

## Dynamic response of simple bridge due to moving vehicles in both along opposite directions



Hoa P. Hoang<sup>1</sup>, Trung D. Pham<sup>2,\*</sup>, Oanh T. K. Do<sup>2</sup>, Phuoc T. Nguyen<sup>3</sup>

<sup>1</sup>Department of Construction of Bridge and Road, University of Science and Technology, The University of Danang, Danang, Vietnam

<sup>2</sup>Department of Civil Engineering, Mien Trung University of Civil Engineering, 24 Nguyen Du St., Tuy Hoa, Vietnam

<sup>3</sup>Department of Civil Engineering, Ho Chi Minh City Open University, 97 Vo Van Tan St., Ho Chi Minh, Vietnam

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### ABSTRACT

The purpose of this study is presenting the dynamic response of a simple bridge subjected to moving vehicles using finite element method. The moving vehicles move in both along opposite directions on the simple bridge at different speeds, described by two masses as car body and wheel, respectively. Based on dynamic balance principle, the governing equation of motion of the bridge-vehicle interaction is derived and solved by the Newmark method in the time domain. At the same time, the characteristic parameters of the moving vehicles such as the ratios of initial position and speed of the vehicles are proposed. And then, the influence of the above parameters on the dynamic response of the bridge-vehicle interaction is investigated in detail. The numerical results showed that those parameters affect significantly the dynamic response of the bridge-vehicle interaction. It is evidently more increasing the dynamic response of the bridge-vehicles interaction than other cases. Hence, this study can be considered as meaningful practice document in the problems of design and response analysis of the bridge due to moving traffic load.

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### 1. Introduction

The problem of dynamic response analysis of structure due to moving load is always one of the most interesting subjects. Especially, dynamic response of the structural bridge considered as beam subjected to moving vehicles has been attracted many researchers in during past decades. In this problem, the moving loads are considered as moving concentrate forces, moving masses or moving vehicles which move on the surface of the structure in the only direction from the left end to right end, presented in many previous types of research (Neves et al., 2012; Michaltsos, 2002; Wu and Chiang, 2004; Jun et al., 2010; 2017; Zhang et al., 2013; Reis and Pala, 2009; Yang and Yau, 1997; Yueqin and Wei, 2005; Yue et al., 2005; Ye et al, 2010; Yin et al., 2010; Yang et al, 2004; Yan et al., 2013; Sun and Zhang, 2014; Vaidya and Chatterjee, 2017; Yu et al., 2018;

An et al., 2016; Pham et al., 2018; Hoang et al., 2019). But, in reality, the vehicles can completely move in the both along the opposite directions which were overlooked in most previous works rated to analyze the bridge-vehicle dynamic response. Therefore, it will affect the dynamic response of the structure and it can cause a more increasing dynamic response of the structure than in the case moving in the only direction.

It can be seen that the dynamic response of the structure due to moving vehicles in the both along the opposite direction is not still attention in the recent time. Hence, this paper studies the dynamic response of the simple bridge due to moving vehicles in the both along opposite direction using the finite element method. The moving vehicles include two vehicles with different moving velocities, one moves along a direction from the left end to the right end, and the rest moves along opposite direction. And then, the governing equation of motion of the bridge-vehicles interaction is established based on the dynamic balance principle and solved by the Newmark method in the time domain. The characteristic parameters for the moving velocity and initial position of the vehicles are proposed for analyzing the dynamic response of the bridge-vehicles interaction and discussed detail.

\* Corresponding Author.

Email Address: [phamdinhtung@muce.edu.vn](mailto:phamdinhtung@muce.edu.vn) (T. D. Pham)

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Corresponding author's ORCID profile:

<https://orcid.org/0000-0001-8629-0640>

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### 2. The problem model

Considering a simple bridge as an Euler-Bernoulli beam subjected to moving vehicles in both along opposite direction, plotted in Fig. 1. Each vehicle moves with different initial velocity and departure time at each end bridge is also not similar. The characteristic parameter of the initial position of the moving vehicles is proposed as.

$$\kappa = \frac{v_1}{v_2} \tag{1}$$

where  $\kappa$  is defined as the ratio of the initial velocity of the first moving vehicle to the initial velocity of the second vehicle. At the same time, the second moving vehicle will be a departure at the right end of the bridge after the first vehicle moves on the bridge in during the time  $t$  corresponding with position  $x_0 \leq L$ , plotted in Fig. 1.

