The effectiveness of improved tuned liquid column damper on the dynamic response of the structure under earthquake excitations

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ABSTRACT

The effectiveness of the improve Tuned Liquid Column Damper (iTLCD), as known as one of liquid damper types, which overcomes the practical difficulties involved in the implementation of the conventional TLCD for short period structures by tuning the frequency of the damper to the structural frequency, is studied for reducing the dynamic response of the structure due to earthquake excitations in this paper. The formulations of the iTLCD connected by a spring to the short period stiff structure due to seismic excitation are established based on the TLCD. And then, the characteristic parameters of the iTLCD are chosen based on the optimal ratios in each studying case. Therefore, the effectiveness of the iTLCD is investigated in three cases of earthquake excitations with and without the damper. The results show that the iTLCD affects significantly on the dynamic response of the structure under seismic excitations, and then it is more decreasing the dynamic response than without the damper. However, the effectiveness of the iTLCD depends on the dynamic character of the structure, excitation loads, and the characteristic parameters of the iTLCD. Hence, in the practical design problem, the property parameters of the structure will be fixed, and then the characteristic parameters of the iTLCD will be chosen based on the optimal ratio to have the best performance for mitigating the dynamic response of the structure under dynamic excitations.

1. Introduction

Studying new types of cost-efficient dampers for mitigating dynamic response of structures due to dynamic load-induced vibrations such as wind or earthquake excitation have been always attracted many attentions in many past decades. Therefore, some auxiliary damping devices were proposed for structural vibration control under different dynamic loading conditions. And, Tuned Mass Damper (TMD) (Guo and Chen, 2007; Han and Li, 2006; Love and Tait, 2015; Sun et al., 2014; Lewandowski and Grzymisławska, 2009; Varadarajan and Nagarajaiah, 2004) consisting of a mass attached to the building through a spring and a dashpot is known as the most common of auxiliary damping devices. Besides, it is well known that Tuned Liquid Damper (TLD) is a type of Tuned Mass Damper (TMD) where the mass

is replaced by a liquid (Cassolato et al., 2011; Felix et al., 2005; Fujino et al., 1992; Kim et al., 2006; Li et al., 2004; Maravani and Hamed, 2011; Nguyen et al., 2018; Wu et al., 2008), and Tuned Liquid Column Damper (TLCD) is a special type of TLD consisting of tube-like containers filled with liquid (usually water) where energy is dissipated by the movement of the liquid through an orifice to counteract the forces acting on the structure. Therefore, the TLCD is also considered as one of the types of passive energy-absorbing devices and there are even more advantaged than TMD such as low cost and maintenance. Especially, it can be utilized for water supply in building, unlike a TMD where the dead weight of the mass has no other functional use. Hence, the TLCD has been suggested for vibration control of structures under different dynamic loading conditions such as wind or earthquake excitation.

Although, almost studying indicate that the TLCD having significant practical advantages is clearly effective in reducing the dynamic response of structures due to dynamic loading conditions corresponding to the properly selected parameters of its (Shum and Xu, 2002; Wu et al., 2005; Xue et al.,

2000; Yalla et al., 2001) but the TLCD is still difficult in obtaining a feasible length of the column tube for tuning the TLCD to a short period structure. Hence, an improved TLCD (iTLCD) connected by a spring to short period stiff structures subjected to seismic excitations was proposed by Ghosh and Basu (2004). The proposed iTLCD can overcome the practical difficulties involved in the implementation of the conventional TLCD for short period structures by tuning the frequency of the damper (including the container mass and the mass of the liquid) to the structural frequency. Hence, in order to demonstrate clearly adaptive of the iTLCD with the variation of dynamic loading conditions such as earthquake excitations, the effectiveness of the iTLCD on the dynamic response of the structures due to ground motion is investigated in detail. The analysis results including displacement, velocity, acceleration and shear force of story are computed in two cases: without iTLCD and the improved iTLCD due to various earthquake excitations such as El-Centro, Northridge, and Superstition earthquake excitation based on the detail of formulations in the following sections.

