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Derivation of simplified model governing behavior of Mindlin plate with elastic support traversed by partially distributed moving load

Michael C. Agarana [1, 2, *], Esther T. Akinlabi [2], Olasunmbo O. Agboola [1]

[1]Department of Mathematics, Covenant University, Ota, Ogun State, Nigeria
[2]Department of Mechanical Engineering Science, University of Johannesburg, Johannesburg, South Africa

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ABSTRACT

The practical importance of dynamic response of elements of structures such as plates when load moves on them cannot be overemphasized in both engineering and applied sciences. The dynamic behavior of an elastic plate resting on a subgrade and traversed by uniform partially distributed moving load is considered in this paper and its simplified governing equations derived. The elastic plate is Mindlin rectangular plate. In particular, the model governing such moving load problem is simplified analytically. The simplified governing model derived is easier to handle. Numerical methods can easily be applied to this simplified model and a lot of computational time is saved.

1. Introduction

The investigation of moving load issue is by and large of reasonable significance in Engineering and Applied Sciences. Such reviews are significant while considering the unwavering quality, wellbeing and execution of present day structures over which loads like vehicles and train move (Gbadeyan and Agarana, 2014; Mindlin, 1951). The arrangement of such moving load issue under thought, requests the displaying of the mechanical conduct of the soil as flexible subgrade, and the type of collaboration between the plate and the soil. In this sort of framework, it is important to couple practical models of the foundation with investigation of the structure. A few foundation models have been accounted for in the writing and examinations on the static deflection; the dynamic reaction and the dynamic stability of plates on elastic foundation have been completed. Many researchers use the Winkler model for soil structure interaction in the static and dynamic analyses of plate resting on elastic foundation where the vertical surface displacement of the plate is assumed to be proportional at any point to the contact pressure at that point (Gbadeyan and Agarana, 2014; Civalek and Yavas, 2006; Nguyen-Thoi et al., 2013; Agarana et al., 2015). In Winkler model, it is accepted that the foundation soil comprises of straight flexible springs which are firmly separated and autonomous of each other (Agarana et al., 2015; Amiri et al., 2013; Kerr, 1964). A few types of administering conditions of the vibration of Mindlin plate under a moving load exist. In this paper endeavor is made to rearrange such existing representing condition to be anything but difficult to deal with by decreasing the computational meticulousness and time. In this rearrangement, both the inertia and the gravitational impacts of the moving load are taken into account (Mindlin, 1951; Boay, 1993; Fryba, 1972). The following assumptions are made:

- The plate is of constant cross-section.
- The moving load moves with a constant speed.
- The moving load is guided in such a way that it keeps contact with the plate throughout the motion.
- The plate is continuously supported by a Winkler foundation.
- The moving load is a partially distributed moving load.
- The rectangular Mindlin plate is elastic.

2. The governing equations

The set of dynamic equilibrium equations which govern the behaviour of Mindlin plate with elastic support and traversed by a partially distributed moving load can be written as (Gbadeyan and

* Corresponding Author.
Email Address: michael.agarana@covenantuniversity.edu.ng (M. C. Agarana)
https://doi.org/10.21833/ijaas.2019.03.004
Corresponding author's ORCID profile:
https://orcid.org/0000-0003-2100-8282

Agarana, 2014; Gbadeyan and Dada, 2001; Agarana and Gbadeyan, 2016):

$$\frac{M_L B}{A}\left[g + \frac{\partial^2 W}{\partial T^2} + 2U\frac{\partial^2 W}{\partial x \partial T} + 2U^2\frac{\partial^2 W}{\partial x^2}\right] = k^2 Gh\left[-\frac{\partial^2 W}{\partial x^2} + \frac{\partial \psi_x}{\partial x} - \frac{\partial^2 W}{\partial y^2} + \frac{\partial \psi_y}{\partial y}\right] - KW - M_f\frac{\partial^2 W}{\partial T^2} + \rho h\frac{\partial^2 W}{\partial T^2} \quad (1)$$

$$\frac{B_{\rho L} h_1^3}{12}\left[\frac{\partial^2 \psi_x}{\partial T^2} + 2U\frac{\partial^2 \psi_x}{\partial x \partial T} + U^2\frac{\partial^2 \psi_x}{\partial x^2}\right] + \frac{\rho h^3}{12}\frac{\partial^2 \psi_x}{\partial T^2} = D\left[\frac{\partial^2 \psi_y}{\partial x^2} + v\frac{\partial^2 \psi_y}{\partial x \partial y}\right] + \frac{(1-v)}{2}D\left[\frac{\partial^2 \psi_x}{\partial y^2} + v\frac{\partial^2 \psi_y}{\partial x \partial y}\right] - k^2 Gh\left(\psi_x - \frac{\partial W}{\partial x}\right) \quad (2)$$

