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A new set of quaternion Laguerre moments for the reconstruction of CT and MRI images

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ABSTRACT

This study proposes a new set of moment functions for the reconstruction of medical computer tomography (CT) and magnetic resonance images (MRI) based on the associated Laguerre polynomials, which are orthogonal over the whole right-half plane. Moreover, the mathematical frameworks of radial associated Laguerre moments and associated rotation invariants are introduced. The proposed radial Laguerre invariants retain the basic form of disc-based moments, such as Zernike moments, pseudo-Zernike moments, Fourier-Mellin moments, and so on. Therefore, the rotation invariants of radial associated Laguerre moments can be easily obtained. In addition, we have also extended the proposed moments and invariants using the algebra of quaternion to avoid losing some significant color information. Finally, we have tested the numerical results performance based on the mean square error technique. The numerical experiment results obtained from both gray-level medical images and color medical images demonstrate that the effectiveness of the proposed ALMs and RALMs could be better according to reconstruction.

1. Introduction

In medical applications, the process of imaging acquisition can be affected by noises and artifacts this will lead to degrading the contrast resolution of images and making medical images difficult to interpret. The denoising of medical image algorithms are needed to achieve best diagnosis without affecting relevant features of the image (Raj and Venkateswarlu, 2012). Orthogonal moment functions are one of the most important tools for the reconstruction of shape descriptors for pattern recognition (Zhang et al., 2010), object classification (Jain et al., 2012), image reconstruction (Yap et al., 2010). Among all kinds of moments, continuous Zernike moments (ZMs), pseudo-Zernike moments (PZMs), orthogonal Fourier-Mellin moments OFMMs and Legendre moments were introduced by Teague (1980) according to their corresponding polynomials as kernel functions. As is known to all, the computation of ZMs, PZMs and OFMMs requires the transformation from image coordinates to a region within the unit circle, and Legendre moments need to transform image coordinates into the interval [-1, 1]. In addition, discretization of the continuous integrals is necessary for the computation of all continuous orthogonal moments. Discrete orthogonal moments give a more accurate reconstruction for medical image features by evaluating moment components directly in the medical image coordinate space (Chen et al., 2011; Singh and Aggarwal, 2017). Single-image super-resolution using orthogonal rotation Invariant moments. Hence, the discrete orthogonal moments eliminate the abovementioned problems associated with the continuous moments by using discrete orthogonal polynomials as kernel functions. Recently, the problem of invariant pattern recognition which undergoes geometric transforms, such as rotation, scaling and translation, has received increased interest in the pattern recognition field. Invariance feature selection always plays an important role in this subject. In recent decades, a number of orthogonal moments-based methods have been reported to reconstruct an invariance feature (Yang and Dai, 2011). Owing to the polar coordinate representation of the kernel functions of disc-based orthogonal moments, these continuous orthogonal moments have better image feature representation capabilities. This is mainly due to the fact that their rotation invariants can be easily constructed. Hence, as far as pattern

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recognition tasks is concerned, the continuous orthogonal moments still outperform discrete moments. Borrowing the specificity of the radial polynomials of disc-based moments, Mukundan (2004) recently introduced the framework of radial Tchebichef moments, which is based on the structure of disc-based moments and is particularly suitable for pattern recognition works requiring rotation invariants. In this paper, we propose another kind of discrete orthogonal moments, called associated Laguerre moments, which are also useful for medical image reconstruction. The proposed ALMs are defined in terms of the associated Laguerre polynomials (Askey and Wimp, 1984) which are orthogonal over the whole right-half plane. The advantage of the proposed ALMs over disc-based continuous orthogonal moments lies in the fact that the computation of these continuous moments requires a coordinate transformation and suitable approximation of the integrals that are not existent in the proposed ALMs. Taking cues from Mukundan's research works (Mukundan, 2004), this study combines the merit of computational advantages of discrete orthogonal moments with phantom reconstruction capabilities of continuous orthogonal moments, and introduces radial associated Laguerre moments RALMs in polar-coordinate form. Thus, the shape descriptors of rotation invariants, which retain the basic form of disc-based continuous moments, can be easily constructed in terms of the magnitude of the proposed RALMs. The advantage of this type of Image reconstruction using Radial Associated Laguerre Moments 3 representation is that a color medical image can be treated as a vector field. The accuracy of the proposed ALMs, RALMs, QALMs, and QRALMs as global feature representation capabilities is assessed by means of medical image reconstruction in noise-free and noisy cases and the results are compared with those of OFMMs.

