)A_1}.$

By using of transformations of (3.1) and (4.47), we can obtain the following complex dark solutions for Eq. 3.1 as

Case 4.1: Family 5

$$u_{13}(x,t) = \left[\sqrt{\frac{2k^2(b^2-c^2)-1-2\beta}{2}}k\sqrt{b^2-c^2}\tanh\left(\frac{\sqrt{b^2-c^2}}{2}\xi(x,t)\right) - \frac{1}{2}\frac{2k^2(b-c)(b+c)^2-(b+c)(2\beta+1)}{\sqrt{b^2-c^2}\,A_1} \times \coth\left(\frac{\sqrt{b^2-c^2}}{2}\xi(x,t)\right)\right] e^{i[((b^2-c^2)k^2-\beta)x+\beta t]}, \quad (4.48)$$

$$v_{13}(x,t) = \frac{1}{2k^2(b^2-c^2)-1-2\beta}\left[\sqrt{\frac{2k^2(b^2-c^2)-1-2\beta}{2}}k\sqrt{b^2-c^2}\tanh\left(\frac{\sqrt{b^2-c^2}}{2}\xi(x,t)\right) - \frac{1}{2}\frac{2k^2(b-c)(b+c)^2-(b+c)(2\beta+1)}{\sqrt{b^2-c^2}\,A_1}\coth\left(\frac{\sqrt{b^2-c^2}}{2}\xi(x,t)\right)\right]^2$$

where $\xi(x,t) = kx - 2k(-\beta + (b2-c2)k2)t$.

Case 4.2: Family 6

$$u_{14}(x,t) = \left[\sqrt{\frac{2k^2b^2-1-2\beta}{2}}kb\tan\left(\frac{1}{2}\arctan\left[\frac{e^{2b\xi(x,t)}-1}{e^{2b\xi(x,t)}+1}, \frac{2e^{b\xi(x,t)}}{e^{2b\xi(x,t)}+1}\right]\right) - \frac{1}{2}\frac{2k^2b^2-(2\beta+1)}{A_1} \times \cot\left(\frac{1}{2}\arctan\left[\frac{e^{2b\xi(x,t)}-1}{e^{2b\xi(x,t)}+1}, \frac{2e^{b\xi(x,t)}}{e^{2b\xi(x,t)}+1}\right]\right)\right] e^{i[((b^2-c^2)k^2-\beta)x+\beta t]}, \quad (4.49)$$

$v_{14}(x,t) = \frac{1}{2k^2b^2-1-2\beta},$

$$\left[\sqrt{\frac{2k^2b^2-1-2\beta}{2}}\,kb\tan\left(\frac{1}{2}\arctan\left[\frac{e^{2b\xi(x,t)}-1}{e^{2b\xi(x,t)}+1},\frac{2e^{b\xi(x,t)}}{e^{2b\xi(x,t)}+1}\right]\right)-\frac{1}{2}\frac{2k^2b^2-(2\beta+1)}{A_1}\times\cot\left(\frac{1}{2}\arctan\left[\frac{e^{2b\xi(x,t)}-1}{e^{2b\xi(x,t)}+1},\frac{2e^{b\xi(x,t)}}{e^{2b\xi(x,t)}+1}\right]\right)\right]^2$$

where $\xi(x, t) = kx - 2k(b2k2 - \beta)t$.

Case 5:

$p=0,\quad a=0,\quad b=b,\quad c=c,\quad \Delta=a^2+b^2-c^2,\quad k=k,\quad \alpha=2k^2(c^2-b^2)-\beta,\quad \beta=\beta,\quad (4.50)$

$A_0=0,\quad A_1=\sqrt{\frac{4k^2(c^2-b^2)-1-2\beta}{2}}\,(b-c)k,\quad B_1=-\frac{1}{2}\frac{4k^2(b-c)(b+c)^2+(b+c)(2\beta+1)}{(b-c)A_1}$

By using of transformations of (3.1) and (4.50), we can obtain the following complex dark solutions for Eq. 3.1 as