$$\frac{B_{\rho L} h_1^3}{12}\left[\frac{\partial^2 \psi_y}{\partial T^2} + 2U\frac{\partial^2 \psi_y}{\partial y \partial T} + U^2\frac{\partial^2 \psi_y}{\partial y^2}\right] + \frac{\rho h^3}{12}\frac{\partial^2 \psi_y}{\partial T^2} = D\left[\frac{\partial^2 \psi_y}{\partial y^2} + v\frac{\partial^2 \psi_y}{\partial y \partial x}\right] + \frac{(1-v)}{2}D\left[\frac{\partial^2 \psi_x}{\partial x \partial y} + v\frac{\partial^2 \psi_y}{\partial x^2}\right] - k^2 Gh\left(\psi_y - \frac{\partial W}{\partial y}\right) \quad (3)$$

where, $\psi_x(x, y, T)$ and $\psi_y(x, y, T)$ are local rotation in the x and y directions respectively. $W(x, y, T)$ is the traversed displacement of the plate at time T. $B = B_x B_y$ such that

$$B_x = \begin{cases} 1 - H\left(x - \xi - \frac{\varepsilon}{2}\right), & 0 \le T \le \frac{\varepsilon}{U} \\ H\left(x - \xi + \frac{\varepsilon}{2}\right) - H\left(x - \xi - \frac{\varepsilon}{2}\right), & \frac{\varepsilon}{U} \le T \le \frac{L_x}{U} \\ H\left(x - \xi - \frac{\varepsilon}{2}\right), & \frac{L_x}{U} \le T < \frac{L_x + +\varepsilon}{U} \\ 0, & \frac{L_x + +\varepsilon}{U} \le T \end{cases} \quad (4)$$

$$B_y = \left\{H\left(y - y_1 + \frac{\mu}{2}\right) - H\left(y - y_1 - \frac{\mu}{2}\right)\right\} \quad (5)$$

$H(x)$ is the Heaviside function defined as:

$$H(x) = \begin{cases} 1. & x > 0 \\ 0.5, & x = 0 \\ 0, & x < 0 \end{cases} \quad (6)$$

U is the velocity of a load of rectangular dimension ε by μ with one of its lines of symmetry moving along $Y = Y_1$.
$\Lambda = \mu\varepsilon$, the area of the load in contact with the plate
The plate is L_x by L_y in dimensions and $= UT + \frac{\varepsilon}{2}$,
h and h_1 are thickness of the plate and load respectively.
ρ and ρ_L are the densities of the plate and load respectively.
G is the modulus of the plate.
D is the flexural rigidity of the plate defined by:

$$D = \frac{1}{2}Eh^2[(1 - v^3)] = Gh^3/6(1 - v) \quad (7)$$

k^2 is the shear correction factor.
v is the poisson's ratio of the plate.
g is the acceleration due to gravity
E is Young modulus of elasticity
M_L is mass of the load.

2.1. Boundary and initial conditions

For a complete formulation of the problem, a simply supported rectangular Mindlin plate is considered as an illustrative example. If the edge $y = 0$ of the plate is simply supported, it then follows that the deflection W along this edge must be zero. At the same time this edge can rotate freely with respect to the $x -$ axis, i.e., there are no bending (M_x) along this edge. Therefore, the boundary conditions can be stated as follows (Gbadeyan and Dada, 2001; Nguyen-Thoi et al., 2013; Amiri et al., 2013; Agarana et al., 2016):

$$\left.\begin{aligned} W(x, y, T) = M_x(x, y, T) = 0, \text{for } x = 0 \text{ and } x = a \\ W(x, y, T) = M_y(x, y, T) = 0, \text{for } y = 0 \text{ and } y = b \end{aligned}\right\} \quad (8)$$

the corresponding initial conditions are

$$W(x, y, T) = 0 = \frac{\partial W}{\partial T}(x, y, 0) \quad (9)$$

3. Simplification of governing equations

The acceleration $\frac{d^2 W}{dT^2}$ is defined as follows (Agarana et al., 2016; Agarana and Gbadeyan, 2016; Agarana and Emetere, 2016):