The rest of this paper as following: Section two gives some mathematical background on the associated Laguerre polynomials and quaternion algebra theory. Section three presents a set of discrete ALMs and RALMs, and investigates the rotational invariance of RALMs. In Section Four presents the definition of QALMs and QRALMs in a holistic manner and Section five presents the experimental results and illustrates the performance of the proposed shape descriptors.

2. Associated Laguerre polynomials

The associated Laguerre polynomials $\{P_n^\alpha\}_{n\geq 0}$, for α > -1, are orthogonal with respect to the weight function $w(t) = t^\alpha e^{-t}$ on the internal$0 \leq t \leq +\infty$, that is,

$$\int_0^\infty e^{-t}\, t^\alpha P_m^\alpha(t) P_m^\alpha(t) dt = \frac{\Gamma(n+\alpha+1)}{n!}\delta_{nm}, m \geq 0 \quad (1)$$

where $\delta_{nm}$is Kroenke's symbol the associated Laguerre polynomials are defined as:

$$P_n^\alpha(t) = \frac{(\alpha+1)_n}{n!} F_1(-n; \alpha+1; t) \quad (2)$$

where the Pochhammer symbol $(\alpha)_k$ is as follows:

$$(\alpha)_k = \alpha(\alpha+1)(\alpha+2)\ldots(\alpha+k-1) \quad (3)$$

with $(\alpha)_0$ and $F_1(-n; \alpha+1; t)$ is a confluent hypergeometric function of the first kind

$$F_1(a; b; z) = 1 + \frac{a}{b}z + \frac{a(a+1)z^2}{b(b+1)2!} + \cdots = \sum_{k=0}^{\infty} \frac{(a)_k}{(b)_k}\frac{z^k}{k!} \quad (4)$$

With Eqs. 4 and 2, the associated Laguerre polynomials can be rewritten as

$$P_n^\alpha(t) = \sum_{k=0}^{n} (-1)^k \frac{(n+\alpha)!}{(n-k)!(k+\alpha)!k!} t^k \quad (5)$$

The associated Laguerre polynomials satisfy the following second-order recurrence relation:

$$tP_n^{\alpha+1}(t) = (n+\alpha)P_{n-1}^\alpha(t) - (n-t)P_n^\alpha(t) \quad (6)$$

3. Associated Laguerre moments

Without loss of generality, the ALMs of an image $I(x, y)$ with order of $n+m$ and size ofNXN are defined by the normalized associated Laguerre orthogonal polynomials $\tilde{P}_n^\alpha(x)$ as follows

$$\tilde{S}_{mn}^\alpha = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} \tilde{P}_m^\alpha(x)\tilde{P}_n^\alpha(y) I(x,y), m, n = 0,1 \ldots, N-1. \quad (7)$$

4. Radial associated Laguerre moments

Rotational invariance is an inherent property of disc-based continuous orthogonal moments. However, as indicated in Eq. 7, the proposed ALMs are defined over the Cartesian coordinates, therefore, it is still not convenient enough to generate rotation invariants. Motivated and aided by the framework of Zernike radial polynomials, this study therefore tries to define the following RALMs of order p and repetition q as

$$\tilde{R}_{pq}^\alpha = \frac{1}{2\pi}\sum_{r=0}^{m-1}\sum_{\theta=0}^{2\pi} \tilde{P}_p^\alpha(r) e^{-jq\theta} I(r,\theta) \quad (8)$$

where the image has a size of NXNpixels and m denotes $N/2$. Since θ is a real quantity measured in radians, one can rewrite Eq. 8 as

$$\tilde{R}_{pq}^\alpha = \frac{1}{n}\sum_{r=0}^{m-1}\sum_{\theta=0}^{n-1} \tilde{P}_p^\alpha(r) e^{-jq\theta} I(r,\theta) \quad (9)$$

where n is at 360. When the image is sampled at one-degree intervals and the coordinates x, y are given by

$$x = \frac{rN}{2(m-1)}\cos\left(\frac{2\pi\theta}{n}\right) + \frac{N}{2}, y = \frac{rN}{2(m-1)}\sin\left(\frac{2\pi\theta}{n}\right) + \frac{N}{2}. \quad (10)$$