Case 5.1: Family 5

$$u_{13}(x,t)=\left[\sqrt{\frac{4k^2(c^2-b^2)-1-2\beta}{2}}\,k\sqrt{b^2-c^2}\tanh\left(\frac{\sqrt{b^2-c^2}}{2}\,\xi(x,t)\right)-\frac{1}{2}\frac{4k^2(b-c)(b+c)^2+(b+c)(2\beta+1)}{\sqrt{b^2-c^2}A_1}\times\coth\left(\frac{\sqrt{b^2-c^2}}{2}\,\xi(x,t)\right)\right]e^{i[(2(c^2-b^2)k^2-\beta)x+\beta t]},\qquad(4.51)$$

$$v_{13}(x,t)=\frac{1}{4k^2(c^2-b^2)-1-2\beta},\left[\sqrt{\frac{4k^2(c^2-b^2)-1-2\beta}{2}}\,k\sqrt{b^2-c^2}\tanh\left(\frac{\sqrt{b^2-c^2}}{2}\,\xi(x,t)\right)-\frac{1}{2}\frac{4k^2(b-c)(b+c)^2+(b+c)(2\beta+1)}{\sqrt{b^2-c^2}A_1}\times\coth\left(\frac{\sqrt{b^2-c^2}}{2}\,\xi(x,t)\right)\right]^2$$

where $\xi(x, t) = kx - 2k(-\beta + 2(c2 - b2)k2)t$.

Case 5.2: Family 6

$$u_{14}(x,t)=\left[\sqrt{\frac{-4k^2c^2-1-2\beta}{2}}\,kb\tan\left(\frac{1}{2}\arctan\left[\frac{e^{2b\xi(x,t)}-1}{e^{2b\xi(x,t)}+1},\frac{2e^{b\xi(x,t)}}{e^{2b\xi(x,t)}+1}\right]\right)-\frac{1}{2}\frac{-4k^2b^2+(2\beta+1)}{A_1}\times\cot\left(\frac{1}{2}\arctan\left[\frac{e^{2b\xi(x,t)}-1}{e^{2b\xi(x,t)}+1},\frac{2e^{b\xi(x,t)}}{e^{2b\xi(x,t)}+1}\right]\right)\right]e^{i[(2(c^2-b^2)k^2-\beta)x+\beta t]},\qquad(4.52)$$

$$v_{14}(x,t)=\frac{1}{-4k^2b^2-1-2\beta},\left[\sqrt{\frac{-4k^2b^2-1-2\beta}{2}}\,kb\,tan\left(\frac{1}{2}arctan\left[\frac{e^{2b\xi(x,t)}-1}{e^{2b\xi(x,t)}+1},\frac{2e^{b\xi(x,t)}}{e^{2b\xi(x,t)}+1}\right]\right)-\frac{1}{2}\frac{-4k^2b^2+(2\beta+1)}{A_1}\times cot\left(\frac{1}{2}arctan\left[\frac{e^{2b\xi(x,t)}-1}{e^{2b\xi(x,t)}+1},\frac{2e^{b\xi(x,t)}}{e^{2b\xi(x,t)}+1}\right]\right)\right]^2,$$

where $\xi(x, t) = kx + 2k(2b2k2 + \beta)t$.

Case 6:

$p=-\frac{a}{b-c},\quad b=b,\quad c=c,\quad \Delta=a^2+b^2-c^2,\quad k=k,\quad \Omega=(b-c)p^2+b+c,\quad \alpha=-\frac{1}{2}k^2\Omega(b-c)-\beta,\quad (4.53)$

$\beta=\beta,\quad A_0=0,\quad A_1=(b-c)k\sqrt{\frac{2\alpha-1}{2}},\quad B_1=0.$

By using of transformations of (3.1) and (4.53), we can obtain the following complex dark solutions for Eq. 3.1 as