$$\frac{d^2 W}{dT^2} = \frac{\partial^2 W}{\partial x^2}\left(\frac{dx}{dT}\right)^2 + \frac{\partial^2 W}{\partial y^2}\left(\frac{dy}{dT}\right)^2 + \frac{\partial^2 W}{\partial T^2} + 2\frac{\partial^2 W}{\partial x \partial y}\frac{dx}{dT}\frac{dy}{dT} + 2\frac{\partial^2 W}{\partial x \partial T}\frac{dx}{dT} + 2\frac{\partial^2 W}{\partial y \partial T}\frac{dy}{dT} + \frac{\partial W}{\partial x}\frac{d^2 x}{dT^2} + \frac{\partial W}{\partial y}\frac{d^2 y}{dT^2} \quad (10)$$

assuming that $x = x(T)$ and $y = y(T)$.

Furthermore, assuming the eternal load moves along a straight line parallel to x-axis with a constant velocity U, then it follows respectively that (Agarana and Gbadeyan, 2016; Agarana and Emetere, 2016)

$$\frac{dy}{dT} = 0$$

and

$$\frac{d^2 x}{dT^2} = 0.$$

Hence, Eq. 10 becomes

$$\frac{d^2 W}{dT^2} = \frac{\partial^2 W}{\partial T^2} + 2U\frac{\partial^2 W}{\partial x \partial T} + U^2\frac{\partial^2 W}{\partial x^2}, \quad (11)$$

similarly,

$$\frac{d^2 \psi_x}{dT^2} = \frac{\partial^2 \psi_x}{\partial T^2} + 2U\frac{\partial^2 \psi_x}{\partial x \partial T} + U^2\frac{\partial^2 \psi_x}{\partial x^2} \quad (12)$$

$$\frac{d^2 \psi_y}{dT^2} = \frac{\partial^2 \psi_y}{\partial T^2} + 2U\frac{\partial^2 \psi_y}{\partial x \partial T} + U^2\frac{\partial^2 \psi_y}{\partial x^2}, \quad (13)$$

hence, the expression on the left-hand side of Eq. 1 finally reduces to

$$-\frac{1}{\mu\varepsilon}\left[-M_L g - M_L\frac{d^2 W}{dT^2}\right]B = P(x, y, T) \quad (14)$$

where $P(x, y, T)$ is the moving load.

Substituting (11), (12) and (13) into (1), (2) and (3) respectively we have

$$-\frac{1}{\mu\varepsilon}\left[-M_L g - M_L\frac{d^2 W}{dT^2}\right]B = k^2 Gh\left[-\frac{\partial^2 W}{\partial x^2} + \frac{\partial \psi_x}{\partial x} - \frac{\partial^2 W}{\partial y^2} + \frac{\partial \psi_y}{\partial x}\right] + \rho h\frac{\partial^2 W}{\partial T^2} - kW - M_f\frac{\partial^2 W}{\partial T^2} \quad (15)$$

$$\frac{B\rho_L h_1^3}{12}\left(\frac{d^2\psi_x}{dT^2}\right)+\frac{\rho h^2}{12}\frac{\partial^2\psi_x}{\partial T^2}=D\frac{\partial}{\partial x}\left[\frac{\partial\psi_x}{\partial x}+v\frac{\partial\psi_x}{\partial y}\right]+\frac{D(1-v)}{2}\frac{\partial}{\partial y}\left[\frac{\partial\psi_x}{\partial y}+\frac{\partial\psi_y}{\partial x}\right]-k^2Gh\left(\psi_x-\frac{\partial W}{\partial x}\right) \quad (16)$$

$$\frac{B\rho_L h_1^3}{12}\left(\frac{d^2\psi_x}{dT^2}\right)+\frac{\rho h^2}{12}\frac{\partial^2\psi_x}{\partial T^2}=D\frac{\partial}{\partial x}\left[\frac{\partial\psi_x}{\partial y}+v\frac{\partial\psi_x}{\partial x}\right]+\frac{D(1-v)}{2}\frac{\partial}{\partial x}\left[\frac{\partial\psi_x}{\partial y}+\frac{\partial\psi_y}{\partial x}\right]-k^2Gh\left(\psi_x-\frac{\partial W}{\partial y}\right) \quad (17)$$