2. Formulation and control algorithm

2.1. Modeling of the SDOF-iTLCD system

Considering the model of the structure–sTLCD system due to earthquake excitation is shown in Fig. 1. The iTLCD is composed of a tube-like container of arbitrary configuration with an orifice(s) installed in it. The tube has a cross-sectional area A, horizontal dimension B, the total length of the liquid column in the tube L and density of the liquid the ρ. The coefficient of head loss, controlled by the opening ratio of the orifice(s), is denoted by ξ The time-varying damping and stiffness coefficients of the spring connecting the iTLCD to the primary system are denoted by c_2 and k_2, respectively. The total mass of the damper system including the mass of the container of iTLCD denoted by M_c and the mass of column liquid may be expressed as $(\rho AL + M_c)$ (Ghosh and Basu, 2004; Sonmez et al., 2016). The ground acceleration at the time (t) is denoted by $\ddot{z}(t)$. The mass, damping, and stiffness parameters of the primary structure system as single degrees of freedom (SDOF) are denoted by m, c, and k, respectively. Based on the formulation of the iTLCD given in detail by Ghosh and Basu (2004), the equivalent linear equation of motion of the iTLCD can be written as:

$$\ddot{u}(t) + 2\frac{C_p}{L}\dot{u}(t) + \omega_L^2 u(t) = -\alpha\{\ddot{y} + \ddot{x} + \ddot{z}(t)\} \quad (1)$$

where $\omega_L = \sqrt{2g/L}$ denotes the natural frequency of the iTLCD, $\alpha = B/L$ is the ratio of the horizontal length to the total length, and C_p represents the equivalent linearized damping coefficient, given below

$$C_p = \frac{\sigma_{\dot{u}}\xi}{\sqrt{2\pi}} \quad (2)$$

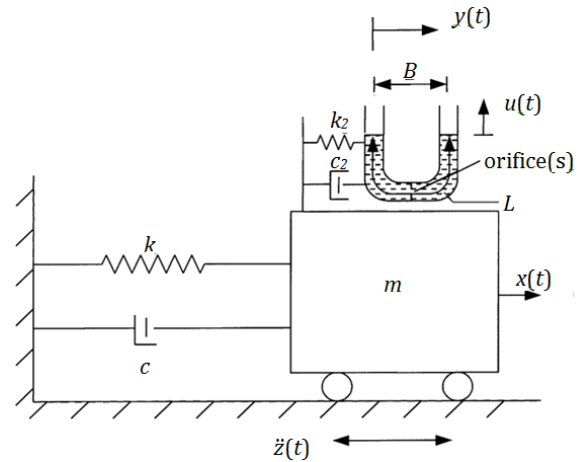which $\sigma_{\dot{u}}$ is the standard deviation of $\dot{u}(t)$.

Fig. 1: The semi-active Tuned Liquid Column Damper model

The normalized equation of motion for the damper system can be also expressed as follows

$$\{\ddot{y} + \ddot{x} + \ddot{z}(t)\} + \frac{\alpha}{1+\tau}\ddot{u}(t) + 2\zeta_2\omega_2\dot{y}(t) + \omega_2^2 y(t) = 0 \quad (3)$$

where $\tau = M_c/\rho AL$ denotes the ratio of the container mass to liquid mass, $\omega_2 = \sqrt{k_2/(\rho AL + M_c)}$ and $\zeta_2 = c_2/2\omega_2(\rho AL + M_c)$ denote the natural frequency and damping ratio of the damper system, respectively.

The normalized equation of motion for the primary structure is given by

$$\{\ddot{x} + \ddot{z}(t)\} + 2\zeta_1\omega_1\dot{x}(t) + \omega_1^2 x(t) = \mu\{2\zeta_2\omega_2\dot{y}(t) + \omega_2^2 y(t)\} \quad (4)$$

where $\mu = (\rho AL + M_c)/m$ is the ratio of the total mass of damper to the mass of the primary structure system, ζ_1 and ω_1 denote the damping ratio and natural frequency of the primary structure, respectively.

Continuously, the governing equation of motion of the whole SDOF-iTLCD system at the time (t) due to earthquake excitation can be rewritten in matrix form as follows (Sonmez et al., 2016)

$$\begin{bmatrix} m & 0 & 0 \\ \rho AL + M_c & \rho AL + M_c & \rho AB \\ \rho AB & \rho AB & \rho AL \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{u} \end{Bmatrix} +$$
$$\begin{bmatrix} c & -c_2 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & 2\rho AC_p \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{u} \end{Bmatrix} + \begin{bmatrix} k & -k_2 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & 2\rho Ag \end{bmatrix}\begin{Bmatrix} x \\ y \\ u \end{Bmatrix} =$$
$$\begin{Bmatrix} -m \\ -(\rho AL + M_c) \\ -\alpha AL \end{Bmatrix}\ddot{z}(t) \quad (5)$$

It can be seen that the selection of parameters such as the frequency of the whole damper system ω_2, the frequency of the iTLCD ω_L, and the coefficient of orifice damping of the iTLCD ξ (in the Eq .2) have an important role for choosing design parameters of the iTLCD which effect significantly on the performance of the iTLCD. Therefore, the optimum

tuning ratio $\gamma_{opt} = 1/(1 + \mu)$ and the optimum value of ξ/L proposed by Ghosh and Basu (2004) based on the minimization of the displacement of an undamped structure are used in this study. In case of the optimum values are obtained, the characteristic parameters of iTLCD can be determined.