Case 6.1: Family 1

$$u_{15}(x,t)=-k\sqrt{\frac{(1-2\alpha)\Delta}{2}}\tan\left(\frac{\sqrt{-\Delta}}{2}\xi(x,t)\right)\,e^{i\left[\left(-\frac{1}{2}k^2\Omega(b-c)-\beta\right)x+\beta t\right]},\qquad(4.54)$$

$v_{15}(x,t)=-\frac{k^2\Delta}{2}\,tan^2\left(\frac{\sqrt{-\Delta}}{2}\xi(x,t)\right),\quad \xi(x,t)=kx+2k\left(\frac{1}{2}\,k^2\Omega(b-c)+\beta\right)t.$

Case 6.2: Family 2

$$u_{16}(x,t)=k\sqrt{\frac{(2\alpha-1)\Delta}{2}}\tanh\left(\frac{\sqrt{\Delta}}{2}\xi(x,t)\right)\,e^{i\left[\left(-\frac{1}{2}k^2\Omega(b-c)-\beta\right)x+\beta t\right]},\qquad(4.55)$$

$v_{16}(x,t)=\frac{k^2\Delta}{2}\,tanh^2\left(\frac{\sqrt{\Delta}}{2}\xi(x,t)\right),\quad \xi(x,t)=kx+2k\left(\frac{1}{2}\,k^2\Omega(b-c)+\beta\right)t.$

Case 6.3: Family 6

$$u_{17}(x,t)=bk\sqrt{\frac{2\alpha-1}{2}}\,tan\left(\frac{1}{2}arctan\left[\frac{e^{2b\xi(x,t)}-1}{e^{2b\xi(x,t)}+1},\frac{2e^{b\xi(x,t)}}{e^{2b\xi(x,t)}+1}\right]\right)e^{i\left[\left(-\frac{1}{2}k^2b^2-\beta\right)x+\beta t\right]},\qquad(4.56)$$

$$v_{17}(x,t)=\frac{k^2b^2}{2}tan^2\left(\frac{1}{2}arctan\left[\frac{e^{2b\xi(x,t)}-1}{e^{2b\xi(x,t)}+1},\frac{2e^{b\xi(x,t)}}{e^{2b\xi(x,t)}+1}\right]\right),\quad \xi(x,t)=kx+2k\left(\frac{1}{2}\,k^2b^2+\beta\right)t.$$

Case 6.4: Family 11

$$u_{18}(x,t)=-(b-a)k\sqrt{\frac{2\alpha-1}{2}}\,\frac{(3p-1)be^{b\xi(x,t)}-(1-p)^2}{be^{b\xi(x,t)}-(1-p)}\,e^{i\left[\left(-\frac{1}{2}k^2\Omega(b-a)-\beta\right)x+\beta t\right]},\qquad(4.57)$$

$v_{18}(x,t)=\frac{k^2(b-a)^2}{2}\left(\frac{(3p-1)be^{b\xi(x,t)}-(1-p)^2}{be^{b\xi(x,t)}-(1-p)}\right)^2,\quad \xi(x,t)=kx+2k\left(\frac{1}{2}\,k^2\Omega(b-a)+\beta\right)t.$

Case 6.5: Family 17

$$u_{19}(x,t)=\sqrt{\frac{2\alpha-1}{2}}\left(\frac{2k}{\xi(x,t)}\right)\,e^{i[-\beta x+\beta t]},\quad v_{19}(x,t)=\frac{2k^2}{\xi^2(x,t)},\quad \xi(x,t)=kx-2k\beta t.\qquad(4.58)$$

Case 7:

$p=p,\quad a=a,\quad b=b,\quad c=c,\quad \Delta=a^2+b^2-c^2,\quad k=k,\quad \alpha=\frac{1}{2}+\frac{{A_1}^2}{k^2(b-c)^2},\quad \beta=-\alpha-\frac{1}{2}k^2\Delta,\quad (4.59)$

$A_0=-\frac{a+p(b-c)}{b-c}A_1,\quad A_1=A_1,\quad B_1=0.$

By using of transformations of (3.1) and (4.59), we can obtain the following complex dark solutions for Eq. 3.1 as