The definitions for moments along x and y axes, twisting moment and shear deformation along x and y axes are given as follows respectively (Agarana and Emetere, 2016)

$$M_x=-D\left(\frac{\partial\psi_x}{\partial x}+v\frac{\partial\psi_y}{\partial y}\right) \quad (18)$$

$$M_y=-D\left(\frac{\partial\psi_x}{\partial y}+v\frac{\partial\psi_x}{\partial x}\right) \quad (19)$$

$$M_{xy}=-D\frac{(1-v)}{2}\left(\frac{\partial\psi_x}{\partial y}+\frac{\partial\psi_y}{\partial x}\right) \quad (20)$$

$$Q_x=-k^2Gh\left(\psi_x-\frac{\partial W}{\partial x}\right) \quad (21)$$

$$Q_y=-k^2Gh\left(\psi_y-\frac{\partial W}{\partial y}\right) \quad (22)$$

Substituting Eqs. 17–21 into Eqs. 14–16, the simplified set of the governing equations can now be written as

$$-P(x,y,T)=k^2Gh\left[-\frac{\partial^2W}{\partial x^2}+\frac{\partial\psi_x}{\partial x}-\frac{\partial^2W}{\partial y^2}+\frac{\partial\psi_y}{\partial y}\right]+\rho h\frac{\partial^2W}{\partial T^2}-kW-M_f\frac{\partial^2W}{\partial T^2} \quad (23)$$

$$\frac{B\rho_L h_1^3}{12}\left(\frac{d^2\psi_x}{dT^2}\right)+\frac{\rho h^2}{12}\frac{\partial^2\psi_x}{\partial T^2}=-\frac{\partial M_x}{\partial x}-\frac{\partial M_{xy}}{\partial y}-k^2Gh\left(\psi_x-\frac{\partial W}{\partial x}\right) \quad (24)$$

$$\frac{B\rho_L h_1^3}{12}\left(\frac{d^2\psi_x}{dT^2}\right)+\frac{\rho h^2}{12}\frac{\partial^2\psi_x}{\partial T^2}=-\frac{\partial M_y}{\partial y}-\frac{\partial M_{xy}}{\partial x}-k^2Gh\left(\psi_x-\frac{\partial W}{\partial y}\right). \quad (25)$$

Eq. 22 can be written as

$$-k^2Gh\left(\frac{\partial^2W}{\partial x^2}+\frac{\partial\psi_x}{\partial x}\right)+-k^2Gh\left(-\frac{\partial^2W}{\partial y^2}+\frac{\partial\psi_y}{\partial y}\right)-\rho h\frac{d^2\psi_x}{dT^2}+kW+M_f\frac{d^2W}{dT^2}=P(x,y,T) \quad (26)$$

which can be expressed as

$$\frac{\partial}{\partial x}\left[-k^2Gh\left(-\frac{\partial^2W}{\partial x^2}+\frac{\partial\psi_x}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[-k^2Gh\left(-\frac{\partial^2W}{\partial y^2}+\frac{\partial\psi_y}{\partial y}\right)\right]-\rho h\frac{\partial^2W}{\partial T^2}+kW+M_f\frac{\partial^2W}{\partial T^2}=P(x,y,T). \quad (27)$$

Now, substituting Eq. 20 and Eq. 21 into Eqs. 23, 24, and 26, we have:

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}-\rho h\frac{\partial^2W}{\partial T^2}+kW+M_f\frac{\partial^2W}{\partial T^2}=P(x,y,T) \quad (28)$$

$$\frac{B\rho_L h_1^3}{12}\left(\frac{\partial^2\psi_y}{\partial T^2}\right)+\frac{\rho h^3}{12}\frac{\partial^2W}{\partial T^2}=-\frac{\partial M_x}{\partial x}-\frac{\partial M_{xy}}{\partial y}+Q_x \quad (29)$$

$$\frac{B\rho_L h_1^3}{12}\left(\frac{\partial^2\psi_y}{\partial T^2}\right)+\frac{\rho h^3}{12}\frac{\partial^2\psi_y}{\partial T^2}=-\frac{\partial M_x}{\partial y}-\frac{\partial M_{xy}}{\partial x}+Q_y \quad (30)$$