2.2. Governing equation

Based on the formulation of the modeling of SDOF-iTLCD system, a primary structural system as multiple degrees of freedom (MDOF) due to earthquake excitation is considered, as shown in Fig. 2. The mass, damping, and stiffness parameters of the *ith* story are denoted by m_i, c_i, and k_i, respectively. The iTLCD is attached at the top floor of the primary structure.

By meaning finite element method, the governing equation of motion of the whole MDOF-iTLCD system under earthquake excitation at the time (t) can be expressed as follows (Sonmez et al., 2016)

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} & \mathbf{0} \\ \{0 \ldots \rho AL + M_c\} & \rho AL + M_c & \rho AB \\ \{0 \ldots \rho AB\} & \rho AB & \rho AL \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{u} \end{Bmatrix} +$$

$$\begin{bmatrix} \mathbf{C} & \begin{Bmatrix} 0 \\ \vdots \\ -c_2 \end{Bmatrix} & \mathbf{0} \\ \mathbf{0}^T & c_2 & 0 \\ \mathbf{0}^T & 0 & 2\rho AC_p(t) \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{u} \end{Bmatrix} + \begin{bmatrix} \mathbf{K} & \begin{Bmatrix} 0 \\ \vdots \\ -k_2 \end{Bmatrix} & \mathbf{0} \\ \mathbf{0}^T & k_2 & 0 \\ \mathbf{0}^T & 0 & 2\rho Ag \end{bmatrix} \begin{Bmatrix} x \\ y \\ u \end{Bmatrix} =$$

$$\begin{Bmatrix} -\mathbf{M1} \\ -(\rho AL + M_c) \\ -\alpha AL \end{Bmatrix} \ddot{z}(t) \qquad (6)$$

where, **0** and **1** are the column vectors of N (number story of the primary structure corresponding with number degree of freedom) zeros or ones.
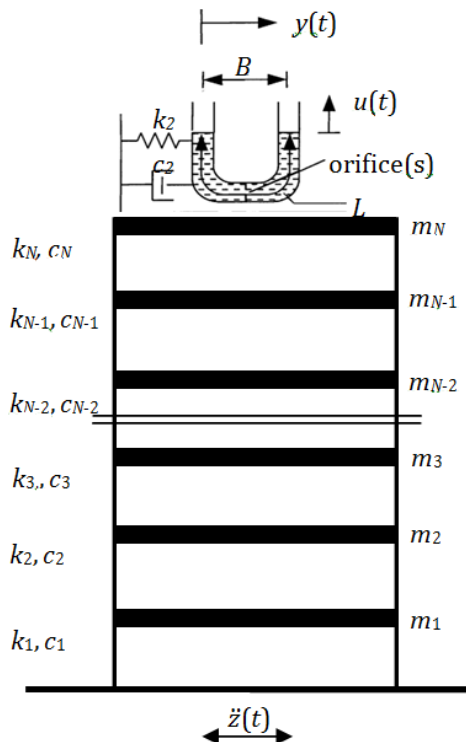

Fig. 2: The MDOF-iTLCD system due to earthquake excitation

However, the foregoing equation can be also rewritten in the general form of the governing equation of motion of the dynamic problem at the time (t), given by

$$\mathbf{M}\ddot{u} + \mathbf{C}\dot{u} + \mathbf{K}u = F \qquad (7)$$

where, matrices M and K denote the global mass and stiffness matrices of the whole MDOF-iTLCD system, respectively; F is the global force vector and C denotes the global damping matrix determined based on Rayleigh damping. It can be seen that the characteristic time-varying parameters of the iTLCD will be easily determined at the time (t) and those parameters will be also assembled in the global governing equation of motion of the whole MDOF-iTLCD system due to earthquake excitation which is solved by Newmark's method. And then, the effectiveness of the iTLCD on the dynamic response of the structure under earthquake excitations will be analyzed in detail in the next section.

3. Numerical results

3.1. The parameters of the MDOF-iTLCD system

A real building is considered for analyzing the effectiveness of the siTLCD on the dynamic response of the structure due to the earthquake excitations. The property parameters of the structure are given in Table 1, having the first dominant frequency 0.58Hz. The damping ratio for the first two modes of the structure is taken 5%.

