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Mathematical analysis of the queuing system and application

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ABSTRACT

The mathematical study of waiting lines is mainly concerned with queue performance measures where several applications have been drawn in past studies. This work studies the mathematical analysis of queuing models formed from observed data of a shopping mall checkout service unit. The efficiency of the models in terms of utilization and waiting length is carried out by administering questionnaires so as to rate the system performance. The results show that the inconsistency of the arrival and service process is a major cause for needless long queues formed.

1. Introduction

Queues are waiting lines formed as a result of high demand for service and limited resources to provide adequate service. Queuing theory is the mathematical study of waiting lines. This theory allows for the calculation of several performance measures. The study of queuing theory started with the works of Erlang (1909), whose primarily concern was to study the behaviour of traffic at telephone exchanges. Erlang (1909) derived the probability of the different number of calls waiting, equilibrium waiting time for calls and probability for a call loss. This stimulated further research in this area, and many mathematical ideas such as link systems, where a set of sources may have limited access to a set of destinations were introduced. He modeled the number of telephone calls arriving at an exchange station by a Poisson process and solved the M/D/1 and M/D/K queues. Further studies were carried by many other researchers such as the M/M/1 queue on Poisson arrival, exponential service, single server, etc. (Patel et al., 2012; Priyangika and Cooray, 2016) used multiple servers to study waiting times and length of queue(s) in supermarkets, Big bazar in Sri Lanka and Thailand respectively (Augustine, 2013; Aradhye and Kallurkar, 2014; Bajpai, 2013; Boucher and Couture-Piché, 2015; Bishop et al., 2018).

Most business managers (Bajpai, 2013; Rafaeli et al., 2002) understand that effective and efficient service quality is key for success, and key to remaining at the apex of a competitive business world. For some, more servers mean more investment, while few servers may result in waiting for too long thereby affecting service quality. This research therefore, aims at using queuing theory to evaluate the parameters involved in the service unit for the sales checkout unit in a shopping mall in Lagos State, Nigeria. The study derives and calculates several performance measures which includes the average waiting time in the queue, the expected number of customers waiting or receiving service and the probability of the state being empty, full, etc.

The study will among others, help form decision making, improve service, reduce time wastage, maximize resources and save lives and property which can result from bad management of queues. This work can also be applied to queues formed at ATM machines, hospitals, bus stations, and other sales checkout services. The works of Augustine (2013), Aradhye and Kallurkar (2014), Bajpai (2013), Boucher and Couture-Piché (2015), Bishop et al., (2018), and Priyangika and Cooray (2016) cannot be applied directly because of the prevailing forces associated with the choice of geographical location and some assumptions of the model.

2. Preambles

Queuing models are commonly labeled as $M_1/M_2/S/K$, where M_1, M_2 represents the exponential distribution of inter-arrival times and service times respectively. S, represents a positive constant representing the number of servers

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(deterministic). K is the specific number of customers in the queue. If K = ∞, then the model becomes M/M/c/∞. Here we adopt the M/M/S model with S ≥ 1. The models incorporate both the deterministic and stochastic components of the queuing system. In these models, the following are defined:

- *Arrival Pattern*: is the number of customers arriving and entering appropriate queues
- *Waiting Processes*: length of queues, servers' discipline (FIFO).
- *Server Process:* server type, serving rate and servicing time.
- *Assumptions:*
  1. Arrivals of customers follow a Poisson process (Stochastic process).
  2. Inter-arrival times are exponentially distributed and independent of each other with mean 1/μ.
  3. Identical service facilities.
  4. Service times are exponentially distributed.
  5. Infinite number of customers in queuing system
  6. All customers are served before leaving the line.
  7. FIFO (First in first out).
  8. System is in a steady-state condition
  9. Items 1 and 4 are independent and identically distributed (IID) with N (0, 1).

3. Results and discussion

The sales check out unit has six (6) servers, however only 2 were observed on week days and weekends. Customers are served according to assumption 7 above. By using questionnaires, the data was collected for only two servers. It was assumed that all the servers are functional. More crowd were observed at weekends (Fridays through Sundays). Hence, each server was treated as single server with single queues.

Confidence intervals

Confidence intervals are as follows:

a. 95% confidence level for arrival rate can be computed as follows:

$$[(\mu + 1.96\, SE\ (\mu))^{-1}, (\mu - 1.96\, SE\ (\mu))^{-1}],$$

where

$$SE\ (\mu)\ =\ SD\ (\mu)/\sqrt{n}$$

b. 95% confidence level for service rate can be computed as follows:

$$[(\lambda + 1.96\, SE\ (\lambda))^{-1}, (\lambda\ - 1.96\, SE\ (\lambda))^{-1}],$$

where

$$SE\ (\lambda)\ =\ SD\ (\lambda)/\sqrt{n}.$$

where μ = mean arrival time, λ = mean service time and n is the total number of customers in the queue. Let service time = ST, arrival time = AT, DS = Standard deviation and SE = Standard error.