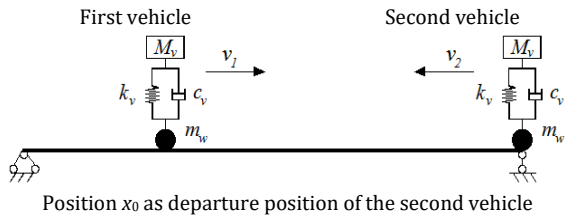


Fig. 1: The simple bridge subjected to moving vehicles in the both along the opposite direction

The moving vehicle is regarded as a two-node system, with one node associated with each of two concentrated masses having the stiffness and damping coefficients of the moving vehicle denoted by  $k_v$  and  $c_v$ , and the mass of the wheel and the mass lumped from the car body by  $m_w$  and  $M_v$ , respectively. By assuming the no-jump condition for the moving vehicle, the equation of motion of the vehicle system can be written as follows

$$\begin{bmatrix} M_v & 0 \\ 0 & m_w \end{bmatrix} \begin{Bmatrix} \dot{z}_v \\ \dot{z}_w \end{Bmatrix} + \begin{bmatrix} c_v & -c_v \\ -c_v & c_v \end{bmatrix} \begin{Bmatrix} \dot{z}_v \\ \dot{z}_w \end{Bmatrix} + \begin{bmatrix} k_v & -k_v \\ -k_v & k_v \end{bmatrix} \begin{Bmatrix} z_v \\ z_w \end{Bmatrix} = \begin{bmatrix} f_c - (M_v + m_w)g \\ 0 \end{bmatrix} \tag{2}$$

where  $f_c$  is the contact force, given by

$$f_c = (m_w + M_v)g + m_w \ddot{z}_w + M_v \ddot{z}_v \tag{3}$$

in which  $z_v$  and  $z_w$  denote the vertical displacements of two nodes, respectively.

### 3. Governing equation

Based on the Euler-Bernoulli theory, the simple bridge is disjointed based on the finite element method of the two-node beam element. Each node has two global degrees of freedom including one displacement axes and one rotation, presented in many documents rated to the finite element method (Pham et al., 2018). And then, the matrices of the bridge element in the global coordinates are obtained by

$$K_e = \begin{bmatrix} \frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\ & \frac{12EI}{l^3} & \frac{6EI}{l^2} & 0 & -\frac{12EI}{l^3} & \frac{6EI}{l^2} \\ & & \frac{4EI}{l} & 0 & -\frac{6EI}{l^2} & \frac{2EI}{l} \\ & & & \frac{EA}{l} & 0 & 0 \\ & & & & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ \text{syms} & & & & & \frac{4EI}{l} \end{bmatrix} \tag{4}$$

$$M_e = \rho Al \begin{bmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ & \frac{13}{35} & \frac{11}{210}l & 0 & \frac{9}{70} & -\frac{13}{420}l \\ & & \frac{1}{105}l^2 & 0 & \frac{13}{420}l & -\frac{1}{140}l^2 \\ & & & \frac{1}{3} & 0 & 0 \\ & & & & \frac{13}{35} & -\frac{11}{210}l \\ \text{syms} & & & & & \frac{1}{105}l^2 \end{bmatrix} \tag{5}$$

Based on dynamic balance principle, the governing equation of the bridge-vehicles interaction element at each time step can be expressed as follows

$$M_e \ddot{q}_e + C_e \dot{q}_e + K_e q_e = \sum F_{e,i} \quad (i = 1,2) \tag{6}$$

where  $M_e$  and  $K_e$  is the mass matrix and stiffness matrix of the beam element presented in many previous works. By adopting Rayleigh damping, the damping matrix  $C_e$  can be obtained as follows

$$C_e = \alpha_0 M_e + \alpha_1 K_e \tag{7}$$

and  $F_{e,i}$  is the vector of consistent nodal forces caused by the contact force of each vehicle in the global coordinates, given by

$$F_{e,i} = -\delta(\cdot)_i N_{v,i}^T f_{c,i} \tag{8}$$

in which  $\delta(\cdot)_i$  is the Dirac-delta function and  $N_{v,i}$  denote the shape function of the beam element corresponding with the position of each the moving vehicle, denoted by

$$N_{v,i} = [0 \quad N_1 \quad N_2 \quad 0 \quad N_3 \quad N_4]_i \tag{9}$$

where  $N_i$  can be expressed as follows

$$\begin{aligned} N_1 &= 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3}, N_2 = x - \frac{2x^2}{l} + \frac{x^3}{l^2}, \\ N_3 &= \frac{3x^2}{l^2} - \frac{2x^3}{l^3}, N_4 = -\frac{x^2}{l} + \frac{x^3}{l^2} \end{aligned} \tag{10}$$

By assembling the element matrices in the global coordinates, the governing equation of motion of the bridge-vehicles interaction can be written as

$$M \ddot{u} + C \dot{u} + K u = F \tag{11}$$

where  $M$  and  $K$  denote the global mass and stiffness matrix, respectively,  $u$  is the global displacement vector,  $F$  denotes the global load vector, and the damping matrix  $C$  can be obtained by adopting Rayleigh damping.