Table 1: The property parameters of the structure

| Story	Mass [kg]	Stiffness [N/m]
1	2.83×10^5	3.31×10^8
2-4	2.76×10^5	1.06×10^9
5-7	2.76×10^5	6.79×10^8
8-10	2.76×10^5	6.79×10^8
11-13	2.76×10^5	5.84×10^8
14-16	2.76×10^5	3.86×10^8
17-19	2.76×10^5	3.47×10^8
20	2.92×10^5	2.29×10^8

The parameters of the iTLCD are also given such as the ratio of the total mass of damper to the structure is taken as $\mu = 0.02$, the ratio of the horizontal length to the total length of the liquid column tube is set to $\alpha = 0.9$, the tuning ratio of the damper system is selected as $\gamma_{opt} = 1/(1 + \mu)$ for iTLCD (for the parametric study), the liquid column mass is assumed to be same as the LCD container mass $\tau = 1$, and the coefficient of head loss is given as ξ. These parameter will be a parametric study for analysis the performance of iTLCD on the dynamic response of the structure under earthquake excitations in the following sections.

3.2. The earthquake excitations

Considering three various earthquake excitations including El-Centro, Superstition, and Northridge earthquake which the time history of ground acceleration are presented in Fig. 3, having the peak

of dominant frequencies from 1.8Hz to 2.6Hz, 0.48Hz to 2Hz, and 0.47 to 2.4Hz, respectively.
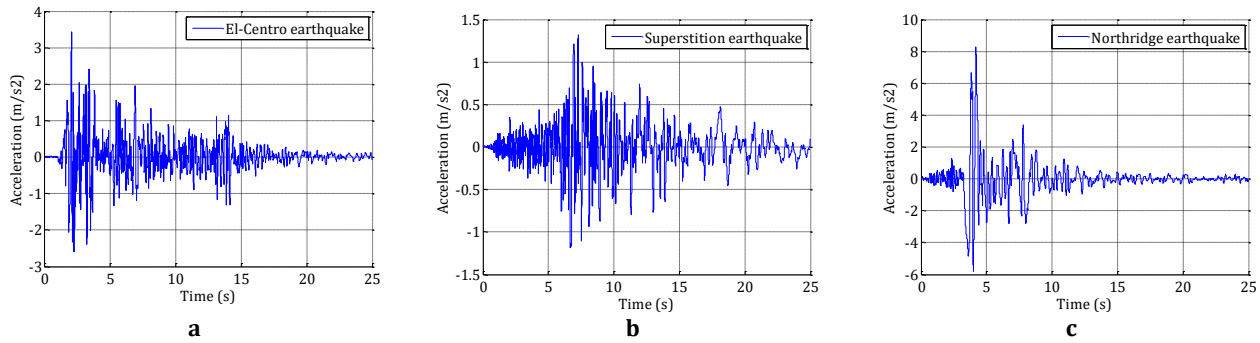
Fig. 3: Ground acceleration: a) El-Centro earthquake, b) Superstition earthquake, c) Northridge earthquake

3.3. Effect of the ratio ξ/L

In this section, the influence of the ratio ξ/L of the iTLCD on the displacement response of the structure due to the above various earthquake excitations has been studied. The results are presented from Figs. 4-6. It can be seen that with an increase of the ratio ξ/L decreases the dynamic resposne of the structure, though the rate of decrease is very gradual. However, for each case, the influence of the ratio ξ/L on the displacement response of the structure is different; it depends on the dynamic characteristic of earthquake excitaion. But, it can be also seen that with the change in the ratio ξ/L demonstrates the influence of the relative motion of the liquid column and the orifice damping on the performance of it.
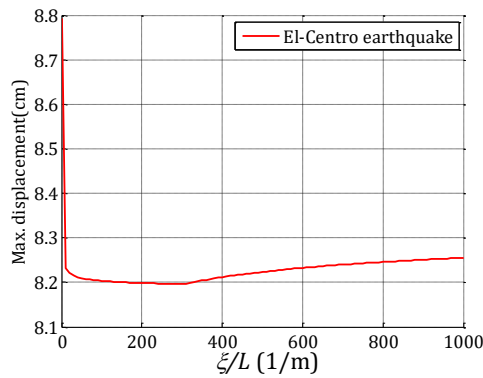
Fig. 4: The influence of the ratio ξ/L on the maximum displacement of the structure due to El-Centro earthquake
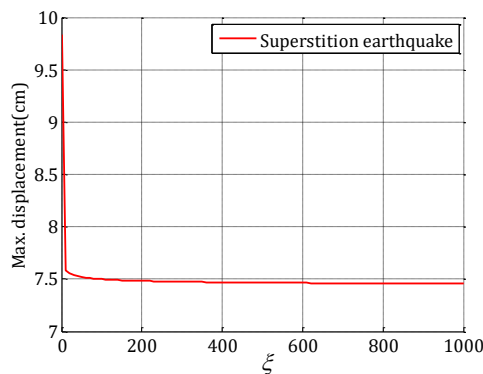
Fig. 5: The influence of the ratio ξ/L on the maximum displacement of the structure due to Superstition earthquake
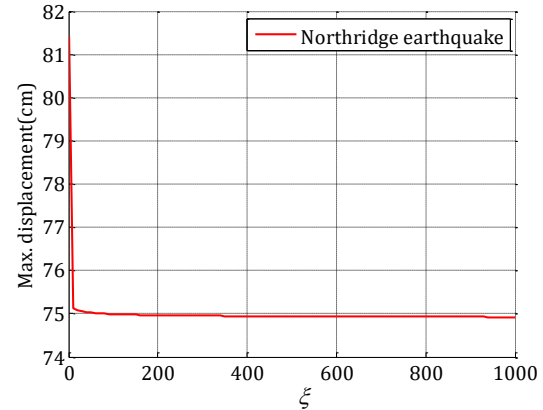
Fig. 6: The influence of the ratio ξ/L on the maximum displacement of the structure due to Northridge earthquake