The structure of the RALMs is very similar to that of disc-based moments. It can also be easily found that the definition in Eq. 9 yields a set of moments that is orthogonal in the discrete polar coordinate space of the image. Moreover, the main purpose of this type of representation is that the rotation invariants can be easily derived. Given an image (r, θ), after rotation by an angle α, the image is $I(r, \theta + \emptyset)$. One has

$$\tilde{R}^{\alpha}_{pq} = \frac{1}{2\pi}\int_0^1\int_0^{2\pi} r\tilde{P}^{\alpha}_p(r) f(r, \theta + \emptyset) e^{-jq(\theta-\emptyset)} dr d\theta$$
$$= e^{-jq\theta}\frac{1}{2\pi}\int_0^1\int_0^{2\pi} r\tilde{P}^{\alpha}_p(r) I(r, \theta) e^{-jq\theta} dr d\theta$$
$$= e^{-jq\emptyset}\tilde{R}^{\alpha}_{pq} \quad (11)$$

after applying normal operations, we have

$$\hat{R}^{\alpha}_{pq} = e^{-jq\emptyset}\tilde{R}^{\alpha}_{pq} = \left\|e^{-jq\emptyset}\right\|.\left\|\tilde{R}^{\alpha}_{pq}\right\| = \left\|\tilde{R}^{\alpha}_{pq}\right\|. \quad (12)$$

Comparing Eq. 11 with Eq. 12, it is not difficult to see that $\left\|\tilde{R}^{\alpha}_{pq}\right\|$ is with rotational invariance. The corresponding inverse moments transform is given by the following equation:

$$\hat{I}(r, \theta) = \sum_{p=0}^{P}\sum_{q=0}^{Q}\tilde{R}^{\alpha}_{pq}\tilde{P}^{\alpha}_p(r)e^{jq\theta} \quad (13)$$

5. Quaternion algebra

A quaternion has four components (one real part and three imaginary parts) and can be represented in a Mukundan (2004)

$$q = a + b.i + c.j + d.k. \quad (14)$$

where $a, b, c, d \in R$, and i, j, k obey the following multiplication rules:

$$i^2 = j^2 = k^2 = -1, i \times j = -j \times i = k$$
$$j \times k = -k \times j = i, k \times i = -i \times k = j \quad (15)$$

The modulus of a quaternion q follows the definition for complex numbers as

$$|q| = \sqrt{a^2 + b^2 + c^2 + d^2} \quad (16)$$

It is often useful to consider a quaternion as the sum of a scalar part and a vector part, which is represented as

$$q = S(q) + V(q) \quad (17)$$

where $S(q) = a$ and $V(q) = b.i + c.j + d.k$. If $S(q) = 0$, then q is reduced to a pure quaternion.

6. Quaternion associated Laguerre moments

Let $I(r, \theta) \equiv I_R i + I_G j + I_B k = I_R(r, \theta)i + I_G(r, \theta)j + I_B(r, \theta)k$ be an RGB image defined in polar coordinates. Therefore, the forward QALMs can be defined as

$$\widetilde{QS}^{\alpha}_{mn} = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\tilde{P}^{\alpha}_m(x)\tilde{P}^{\alpha}_n(y)(iI_R + jI_G + kI_B)\mu =$$
$$-\frac{1}{\sqrt{3}}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\tilde{P}^{\alpha}_m(x)\tilde{P}^{\alpha}_n(y)(iI_R + jI_G + kI_B)(i + j + k)$$
$$= -\frac{1}{\sqrt{3}}\left[\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\tilde{P}^{\alpha}_m(x)\tilde{P}^{\alpha}_n(y)\,(I_R + I_G + I_B)\right]$$
$$= -\frac{1}{\sqrt{3}}\,i\left[\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\tilde{P}^{\alpha}_m(x)\tilde{P}^{\alpha}_n(y)\,(I_G - I_B)\right]$$
$$-\frac{1}{\sqrt{3}}\,j\left[\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\tilde{P}^{\alpha}_m(x)\tilde{P}^{\alpha}_n(y)\,(I_B - I_R)\right]$$
$$-\frac{1}{\sqrt{3}}\,k\left[\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\tilde{P}^{\alpha}_m(x)\tilde{P}^{\alpha}_n(y)\,(I_R - I_G)\right] \quad (18)$$