The above dynamic equation (11) is used for studying the dynamic response of the bridge-

vehicles interaction and solved by means of the direct integration method based on Newmark algorithm, plotted in Fig. 2. Therefore, the computer program using MATLAB language is developed by the authors based on the above flowchart and the accuracy of the algorithm is verified by comparing the numerical results with the other numerical results in the literature.

**4. Numerical investigation**

**4.1. Verification**

In the verification, the dynamic responses of the simple bridge subjected to the moving vehicle in the only direction are compared with the results obtained by Yang et al. (2004). The time history of the vertical displacement of the middle of the bridge and car body are plotted in Fig. 3.

It can be seen that this numerical verification based on the suggested formulation are quite good agreement with numerical results in the literature. Therefore, the algorithm which used to analyze the dynamic response of the bridge-vehicles interaction in the next section is reliable.

**4.2. Numerical results**

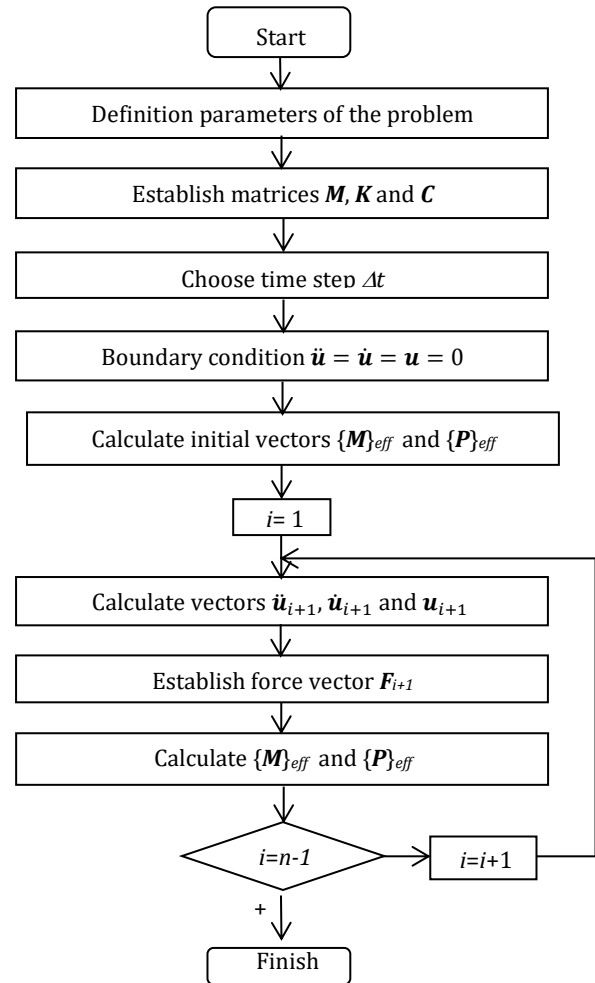
In this section, the dynamic response of the bridge due to the moving vehicles in the both along opposite direction are investigated in detail. The properties of the bridge-vehicles interaction are given in Table 1.

**Table 1:** The properties of bridge-vehicles interaction

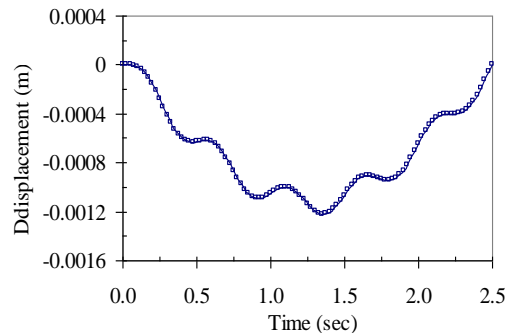
| Parameters                  | Unit              | Value   |
|-----------------------------|-------------------|---------|
| Length $L$                  | m                 | 25      |
| Young's modulus $E$         | Nm <sup>-2</sup>  | 2.87E9  |
| Moment of inertia $I$       | m <sup>4</sup>    | 2.9     |
| Mass per unit length $\rho$ | kgm <sup>-1</sup> | 2303    |
| Damping ratio $\xi$         |                   | 0.02    |
| Body mass $M_v$             | kg                | 5750    |
| Spring stiffness $k_v$      | Nm <sup>-1</sup>  | 1.595E6 |
| Damping coefficient $c_v$   | Nsm <sup>-1</sup> | 4.5E3   |
| Wheel mass $m_w$            | kg                | 250     |