Case 7.1: Family 1

$$u_{20}(x,t)=-\frac{A_1\sqrt{-\Delta}}{b-c}\tan\left(\frac{\sqrt{-\Delta}}{2}\xi(x,t)\right)\,e^{i\left[\left(\frac{1}{2}+\frac{{A_1}^2}{k^2(b-c)^2}\right)x-\left(\frac{1}{2}+\frac{{A_1}^2}{k^2(b-c)^2}+\frac{1}{2}k^2\Delta\right)t\right]},\qquad(4.60)$$

$v_{20}(x,t)=\frac{-k^2\Delta}{2}\,tan^2\left(\frac{\sqrt{-\Delta}}{2}\xi(x,t)\right),\quad \xi(x,t)=kx-k\left(1+\frac{2{A_1}^2}{k^2(b-c)^2}\right)t.$

Case 7.2: Family 2

$$u_{21}(x,t)=\frac{A_1\sqrt{\Delta}}{b-c}\tanh\left(\frac{\sqrt{\Delta}}{2}\xi(x,t)\right)e^{i\left[\left(\frac{1}{2}+\frac{{A_1}^2}{k^2(b-c)^2}\right)x-\left(\frac{1}{2}+\frac{{A_1}^2}{k^2(b-c)^2}+\frac{1}{2}k^2\Delta\right)t\right]},$$

$$v_{21}(x,t)=\frac{k^2\Delta}{2}tanh^2\left(\frac{\sqrt{\Delta}}{2}\xi(x,t)\right),\quad \xi(x,t)=kx-k\left(1+\frac{2{A_1}^2}{k^2(b-c)^2}\right)t. \tag{4.61}$$

Case 7.3: Family 6

$$u_{22}(x,t)=A_1\tan\left(\frac{1}{2}arctan\left[\frac{e^{2b\xi(x,t)}-1}{e^{2b\xi(x,t)}+1},\frac{2e^{b\xi(x,t)}}{e^{2b\xi(x,t)}+1}\right]\right),\ e^{i\left[\left(\frac{1}{2}+\frac{{A_1}^2}{k^2b^2}\right)x-\left(\frac{1}{2}+\frac{{A_1}^2}{k^2b^2}+\frac{1}{2}k^2\mathrm{b}^2\right)t\right]} \tag{4.62}$$

$$v_{22}(x,t)=\frac{k^2}{2}\ tan^2\ \left(\frac{1}{2}arctan\left[\frac{e^{2b\xi(x,t)}-1}{e^{2b\xi(x,t)}+1},\frac{2e^{b\xi(x,t)}}{e^{2b\xi(x,t)}+1}\right]\right),\quad \xi(x,t)=kx-k\left(1+\frac{2{A_1}^2}{k^2b^2}\right)t.$$

Case 7.4: Family 12

$$u_{23}(x,t)=A_1\left\{-\frac{c}{b-c}+\frac{(b+c)e^{b(\xi+C)}+1}{(b-c)e^{b(\xi+C)}-1}\right\}e^{i\left[\left(\frac{1}{2}+\frac{{A_1}^2}{k^2(b-c)^2}\right)x-\left(\frac{1}{2}+\frac{{A_1}^2}{k^2(b-c)^2}+\frac{1}{2}k^2b^2\right)t\right]}, \tag{4.63}$$

$$v_{23}(x,t)=\frac{k^2(b-c)^2}{2}\left\{-\frac{c}{b-c}+\frac{(b+c)e^{b(\xi+C)}+1}{(b-c)e^{b(\xi+C)}-1}\right\}^2,\quad \xi)(x,t)=kx-k\left(1+\frac{2{A_1}^2}{k^2(b-c)^2}\right)t.$$