Additionally, the curve results are also useful for practical design. The response reduction of the structure due to the incorporation of the damper can be completely estimated for a given ratio of the ratio ξ/L, which may have been fixed from other practical considerations. Another way, the investigated results also indicated that the whole damper system has been kept tuned to the ratio frequency as TMD, and then the whole damper was more increasing the dynamic response of the the sturcture than without the damper in the structure due to earthquake excitations.

3.4. Effect of the mass ratio μ

Continuously, the ratio of the total mass of damper to the structure μ as one of the important parameters for characteristic performance of the iTLCD which effects directly and strongly on the dynamic response of the structure will be also studied in this section. The results are presented from Figs. 7-9.

It can be seen that the mass ratio μ is very sensitive to the effectiveness of the iTLCD. The influence of the mass ratio μ on dynamic response reduction of the structure is not according to a similar law. In the El-Centro earthquake case, with an increase of the mass ratio μ will decrease strongly the dynamic response of the structure, shown in Fig. 7. However, also with those will cause significantly
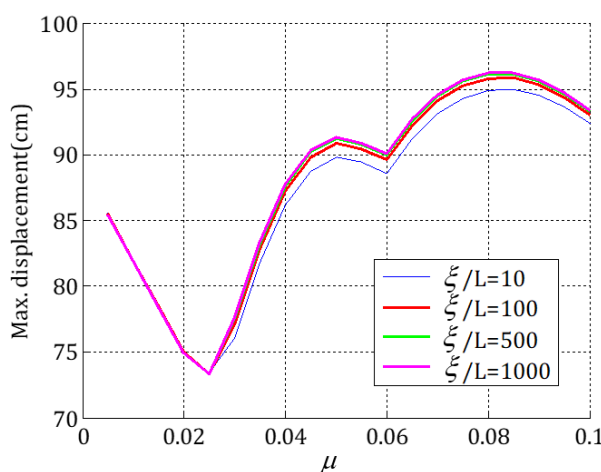
increase the general mass of the whole structure, and then it will also increase the complication of the structural foundation.
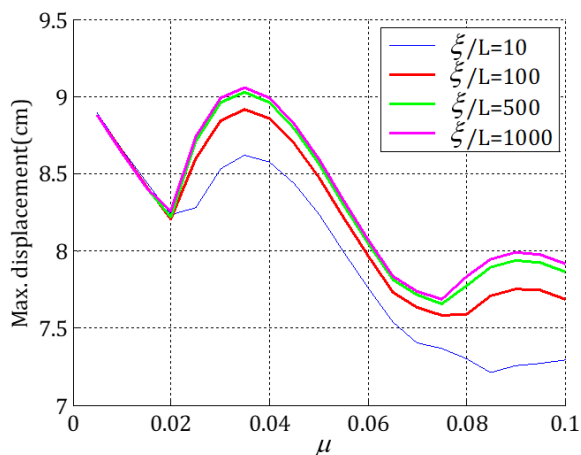
Fig. 7: The influence of the ratio μ on the maximum displacement of the structure due to El-Centro earthquake
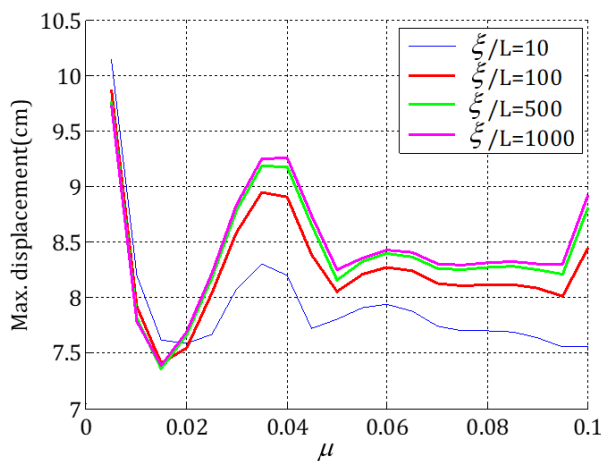
Fig. 8: The influence of the ratio μ on the maximum displacement of the structure due to Superstition earthquake