In the first investigation, the time history of dynamic response of the bridge is studied. Fig 4 and Fig. 5 present the influence of the departure time of the second vehicle corresponding with the position of the first vehicle  $x_0$  on the time history of the vertical displacement of the middle of the bridge with various values of the ratio of initial velocity. It can be seen that the departure position effects on the time history displacement of the bridge, it is more increasing the dynamic response of the bridge than in the case having only moving vehicle. When the departure position is near the right bridge support as the first vehicle moved near to the end-point bridge and then the second vehicle will begin moving into the bridge. Hence, the influence of both vehicles on the dynamic response of the bridge will be not significant, therefore it is not more increasing significantly the time history displacement of the bridge than another case, shown in Fig. 4 and Fig. 5.

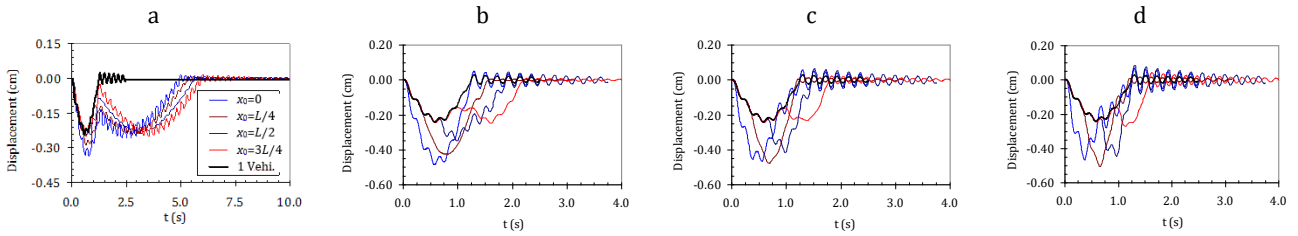
Additionally, the influence of the ration of initial velocity on the time history displacement of the bridge is also plotted in Fig. 6 and Fig. 7. It can be easily seen that the ratios of initial velocity increase the time history displacement of the bridge with an increase of its  $\kappa$ ; these increases are evident when the departure position near the left bridge support. Besides, the influence of the above parameters on time history displacement of the car body is also plotted in Fig. 8 and Fig. 9. The results show that these parameters have also effects similar to the above results.



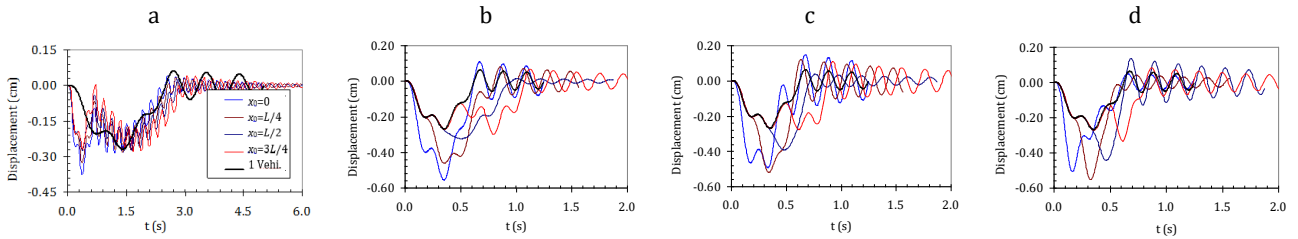
**Fig. 2:** Flowchart for analyzing the dynamic response of the bridge-vehicles interaction