Case 7.5: Family 15

$$u_{24}(x,t)=-\frac{2A_1}{c\xi(x,t)}e^{i\left[\left(\frac{1}{2}+\frac{{A_1}^2}{k^2c^2}\right)x-\left(\frac{1}{2}+\frac{{A_1}^2}{k^2c^2}\right)t\right]},\quad v_{24}(x,t)=\frac{2}{\left(x-\left(1+\frac{2{A_1}^2}{k^2c^2}\right)t\right)^2} \tag{4.64}$$

5. Simulation and discussion of the solutions

In this section, the numerical simulations of the the nonlinear long-short wave interaction system will be given. Now, we will discuss all possible physical significance for each parameter. By utilizing the balance principle, one can found m = 1, therefore we can write other following equations:

$$U(\xi)=A_0+A_1[p+\tan(\phi/2)]+B_1[p+\tan(\phi/2)]^{-1}, \tag{5.1}$$

$$U'(\xi)=A_1sec^2(\phi/2)-B_1sec^2(\phi/2)[p+\tan(\phi/2)]^{-2}, \tag{5.2}$$

$$U''(\xi)=2A_1\tan(\phi/2)sec^2(\phi/2)-2B_1\tan(\phi/2)sec^2(\phi/2)\,[p+\tan(\phi/2)]^{-2}+2B_1sec^4(\phi/2)[p+\tan(\phi/2)]^{-2} \tag{5.3}$$

where $A1\neq 0$ and $B1\neq 0$. When we use Eqs. 5.1 to 5.3 in Eq. 3.7, we get a system of algebraic equations from the coefficients of polynomial of tan $(\phi/2)$. By solving this system of algebraic equations via Maple 13 software, we can find other different style analytical solutions which can be obtained by using ITEM. We have also obtained the dark, bright and singular soliton solutions of the nonlinear long-short wave interaction system (3.1) by using He's semi-inverse variational method and briefly studied their behavior dynamics. Moreover, by utilizing the ITEM, can found the exact particular solutions containing four types hyperbolic function solution (exact soliton wave solution), trigonometric function solution (exact periodic wave solution), rational exponential solution (exact singular kink-type wave solution) and rational solution (exact singular cupson wave solution). It can be said the ITEM has further merit comparing with other methods. This study will find analytical applications in nonlinear sciences, particularly in the literature we refer to the circular functions, the gravitational potential of a cylinder (Weisstein, 2002), the profile of a laminar jet (Weisstein, 2002), the Langevin function for magnetic polarization (Weisstein, 2002), the longitudinal waves such as in sound, pressure waves and musical instruments waves. In Figs. 1- 12, we plot two and three dimensional graphics of absolute values of (4.33), (4.34), (4.36), (4.37), (4.54) and (4.55) by means of Section 4.2, which denote the dynamics of solutions with appropriate parametric selections. Likewise, after comparing these analytical solutions obtained via He's semi-inverse variational method and ITEM with solutions obtained by authors of (Bekir et al., 2013; Baskonus et al., 2017; Khater et al., 2010), and to the best of our current state of knowledge, we think that complex hyperbolic function, trigonometric function and rational function solutions may have been obtained here for the first time, in the literature.

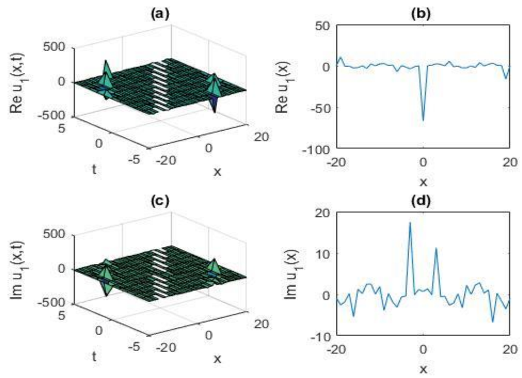

Fig. 1: Graphs of (4.33) ((a) and (b)) real values and ((c) and (d)) imaginary values by considering the values $a=b=B_1=p=k=2, c=3, -20<x<20, -5<t<5$ and $t=0.01$ for 2D surfaces