3.1. Further simplification

Eqs. 27, 28, and 29 can further be simplified. Firstly Eq. 11 can be written as:

$$\frac{d^2W}{dT^2}=\frac{\partial^2W}{\partial T^2}+U\frac{\partial}{\partial T}\left(\frac{\partial W}{\partial x}\right)+U\frac{\partial^2W}{\partial x\partial T}+U^2\frac{\partial}{\partial x}\left(\frac{\partial W}{\partial x}\right), \quad (31)$$

but from Eq. 20

$$\frac{\partial W}{\partial x}=\psi_x-\frac{Q_x}{\alpha Gh} \quad (32)$$

which leads to

$$\frac{d^2W}{dT^2}=\frac{\partial^2W}{\partial T^2}+U\frac{\partial^2W}{\partial x\partial T}+U\left\{\frac{\partial\psi_x}{\partial T}+U\frac{\partial\psi_x}{\partial x}\right\}-\frac{U}{\alpha Gh}\left\{\frac{\partial Q_x}{\partial T}+U\frac{\partial Q_x}{\partial x}\right\} \quad (33)$$

where $=-k^2$. Solving for $\frac{\partial\psi_x}{\partial x}$ in Eq. 17 and Eq. 18 we have

$$\frac{\partial\psi_x}{\partial x}=\frac{M_x-vM_y}{D(v^2-1)}, \quad (34)$$

substituting Eq. 34 into Eq. 33 yields

$$\frac{d^2W}{dT^2}=\frac{\partial^2W}{\partial T^2}+U\frac{\partial^2W}{\partial x\partial T}+U\left\{\frac{\partial\psi_x}{\partial T}+U\frac{(M_x-vM_y)}{D(v^2-1)}\right\}-\frac{U}{\alpha Gh}\left\{\frac{\partial Q_x}{\partial T}+U\frac{\partial Q_x}{\partial x}\right\}, \quad (35)$$

similarly, we have

$$\frac{d\psi_x}{dT^2}=\frac{\partial^2\psi_x}{\partial T^2}+U\frac{\partial^2\psi_x}{\partial x\partial T}+U\frac{\partial}{\partial T}\left(\frac{\partial\psi_x}{\partial x}\right)+U^2\frac{\partial}{\partial x}\left(\frac{\partial\psi_x}{\partial x}\right)=\frac{\partial^2\psi_x}{\partial T^2}+U\frac{\partial^2\psi_x}{\partial x\partial T}+U\frac{\partial}{\partial T}\frac{(M_x-vM_y)}{D(v^2-1)}+U^2\frac{\partial}{\partial x}\frac{(M_x-vM_y)}{D(v^2-1)} \quad (36)$$

by virtue of Eq. 34. Therefore,

$$\frac{d\psi_x}{dT^2}=\frac{\partial^2\psi_x}{\partial T^2}+U\frac{\partial^2\psi_x}{\partial x\partial T}+\frac{U}{D(v^2-1)}\left\{\frac{\partial M_x}{\partial T}+U\frac{\partial M_x}{\partial T}+U\frac{\partial M_x}{\partial x}\right\}-\frac{Uv}{D(v^2-1)}\left\{\frac{\partial M_y}{\partial T}+U\frac{\partial M_y}{\partial T}\right\} \quad (37)$$

similarly, Eq. 13 reduces to

$$\frac{d\psi_y}{dT^2}=\frac{\partial^2\psi_y}{\partial T^2}+U\frac{\partial^2y}{\partial y\partial T}+\frac{U}{D(v^2-1)}\left\{\frac{\partial M_x}{\partial T}+U\frac{\partial M_y}{\partial T}+U\frac{\partial M_y}{\partial y}\right\}-\frac{Uv}{D(v^2-1)}\left\{\frac{\partial M_x}{\partial T}+U\frac{\partial M_x}{\partial y}\right\}, \quad (38)$$

recalling Eq. 14, we have

$$P(x,y,T)=\frac{1}{\mu\varepsilon}\left[-M_Lg-M_L\frac{d^2W}{dT^2}\right]B, \quad (39)$$

which reduces to

$$P(x,y,T)=\frac{M_L}{\mu\varepsilon}\left[g+\frac{d^2W}{dT^2}\right]B \quad (40)$$

$$=\frac{M_LB}{A}\left[g+\frac{d^2W}{dT^2}\right], \quad (41)$$

where

$A=\mu\varepsilon$

substituting Eq. 35 into Eq. 41 we have

$$P(x,y,T)=\frac{-M_L}{A}\left[\left\{g+\frac{\partial^2W}{\partial T^2}+U\frac{\partial^2W}{\partial x\partial T}\right\}+U\left\{\frac{\partial\psi_x}{\partial T}+U\frac{(M_x-vM_y)}{D(v^2-1)}\right\}-\frac{U}{\alpha GH}\left\{\frac{\partial Q_x}{\partial T}+U\frac{\partial Q_x}{\partial x}\right\}\right]B \quad (42)$$

$$=\frac{-M_L}{A}\left[g+\frac{\partial^2W}{\partial T^2}+U\frac{\partial^2W}{\partial x\partial T}+U\left\{\frac{\partial\psi_x}{\partial T}+\frac{UM_x}{D(v^2-1)}-\frac{UvM_x}{D(v^2-1)}\right\}-\frac{U}{\alpha GH}\left\{\frac{\partial Q_x}{\partial T}+U\frac{\partial Q_x}{\partial x}\right\}\right]B \quad (43)$$

Therefore the simplified governing equations as derived from above are gotten by substituting Eq. 43 into Eq. 28 with Eq. 29 and Eq. 30.