7. Image representation using radial associated Laguerre moments

Considering the set of associate Laguerre polynomials, $\left\{P^{(\alpha)}_n\right\}_{n\geq 0}$ is not suitable for defining moments because the range of values of the polynomials expands rapidly with the increase of order. To avoid numerical fluctuation in the moment computation, the current study applies normalized associated orthogonal Laguerre polynomials $\tilde{P}^{\alpha}_n(t)$ to define the proposed ALMs.

$$\tilde{P}^{\alpha}_n(t) = \frac{t^{\alpha}e^{-t}n!}{(n+k)!}P^{\alpha}_n(t) \quad (19)$$

The first few orders of the normalized associated Laguerre polynomials with the parameters $\alpha = 0, 1, 2$ are shown in Fig. 1. From Fig. 1 one can observe clearly that the values of normalized associated polynomials $\tilde{P}^{\alpha}_n(t)$are bounded on a finite interval and have a table difference from the associated polynomials $P^{\alpha}_n(t)$.

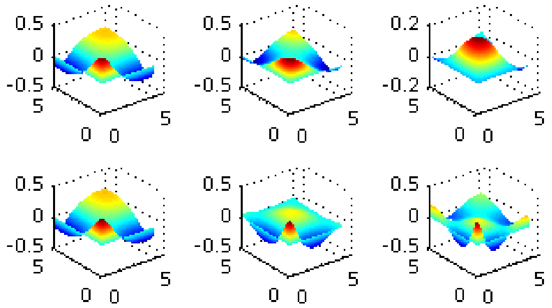

Fig. 1: Shows associated Laguerre polynomials with $n = 1$ and $\alpha = 0, 1, 2$ in the above row and $\alpha = 1$ and $n = 1, 2, 3$ in the second row

Comparing Eq. 1 and Eq. 18, one can obtain the orthogonality condition of normalized associated polynomials $\tilde{P}^{\alpha}_n(t)$ as

$$\int_0^{\infty}\tilde{P}^{\alpha}_n(t)\tilde{P}^{\alpha}_m(t)dt = \delta_{nm}\, , n, m \geq 0. \quad (20)$$

The performance of moment descriptor $\tilde{S}^{\alpha}_n$ is well assessed by means of imagereconstruction. Thanks to the orthogonality and completeness of $\{\tilde{P}^{\alpha}_n(t)\}$, which allow one to represent any square integrable image $I(x, y)$ via a truncated series defined over whole right-half plane, to be written by

$$\hat{I}(x, y) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1}\tilde{S}^{\alpha}_{mn}\,\tilde{P}^{\alpha}_m(x)\tilde{P}^{\alpha}_n(y) \quad (21)$$

It should be expected that the representation in the above formula can converge to the true image by making orders N sufficiently large.

Where μ is a unit pure quaternion chosen as in the current study, Thus, Eq. 21 can be rewritten as

$$\widetilde{QS}^{\alpha}_{mn} = A^R_0 + iA^R_1 + jA^R_2 + kA^R_3 \quad (22)$$

where

$$A_0^R = \frac{1}{\sqrt{3}}\left[\tilde{S}_{mn}^{\alpha}(I_R) + \tilde{S}_{mn}^{\alpha}(I_G) + \tilde{S}_{mn}^{\alpha}(I_B)\right]$$
$$A_1^R = -\frac{1}{\sqrt{3}}\left[\tilde{S}_{mn}^{\alpha}(I_G) - \tilde{S}_{mn}^{\alpha}(I_B)\right]$$
$$A_2^R = -\frac{1}{\sqrt{3}}\left[\tilde{S}_{mn}^{\alpha}(I_B) - \tilde{S}_{mn}^{\alpha}(I_R)\right]$$
$$A_3^R = -\frac{1}{\sqrt{3}}\left[\tilde{S}_{mn}^{\alpha}(I_R) - \tilde{S}_{mn}^{\alpha}(I_G)\right]$$