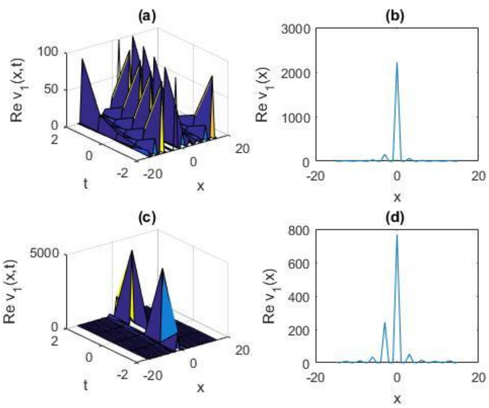

Fig. 2: Graphs of (4.33) real values by considering the values $a=b=B_1=p=k=2, c=3, -20<x<20, -5<t<5$ for ((a) and (b)), values $a=b=p=k=2,\ B_1=0.2,\ c=3, -20<x<20, -5<t<5$ ((a) and (b)) and $t=0.01$ for 2D surfaces

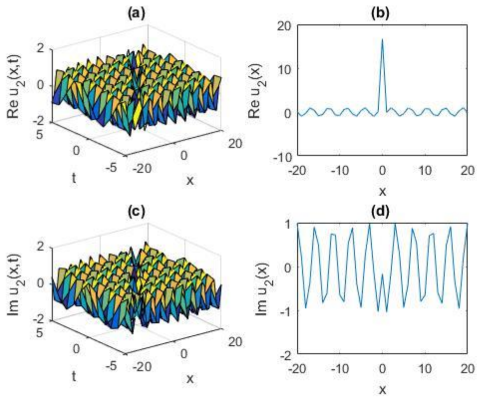

Fig. 3: Graphs of (4.34) ((a) and (b)) real values and ((c) and (d)) imaginary values by considering the values $b = B_1 = p = k = 2, a = c = 3, -20 < x < 20, -5 < t < 5$ and $t = 0.01$ for 2D surfaces

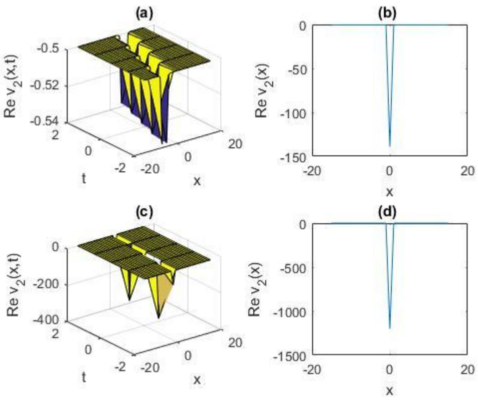

Fig. 4: Graphs of (4.34) real values by considering the values $b = B_1 = p = k = 2, a = c = 3, -20 < x < 20, -5 < t < 5$ for ((a) and (b)), values $b = p = k = 2,\ B_1 = 0.2,\ a = c = 3, -20 < x < 20, -5 < t < 5$ ((a) and (b)) and $t = 0.01$ for 2D surfaces

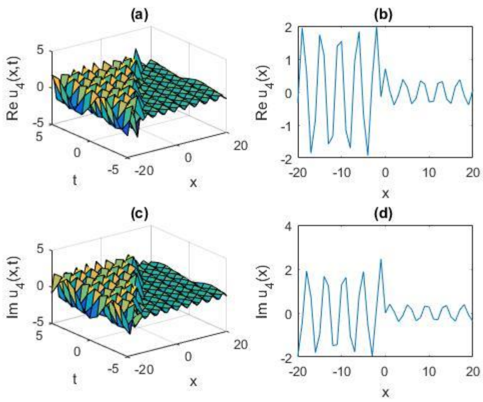

Fig. 5: Graphs of (4.36) ((a) and (b)) real values and ((c) and (d)) imaginary values by considering the values $b = B_1 = p = k = 2, a = c = 3, -20 < x < 20, -5 < t < 5$ and $t = 0.01$ for 2D surfaces

6. Conclusion

This paper presented a study on the nonlinear long-short wave interaction system. The nonlinear long-short wave interaction system is solved by two analytical methods, namely, the improved tan $(\phi/2)$-expansion method and He's semi-inverse variational method, by using the integration tools. Abundant exact traveling wave solutions including solitons, kink, periodic and rational solutions have been found. The obtained results are useful in gaining understanding of the transmission of the soliton wave solutions.