4. Conclusion

Various versions of differential equation(s) governing the behaviour of plates under a moving load appear in literature. However, almost all of them are not easy to handle; a lot of computational time is required. Those that are relatively easy to handle are with many assumptions, like neglecting the effects of both rotatory inertia and shear deformation. Others assumed that the plate is not supported by any subgrade and the effect of damping neglected. The main contribution of this paper is to present a simplified set of partial differential equations modelling the dynamic behaviour of plate under a moving load. In contrast to the models of Gbadeyan and Dada (2006), this model is simple and was derived analytically. Additionally, the exact solution of this model can be sort for instead of an approximate solution. Finally, this simplified model should be considered as a more practical representation of real life situation that is easier to solve less computation time and high level of accuracy.

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Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

References

Agarana MC and Emetere M (2016). Solving non-linear damped driven simple pendulum with small amplitude using a semi analytical method. ARPN Journal of Engineering and Applied Sciences, 11(7): 4478-4484.

Agarana MC and Gbadeyan JA (2016). A comparison of dynamic behaviours of mindlin, shear, rotatory and kirchoff plates supported by subgrade under moving load. Governing, 1(2): 3-13.

Agarana MC, Gbadeyan JA, Agboola OO, Anake TA, and Adeleke OJ (2015). Dynamic response of an inclined railway bridge supported by Winkler foundation under a moving railway vehicle. Australian Journal of Basic and Applied Sciences, 9(11): 355-361.

Agarana MC, Gbadeyan JA, and Ajayi OO (2016). Dynamic response of inclined isotropic elastic damped rectangular Mindlin plate resting on Pasternak foundation under a moving load. In the International MultiConference of Engineers and Computer Scientists, Hong Kong: 2: 1-6.

Amiri JV, Nikkhoo A, Davoodi MR, and Hassanabadi ME (2013). Vibration analysis of a Mindlin elastic plate under a moving mass excitation by eigenfunction expansion method. Thin-Walled Structures, 62: 53-64. https://doi.org/10.1016/j.tws.2012.07.014

Boay CG (1993). Free vibration of rectangular isotropic plates with and without a concentrated mass. Computers and Structures, 48(3): 529-533. https://doi.org/10.1016/0045-7949(93)90331-7

Civalek O and Yavas A (2006). Large deflection static analysis of rectangular plates on two parameter elastic foundations. International Journal of Science and Technology, 1(1): 43-50.

Fryba L (1972). Vibration of solids and structures under moving loads. Noordhoff International Publishing, Groningen, Netherlands. https://doi.org/10.1007/978-94-011-9685-7

Gbadeyan JA and Agarana MC (2014). Dynamic analysis of railway bridges supported by a Winkler foundation under uniform partially distributed moving railway vehicles. WIT Transactions on the Built Environment, 135: 873-883. https://doi.org/10.2495/CR140731

Gbadeyan JA and Dada MS (2001). The dynamic response of plates on Pasternak foundation to distributed moving load. Journal of the Nigerian Association of Mathematical Physics, 5: 185-200.

Gbadeyan JA and Dada MS (2006). Dynamic response of a Mindlin elastic rectangular plate under a distributed moving mass. International Journal of Mechanical Sciences, 48(3): 323-340. https://doi.org/10.1016/j.ijmecsci.2005.09.005

Kerr AD (1964). Elastic and viscoelastic foundation models. Journal of Applied Mechanics, 31(3): 491-498. https://doi.org/10.1115/1.3629667

Mindlin RD (1951). Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates. Journal of Applied Mechanics, 18(14): 31-38.

Nguyen-Thoi T, Luong-Van H, Phung-Van P, Rabczuk T, and Tran-Trung D (2013). Dynamic responses of composite plates on the Pasternak foundation subjected to a moving mass by a cell-based smoothed discrete shear gap (CS-FEM-DSG3) method. International Journal of Composite Materials, 3(6A): 19-27.