Fig. 9: The influence of the ratio μ on the maximum displacement of the structure due to Northridge earthquake

Oppositely, with an increase of the mass ratio μ causes an increase strongly the dynamic response of the structure in remain cases, and even it is more unfavourable than without the iTLCD. But, a general view can be focused that the mass ratio is the second important parameter for choosing a design parameter of the iTLCD. For each loading excitation, there will be an optimal mass ratio but in a arrange from 0.015 to 0.025, and then the effectiveness of the iTLCD is clearest for both the dynamic response reduction and an increase of the general mass of the whole structure, presented from Figs. 7-9. In detail, the optimal mass ratio μ is 0.02 in the El-Centro, 0.015 in the Superstition and 0.025 in the Northridge, respectively.

3.5. Performance of the iTLCD on the dynamic response structure

It can be seen that each design parameter of the iTLCD will give different performance. Hence, choosing the design parameters of the iTLCD under each dynamic excitation load is extremely important for practical structure. Therefore, the property parameters of the structure are fixed, and then the characteristic parameters of the iTLCD will be obtained based on the optimal parameters of the above investigation cases for studying the effectiveness of the iTLCD on the dynamic response of the structure due to earthquake excitation. But, to study adaptive of the iTLCD under different earthquake excitations, the parameters of the iTLCD are taken the ratio $\xi/L = 100$ and the mass ratio $\mu = 0.02$. And then, the investigation results for the effectiveness of the iTLC on the dynamic response of the structure are presented from Figs. 10-14.

It can be seen that the iTLCD changed the dynamic character of the whole structure under earthquake excitations base on tuning the ratio frequency. Therefore, it decreases the time history of displacement of the structure in all investigated cases, as shown in Fig. 10.

Additionally, a decrease of the time history of displacement of the structure is also the reason which is more decreasing the maximum displacement of the structure due to earthquake excitations than without the damper, plotted in Fig. 11.

Moreover, one of the problems which are necessary attention for the safety of the structure due to a dynamic load is a shear force at each story. With the increase of the shear force of the story will be the reason that increases ability destroys of the structure. Hence, in the problem of structural control, proposing the damper to control the dynamic response of the structure for decreasing internal force to reduce ability damage or destruction of the structure is extremely meaningful and valuable.

In this study shows that the iTLCD has significant effectiveness for reducing the internal force of the structure under earthquake excitations, as shown in Fig. 12 and Fig. 13. It can be seen that the iTLCD is more decreasing the time history and the maximum of shear force of the structure than in the case without the damper. The percent decrease is not