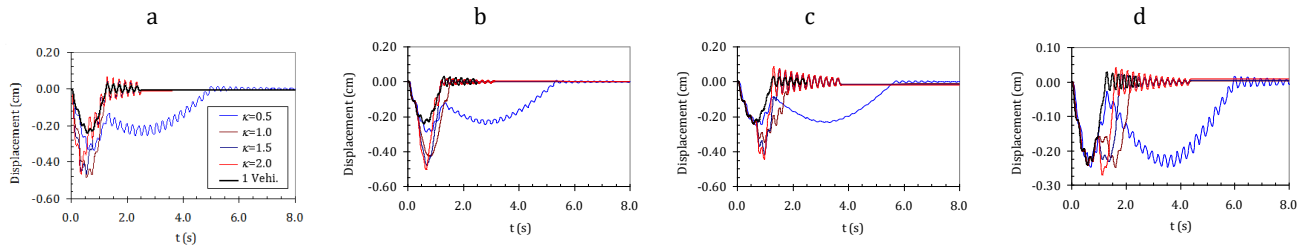
**Fig. 3:** The time history of the midpoint displacement: (-) Present, (□) Yang et al. (2004)



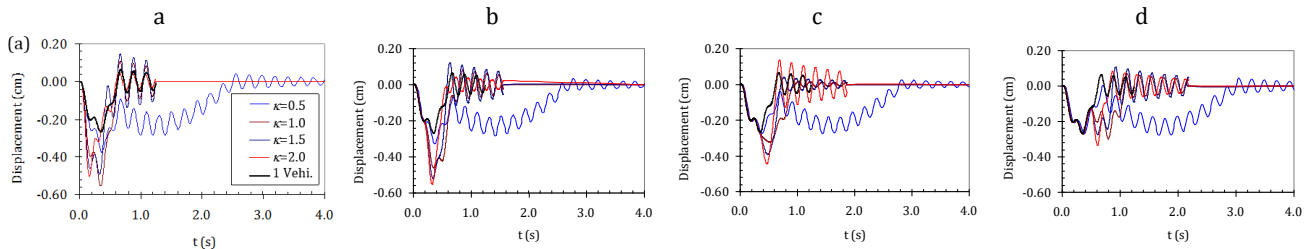
**Fig. 4:** The time history of vertical displacement of the middle of the bridge with  $v_I=20 \text{ ms}^{-1}$ : (a)  $\kappa= 0.25$ , (b)  $\kappa= 1$ , (c)  $\kappa= 1.5$ , (d)  $\kappa= 2$



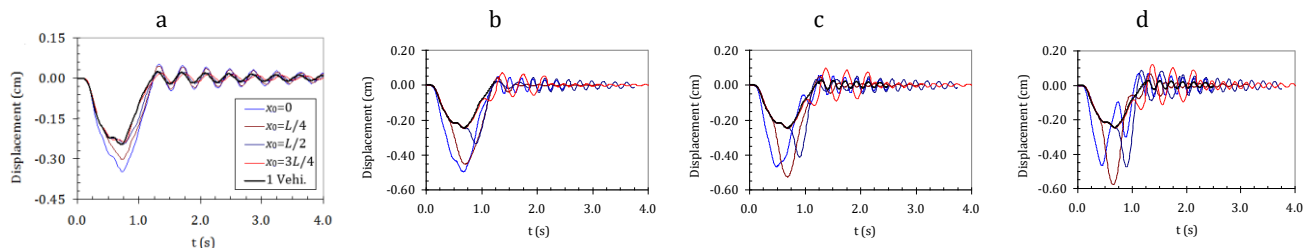
**Fig. 5:** The time history of vertical displacement of the middle of the bridge with  $v_I=40 \text{ ms}^{-1}$ : (a)  $\kappa= 0.25$ , (b)  $\kappa= 1$ , (c)  $\kappa= 1.5$ , (d)  $\kappa= 2$



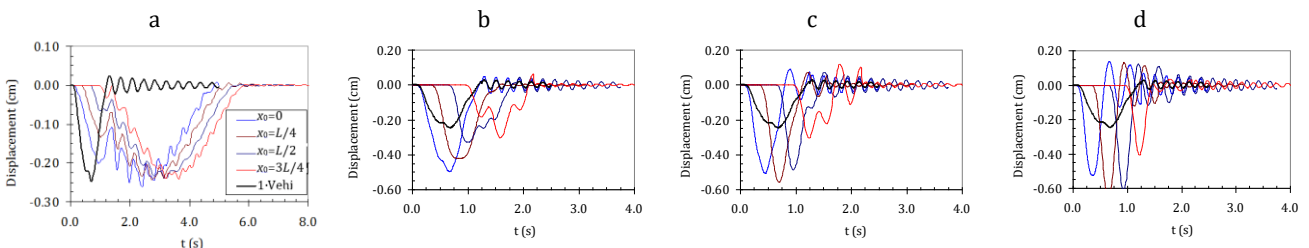
**Fig. 6:** The time history of vertical displacement of the middle of the bridge with  $v_I=20 \text{ ms}^{-1}$ : (a)  $x_0=0$ , (b)  $x_0=L/4$ , (c)  $x_0=L/2$ , (d)  $x_0=3L/4$