As discussed in the above section, since the associated Laguerre polynomials are orthogonal, color images can be estimated from a finite number N of QALMs using the following inverse moment transform:

$$\hat{I}(x,y) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} \widetilde{QS}_{mn}^{\alpha}\tilde{P}_m^{\alpha}(x)\tilde{P}_n^{\alpha}(y)\mu$$
$$= \frac{-1}{\sqrt{3}}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}(A_0^R + kA_3^R)\tilde{P}_m^{\alpha}(x)\tilde{P}_n^{\alpha}(y)(i+jk) \ =$$
$$-\frac{1}{\sqrt{3}}\left[\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} \tilde{P}_m^{\alpha}(x)\tilde{P}_n^{\alpha}(y)\,(A_1^R + A_2^R + A_3^R)\,\right]$$
$$= \frac{-1}{\sqrt{3}}\ i\left[\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} \tilde{P}_m^{\alpha}(x)\tilde{P}_n^{\alpha}(y)\,(A_0^R + A_2^R - A_3^R)\right]$$
$$\frac{-1}{\sqrt{3}}\ j\left[\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} \tilde{P}_m^{\alpha}(x)\tilde{P}_n^{\alpha}(y)\,(A_0^R - A_1^R + A_3^R)\right]$$
$$\frac{-1}{\sqrt{3}}k\left[\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} \tilde{P}_m^{\alpha}(x)\,\tilde{P}_n^{\alpha}(y)\,(A_0^R + A_2^R - A_2^R)\right] \tag{23}$$

The right-side QRALMs of order p with repetition q are defined as follows from such a representation, the rotation invariants are easily achieved by taking the modulus of QRALMs. Similar approaches can be used to obtain left-side QRALMs. Owing to limited space we will omit their discussion.

8. Numerical results

We have tested the performance of the proposed ALMs and RALMs. In the numerical results, we have used three selected test images including phantom images, gray-level images and color images, with a resolution of 256×256 and 128x128 (Fig. 2).

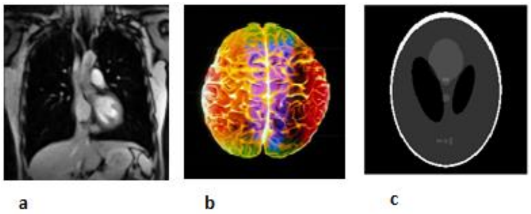

Fig. 2: Original test images, (a) gray-level images (size: 256×256); (b) color images (size: 128×128) and (c) is the artificial image

The mean square error (MSE) was used as the fidelity criteria measuring the resemblance between the reconstructed images and the original ones. It can be written as:

$$MSE = \frac{1}{NM}\sum\left|I(x,y) - \hat{I}(x,y)\right|^2$$

where I(x,y)and Î(x,y) are the exact and the approximate respectively.

The numerical results on test image show the effective of moment functions by image reconstruction. The orthogonality of moments given in Eq. 8, both gray-level and color images can be estimated approximately by Eq. 9. Fig. 3 and Fig. 4 shows the reconstruction result using Associate Laguerre moments (ALMs) and Q-ALMs for various values of the parameter α. Also we have carried out to illustrate the image reconstruction by using the radial ALMs using artificial images with a size of 200×200 pixels, as shown in Fig. 5.