similar to in each case, it depends on the dynamic characteristic of each earthquake excitation, such as the maximum percent of the El-Centro earthquake is about 8 percent and the Northridge earthquake also take about 15 percent. But, in the Superstition earthquake is significant, it goes to 60 percent, as shown in Fig. 14.
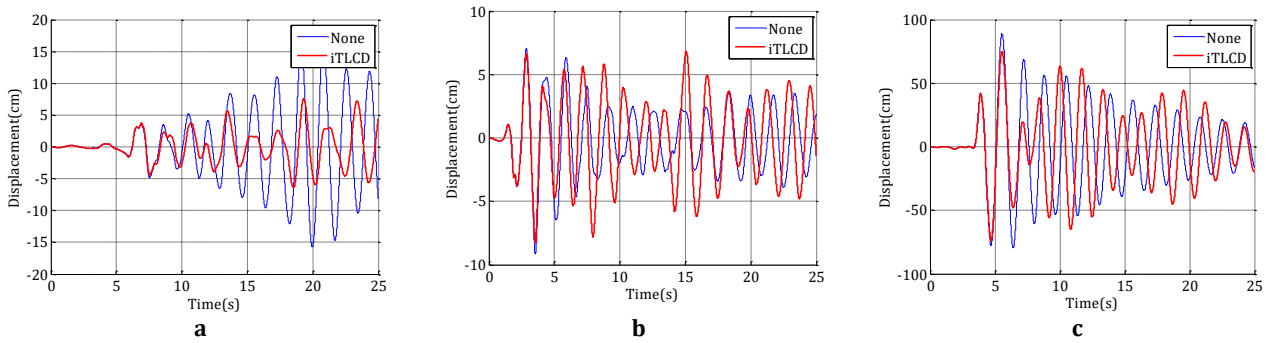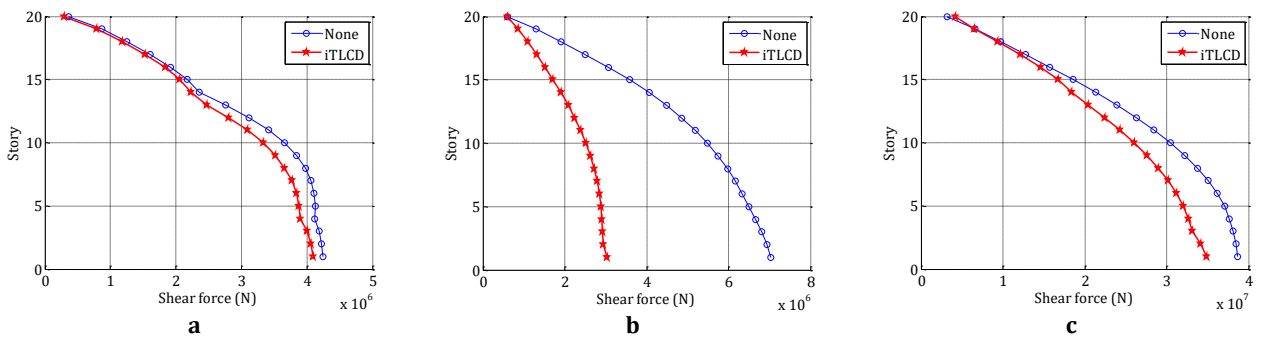

Fig. 10: The time history of displacement at the top floor due to earthquake: (a) El-Centro, (b) Superstition, (c) Northridge
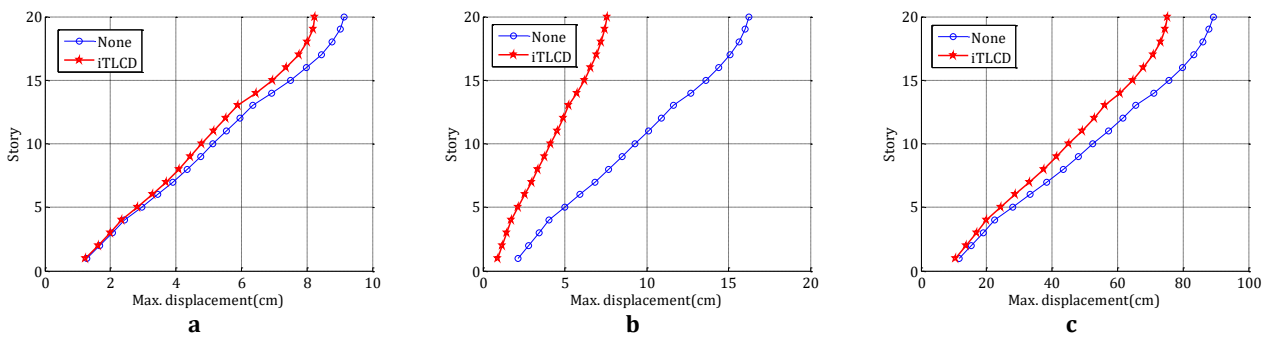
Fig. 11: The maximum displacement at each floor due to earthquake: (a) El-Centro, (b) Superstition, (c) Northridge
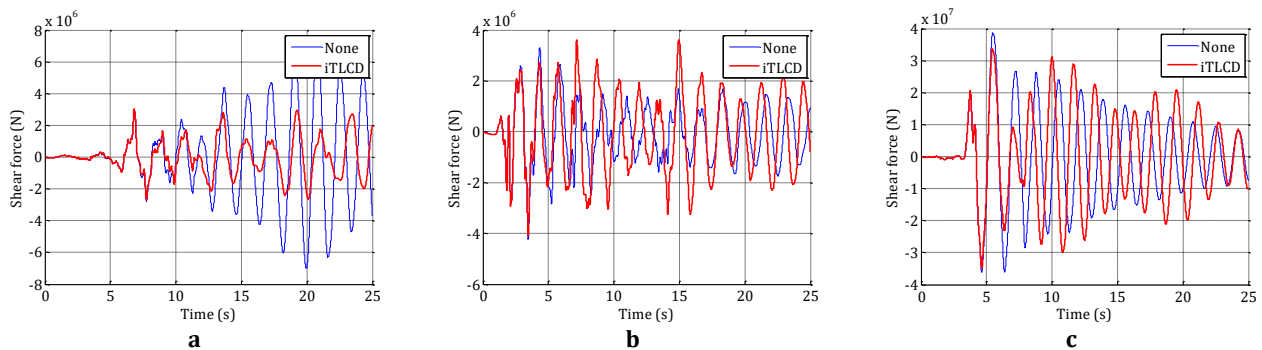
Fig. 12: The time history of shear force of the first story due to earthquake: (a) El-Centro, (b) Superstition, (c) Northridge