**Fig. 7:** The time history of vertical displacement of the middle of the bridge with  $v_I=40 \text{ ms}^{-1}$ : (a)  $x_0=0$ , (b)  $x_0=L/4$ , (c)  $x_0=L/2$ , (d)  $x_0=3L/4$



**Fig. 8:** The time history of vertical displacement of the first car body with  $v_I=20 \text{ ms}^{-1}$ : (a)  $\kappa= 0.25$ , (b)  $\kappa= 1$ , (c)  $\kappa= 1.5$ , (d)  $\kappa= 2$

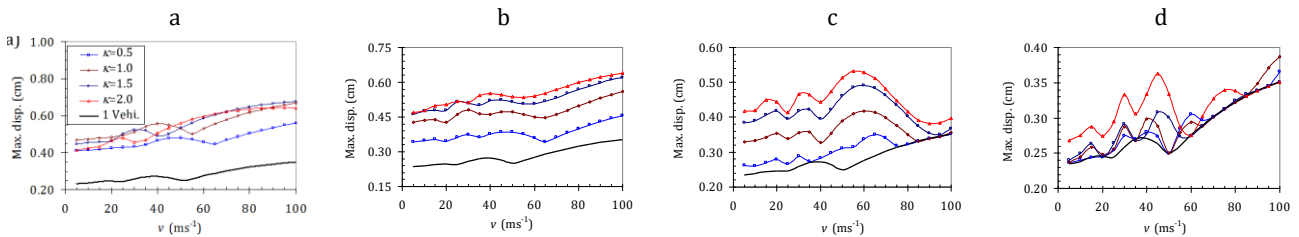


**Fig. 9:** The time history of vertical displacement of the second car body with  $v_I=20 \text{ ms}^{-1}$ : (a)  $\kappa= 0.25$ , (b)  $\kappa= 1$ , (c)  $\kappa= 1.5$ , (d)  $\kappa= 2$

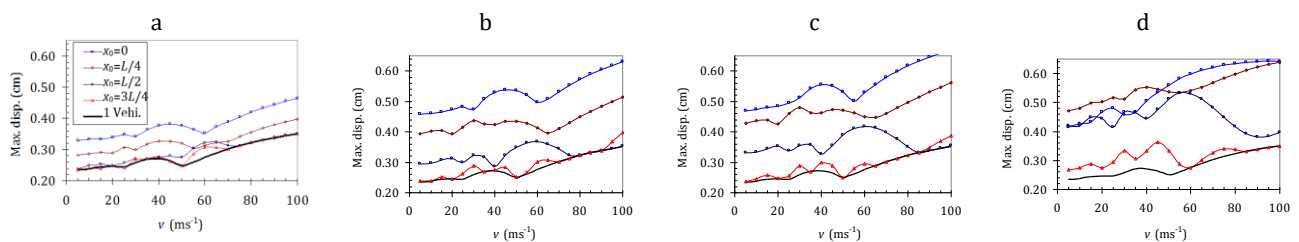
The above results show that the time response of the bridge-vehicles interaction depends on both the ratio of initial velocity and the departure position as the time departure of the second vehicle. In the range of the ratios of initial velocity less than or equal 1, the influence of the time departure of the second vehicle on the time response of the bridge-vehicles interaction has a difference with others, such as the time response of the bridge-vehicles interaction will be the largest corresponding with departure position  $x_0=0$  while it will be the largest with  $x_0=L/4$  in the opposite cases. It can be easily seen that if the initial velocity of two vehicles does not have much unequal, both the vehicles will move to the midpoint of the bridge with the same time departure, and then the acting force of both vehicles on the bridge will be maximum as loading resonance. Therefore, this phenomenon will cause more increasing the time response of the bridge-vehicles interaction than other cases. But, in the case of the initial velocity of two vehicles having difference significantly, both the vehicles will not move to the midpoint of the bridge at the same time if these vehicles have the same time departure. Therefore, the departure position will affect significantly on the time point which both the vehicles move to the central bridge. Hence, it also affects the dynamic response of the bridge-vehicles interactions and the

influence of this parameter also depends on the ratio of the initial velocity, as presented from Fig. 4 to Fig. 9.