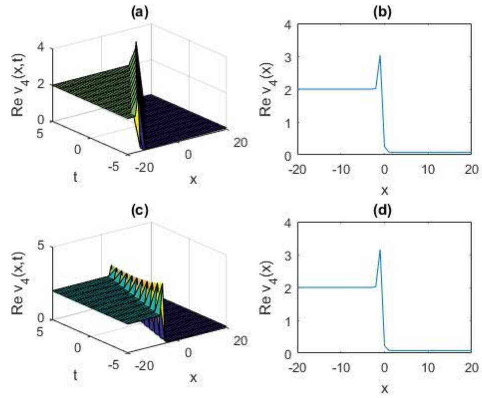

Fig. 6: Graphs of (4.36) real values by considering the values $b = B_1 = p = k = 2, a = c = 3, -20 < x < 20, -5 < t < 5$ for ((a) and (b)), values $b = p = k = 2,\ B_1 = 0.2,\ a = c = 3, -20 < x < 20, -5 < t < 5$ ((a) and (b)) and $t = 0.01$ for 2D surfaces

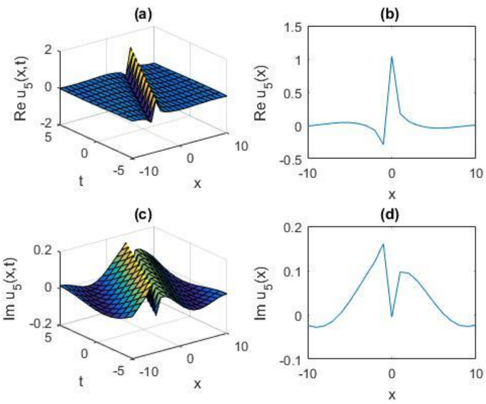

Fig. 7: Graphs of (4.37) ((a) and (b)) real values and ((c) and (d)) imaginary values by considering the values $a = 0$, $b = c = k = 2, B_1 = p = 0.5, -20 < x < 20, -5 < t < 5$ and $t = 0.01$ for 2D surfaces

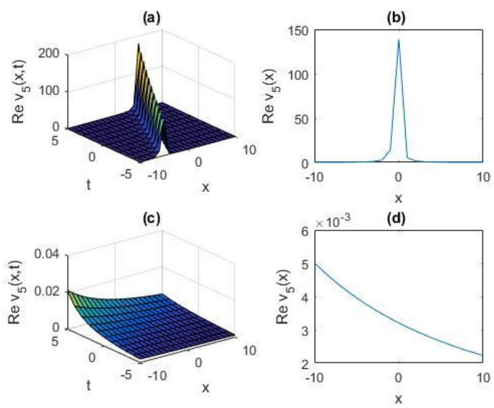

Fig. 8: Graphs of (4.37) real values by considering the values $a = 0,\ b = c = k = 2, B_1 = p = 0.5, -20 < x < 20, -5 < t < 5$ for ((a) and (b)), values $a = 0,\ b = c = k = 2, B_1 = 0.5, p = 5, -20 < x < 20, -5 < t < 5$ ((a) and (b)) and $t = 0.01$ for 2D surfaces

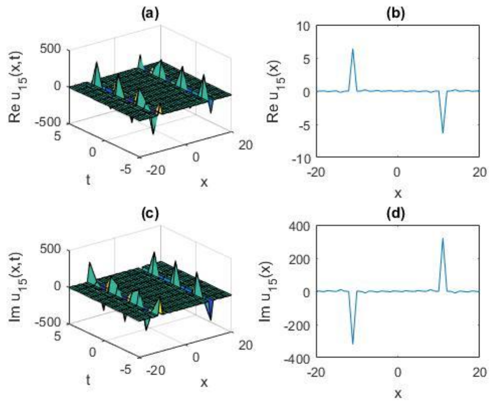

Fig. 9: Graphs of (4.54) ((a) and (b)) real values and ((c) and (d)) imaginary values by considering the values $\beta = b = B_1 = p = k = 2, c = 3, -20 < x < 20, -5 < t < 5$ and $t = 0.01$ for 2D surfaces