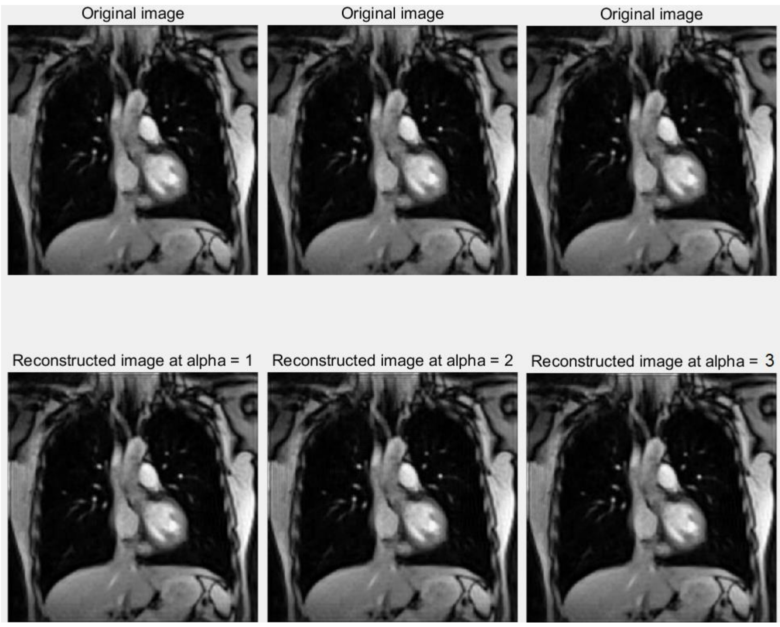

Fig. 3: Reconstructed gray-level images using ALMs for with α = 1, 2, 3 respectively

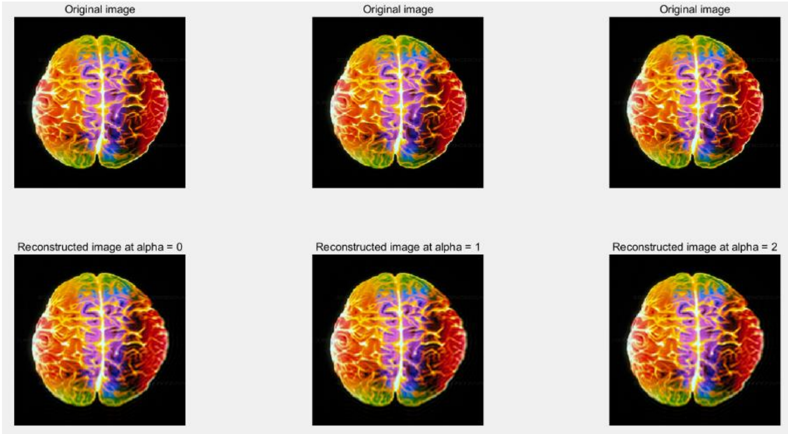

Fig. 4: Reconstructed color images using Q-ALMs with α = 0, 1, 2 respectively

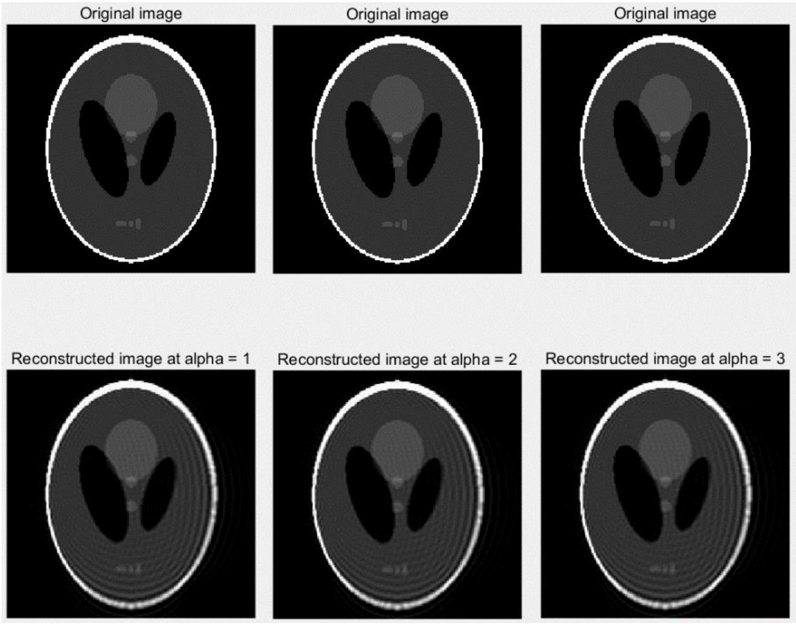

Fig. 5: Reconstruction of artificial image using ALMs and α = 1, 2, 3

9. Conclusion

We have discussed a set of Laguerre orthogonal moments depend on the associated Laguerre polynomials for medical image reconstruction RALMs using methods that are similar to those of disc-based moments. This form makes them particularly suitable for reconstruction tasks requiring rotation invariants. We have also extended the proposed moments and rotation invariants defined for gray-level medical images to color medical images using the theory of quaternion algebra. The numerical experiment results obtained from both gray-level medical images and color medical images demonstrate that the effectiveness of the proposed ALMs and RALMs could be better according to reconstruction.

Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

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