Confidence intervals (CI) for weekends at 95%

Confidence intervals (CI) for weekends at 95% with all times will be in seconds:

$$\lambda\ =\ 66 secs/customer$$
$$SD\ (ST) =\ 60\ seconds, \mu\ =\ 36\ seconds,$$
$$SD\ (AT) =\ 60 seconds, n\ =\ 50.$$

(i) CI for Service Time:

$$[\lambda -\ 1.96\ (SE\ (ST))] =\ 49\ secs/customer,$$
$$[\lambda +\ 1.96\ (SE\ (ST))] =\ 82\ secs/customer,$$

where

$$SE\ (\mu)\ =\ SD\ (\mu)/\sqrt{n}$$

(ii) CI for Service Rate:

$$1/\ [\lambda +\ 1.96\ (SE\ (ST))]\ =\ 0.01230 \\ =\ 44\ customers\ /hour$$
$$1/\ [\lambda -\ 1.96\ (SE\ (ST))]\ =\ 0.02041 \\ =\ 73\ customers\ /hour$$

(iii) CI for Arrival Time:

$$\mu -\ 1.96\ (SE\ (arrival\ time))\ =\ 19\ secs/customer,$$
$$\mu +\ 1.96\ (SE\ (arrival\ time))\ =\ 53\ secs/customer.$$

(iv) CI for Arrival Rate:

$$1/\ [\mu\ + 1.96\ (SE\ (AT))]\ =\ 0.01887 \\ =\ 68\ customers\ /hour,$$
$$1/\ [\mu -\ 1.96\ (SE\ (AT))]\ =\ 0.05263 \\ =\ 189\ customers\ /hour.$$

Interpretation of confidence intervals

The *CI* shows that 68 to 189 customers arrive in 2 – servers' system within an hour while only 44 to 73 are served while the service time is between 49 seconds to 82 seconds for each customer.

Expected queue length

Expected queue length is given by (3+4+...+2+3)/50 = 3 customers/minute within 36 minutes

Queuing analysis

At weekends, customers arrive at an average of 103 customers per hour, while an average of 55 of them are served in every one hour by a server.

Results for queuing model M/M/2

Parameters and Characteristics of M/M/2 are:

S = 2
α = arrival rate = 103 customers/per hour
β = service rate = 55 customers/server/ per hour
Sβ = 2(55) = 110 (service rate for 2 servers)
ρ = α / Sβ = 103/110 = 0.9363
γ = α/β = 103/55 = 1.8727

Overall system utilization = ρ = 93.63% shows good performance of servers. The steady state probability that all servers are empty is given by (P_0) = 0.0329. Average number of customers in the queue is given by L_q = 13 per 2-servers. Average time spent on queue by a customer is given by $W_q = L_q/ \lambda = 7.5$ minutes showing that the server is busy. This estimate is not realistic because the model shows that customers make a single queue and are free to choose available servers. Thus, a single queue single server is adopted to obtain a correct estimate of the queue length.

Results for M/M/1 queue model

Parameters and Characteristics of M/M/1, assuming a steady-state system are:

$$S = 1, \rho = 0.9363, \gamma = 0.9363, n = 50, P_0 = 0.0637$$

Average time spent on queue by a customer is given by L_q= 15 per 1-server, $W_q=L_q/ \lambda$= 8.2 minutes showing that the number of customers is in queue is higher than the model with 2-servers and implies each customer waiting for 16 minutes before been attended to. This means, reducing the number of servers will most likely lead to longer queues.

Quality of service

Fig. 1 shows that out of 50 numbers of customers, 30 say the service is sufficient, 15 say the service is moderately sufficient while 5 say it is insufficient/incomplete.

4. Conclusion

The study reviewed queuing models with single and multiple servers. The average queue length was estimated from the raw data obtained from questionnaires. This can be compared with that of the model. The observed analysis of this queuing model, is that they are not very efficient in terms of resource utilization.

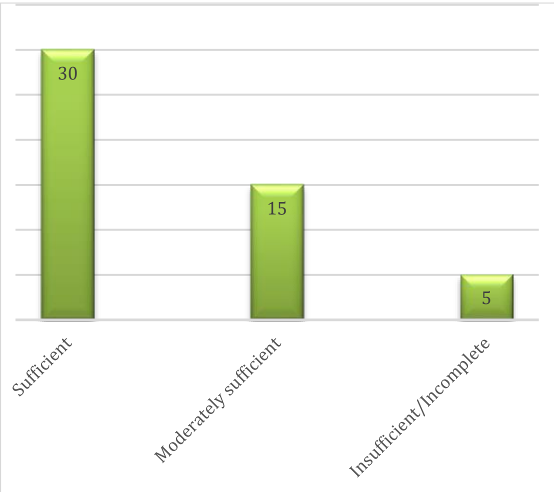

Fig 1: Quality of service

The variability of the arrival and service process is the reason for the unnecessary long queues formed especially at weekends. If the variability can be eliminated, the system could be designed economically so as to eliminate long queues and time wastage.

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Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

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