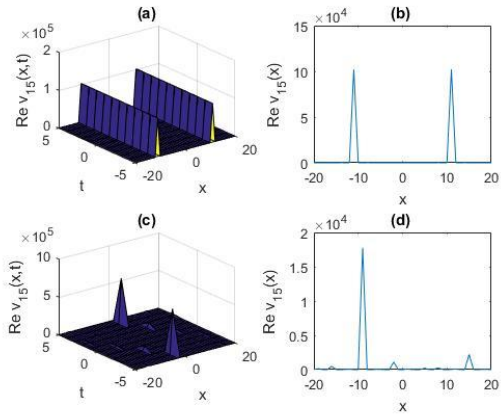

Fig. 10: Graphs of (4.54) real values by considering the values $\beta = b = B_1 = p = k = 2, c = 3, -20 < x < 20, -5 < t < 5$ for ((a) and (b)), values $\beta = b = B_1 = k = 2, c = 3, p = 0.2, -20 < x < 20, -5 < t < 5$ ((a) and (b)) and $t = 0.01$ for 2D surfaces

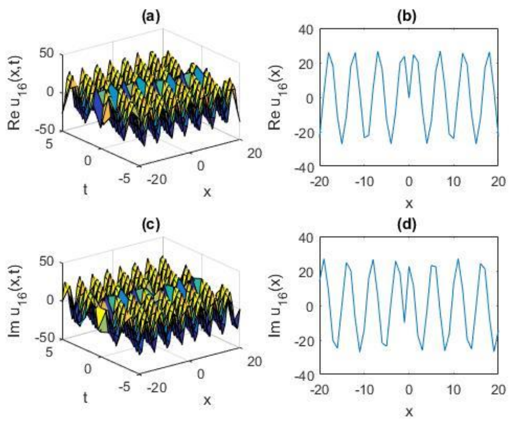

Fig. 11: Graphs of (4.55) ((a) and (b)) real values and ((c) and (d)) imaginary values by considering the values $\beta = c = B_1 = p = k = 2, b = 3, -20 < x < 20, -5 < t < 5$ and $t = 0.01$ for 2D surfaces

It is worth noting that the new solutions obtained by means of aforementioned methods confirm the correctness of those obtained by other methods. Not only, the newly obtained solutions are identical to already published results, but also further solutions have obtained. Therefore, these methods can be applied to study many other nonlinear partial differential equations which frequently arise in mathematical physics and mechanical sciences.

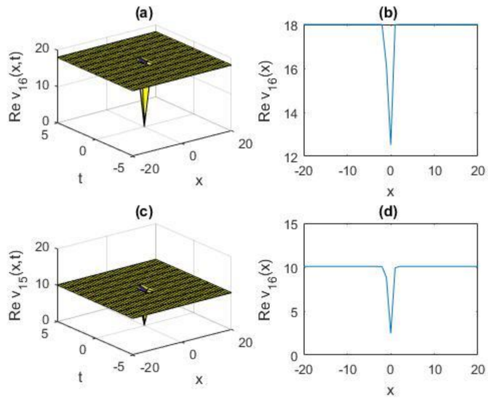

Fig. 12: Graphs of (4.55) real values by considering the values $\beta = b = B_1 = p = k = 2, c = 3, -20 < x < 20, -5 < t < 5$ for ((a) and (b)), values $\beta = c = B_1 = k = 2, b = 3, p = 0.2, -20 < x < 20, -5 < t < 5$ ((a) and (b)) and $t = 0.01$ for 2D surfaces

Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

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