Continuously, to have a general estimation for the influence of these parameters on the dynamic response of the bridge-vehicles interaction, the maximum displacements are investigated in the follows section. Firstly, the influence of the ratio of initial velocity on the maximum vertical displacement of the bridge-vehicles interaction is studied. The effects of the ratio of initial velocity on the maximum vertical displacement of the bridge for various values of the departure position are plotted in Fig. 10. It can be commented that the ratio of initial velocity is one of the most important parameters effected directly strong on the dynamic response of the bridge, with an increases of its increase moving velocity of the second vehicle, and then it increases both influences of vehicles caused more increasing the dynamic response of the bridge than in the case having the only vehicle. At the same time, the increase of dynamic response of the bridge is evident when the departure position as the second important parameter is near the left bridge support meaning the second vehicle will begin almost immediately after the first vehicle moved into the bridge and then it increases both influences of vehicles, shown in Fig. 11.



**Fig. 10:** The influence of the ratio of initial velocity on the maximum vertical displacement of the bridge: (a)  $x_0=0$ , (b)  $x_0=L/4$ , (c)  $x_0=L/2$ , (d)  $x_0=3L/4$



**Fig. 11:** The influence of the departure position on the maximum vertical displacement of the bridge: (a)  $\kappa=0.25$ , (b)  $\kappa=0.75$ , (c)  $\kappa=1$ , (d)  $\kappa=2$

It can be seen that the maximum results are also similar to the above results of the time dynamic response. Both the above parameters affect and cause increase the dynamic response of the bridge-vehicles interaction. The increase is clearly in the range of the ratios of initial velocity less than or equal 1. Especially, when as the initial velocity of two vehicles does not have much unequal and both the departure of the vehicle is the same time as loading resonance, the increase of the dynamic response of

the bridge-vehicles interaction is the clearest, as shown in Fig. 10 and Fig. 11.

Additionally, the influence of the ratio of initial velocity and the departure position on the maximum vertical displacement of the car body are also studied, as shown in Fig. 12 to Fig. 15. The numerical results also show that these parameters affect clearly the dynamic response of the vehicles. Especially, when the phenomenon of loading resonance occurs as both the vehicles move to the midpoint of the bridge at the same time, the dynamic response of the

vehicle is also more increasing evidently than others. It can be seen that the maximum results are also similar to the above results of the dynamic response of the bridge-vehicles interaction. Both the above

parameters cause to increase the dynamic response of the bridge-vehicles interaction. The increase is the clearest in the range of low moving velocity, as shown in Fig. 12 to Fig. 15.

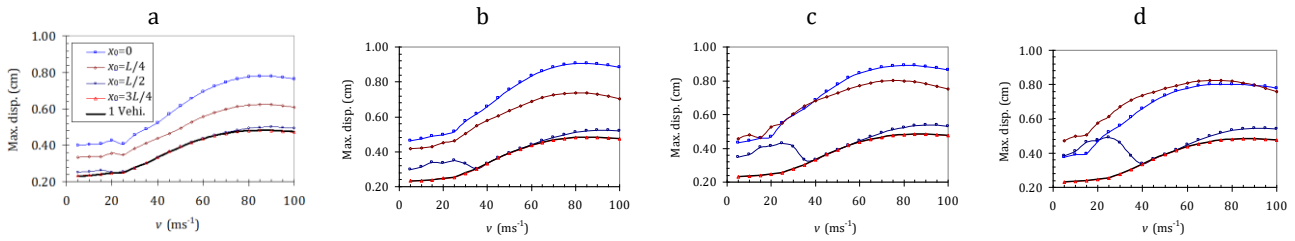


Fig. 12: The influence of the ratio of initial velocity on the maximum vertical displacement of the first car body: (a)  $\kappa=0.25$ , (b)  $\kappa=0.75$ , (c)  $\kappa=1$ , (d)  $\kappa=2$

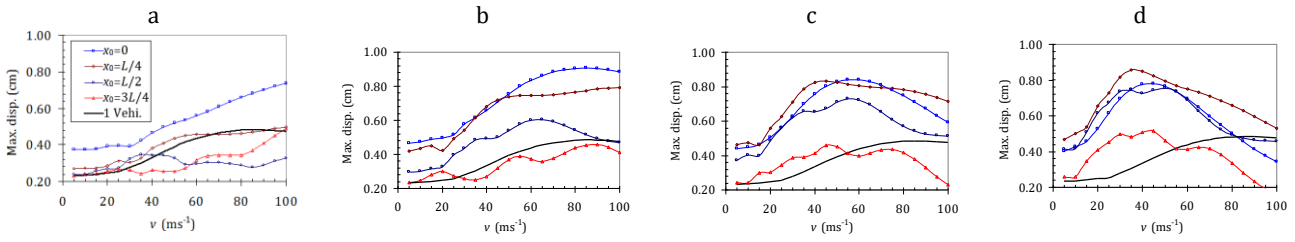


Fig. 13: The influence of the ratio of initial velocity on the maximum vertical displacement of the second car body: (a)  $\kappa=0.25$ , (b)  $\kappa=0.75$ , (c)  $\kappa=1$ , (d)  $\kappa=2$