Fig. 13: The maximum shear force of each story due to earthquake: (a) El-Centro, (b) Superstition, (c) Northridge

The results indicated that the performance of the iTLCD is the clearest when the dominant frequencies between the earthquake excitations and the structure are approximate. Hence, the effectiveness of the iTCLD is the clearest in the Superstition earthquake case (ratio frequencies from 0.8 to 3.4) and the lowest in the El-Centro earthquake case (ratio frequencies from 3.1 to 4.5), as shown in Fig. 14. It can be seen that the iTLCD attached to the structure had a significant effect on the dynamic

response of the structure due to earthquake excitations. It decreases the time history of dynamic response of the structure, and then it is more decreasing the dynamic response than without the damper. Therefore, the iTLCD decreases the internal force in the structure which is extremely useful to protect or reduce damage or destruction of the structure under dynamic loads. The effectiveness of the iTLCD depends on both the dynamic character of

the structure and excitation loads. Moreover, it also depends on the characteristic parameters of the iTLCD.

Hence, in the practical design problem, the property parameters of the structure will be fixed, and then the characteristic parameters of the iTLCD will be chosen based on the optimal ratio to bring the best performance for mitigating the dynamic response of the structure under dynamic excitations.
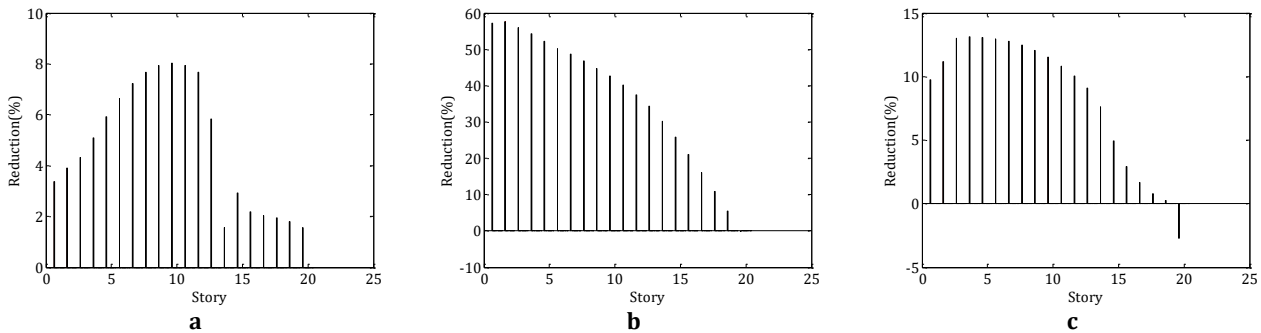
Fig. 14: The shear force reduction of the structure due to earthquake: (a) El-Centro, (b) Superstition, (c) Northridge

4. Conclusion

Based on the formulations and numerical investigation results for the effectiveness of the iTLCD on the reduction of the dynamic response of the structure due to earthquake excitations, the following conclusions can be drawn as follows:

- The property parameters of the iTLCD affect significantly on the dynamic response of the structure. It changes the dynamic character of the whole structure based on tuning the frequency ratio. Therefore, the iTLCD is more decreasing the dynamic response of the structure than without the damper.
- The effectiveness of the iTLCD depends on both the dynamic character of the structure and the excitation load. Additionally, it also depends on the property parameters of the iTLCD. Hence, the property parameters of the structure will be fixed, and then the characteristic parameters of the iTLCD will be obtained based on the optimal ratios to give the best performance of the iTLCD for reducing the dynamic response of the structure.
- It can be seen that the iTLCD attached to the structure had a significant effect on the reduction of the dynamic response of the structure due to earthquake excitations. Hence, this study has meaning practice in the problem of vibration control of the structure due to earthquake excitations.

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Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

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