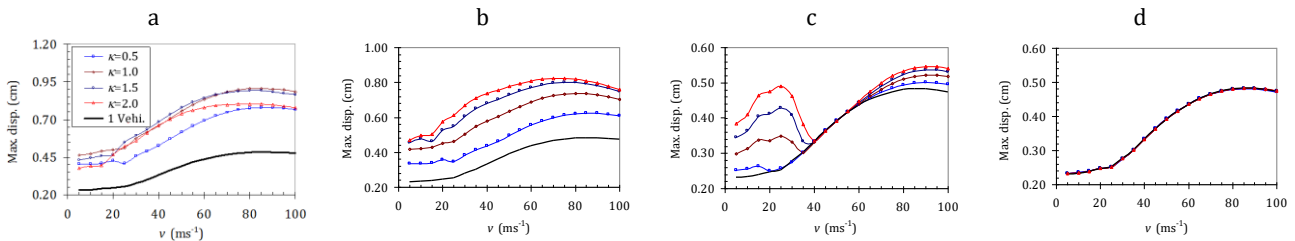


Fig. 14: The influence of the departure position on the maximum vertical displacement of the first car body. (a)  $x_0=0$ , (b)  $x_0=L/4$ , (c)  $x_0=L/2$ , (d)  $x_0=3L/4$

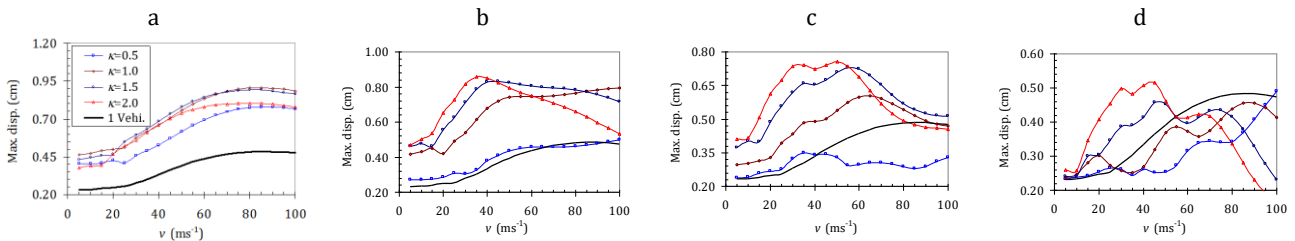


Fig. 15: The influence of the departure position on the maximum vertical displacement of the second car body. (a)  $x_0=0$ , (b)  $x_0=L/4$ , (c)  $x_0=L/2$ , (d)  $x_0=3L/4$

### 5. Conclusion

Based on the formulation and investigation results on the dynamic response of the bridge subjected to moving vehicles in both along the opposite direction using finite element method, some conclusions are drawn as follows:

- The characteristic parameters of the moving vehicles in both along the opposite direction such as the ratio of initial velocity and departure position as two important parameters are proposed for analyzing the dynamic response of the bridge-vehicles interaction.
- The investigation results showed that the ratio of initial velocity affects significantly on the dynamic

response of the bridge-vehicles interaction. It is more increasing the dynamic response of the bridge-vehicle interactions with increases of its  $\kappa$  than in the case having the only vehicle.

- Additionally, the departure position also affects evidently on the dynamic response of the bridge-vehicles interaction. When the departure position is near the left bridge support as mean as the second vehicle will begin almost immediately after the first vehicle moved into the bridge, the dynamic response of the bridge-vehicles interaction is also more increasing significantly than other cases.
- It can be seen that the influence of these parameters on the dynamic response of the bridge-vehicles interaction is significant. It is evidently more increasing the dynamic response of the

bridge-vehicles interaction than other cases. Hence, this study can be considered as meaningful practice document for analyzing dynamic response of the bridge due to the moving vehicles in both along the opposite direction as the real bridge model.

### Compliance with ethical standards

### Conflict of interest

The authors declare that they have no conflict of interest.

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