

Dynamic response of bridge-vehicle three phases interaction considering the effects of sudden heavy braking



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ABSTRACT

The main purpose of this study is to present a new approach for analyzing the dynamic response of bridge-vehicle interaction considering effects of sudden heavy braking force. The interaction between the bridge and vehicle is described by three phases which show quite agree with the real natural response of the moving vehicle, including before braking phases, sudden heavy braking phase and after braking phase. Especially, in the second phase, the vehicle can even stop in a short the time to the time t_2 and the sliding of the wheel also can even occur in this case. From the three above phases, the governing equation of the motion of the bridge-vehicle interaction was established based on the finite element method and dynamic balance principle. And then, the influence of characteristic parameters of the bridge and moving vehicle on the dynamic response of the bridge-vehicle interaction was analyzed detail. The numerical investigation results showed that the new approach for bridge-vehicle interaction is the more increasing dynamic response of the bridge and moving vehicle than previous approaches. It can be seen that the study has meaning practice and it quite agrees to describe the true behavior of structure-vehicle interaction.

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1. Introduction

The problem model for analyzing the dynamic response of structure-vehicle interaction has been still always attracted many researchers in during past decades. Especially, the problem model of bridge-vehicle interaction is one of the best-interested models in the recent time. In almost this model, the moving vehicle model is described by a system of two degrees or multi-degree of freedom with constant speed. It means that the influence of moving acceleration on the dynamic response of the bridge almost overlooked. At the same time, the contact force between bridge and moving vehicle only considers in vertical direction, and therefore, the influence of friction force between wheel and bridge caused by braking vehicle on dynamic behavior of the structure was not also considered in

during the time of moving process of vehicle (Cantero et al., 2014; Daniel and Kortiš, 2014; Esmailzadeh and Jalili, 2003; Liu et al., 2009; Luboš et al., 2016; Pham et al., 2018; Machado and Bernardes, 2007; Vaidya and Chatterjee, 2017; Yang et al., 2004; Zhong et al., 2015).

However, the speed of the vehicle is not always uniform in during the moving time, it can increase or decrease in many cases because of the influence of moving acceleration. Especially, the appearance of the braking force in the short times will also decrease the moving velocity of the vehicle. And then, this causes a significant change in the dynamic response of bridge (Ang et al., 2013; Azimi et al., 2013; Deng and Wang, 2015; Ju and Lin, 2007; Law and Zhu, 2005; Tran et al., 2016; Huang et al., 2018).

Although, it can be seen that the problem model of bridge-vehicle interaction considering to effects of acceleration and braking force had many published works and it also showed that the influence of this is significantly on the dynamic response of the bridge-vehicle interaction. But, one of the most deficiencies of previous works does not describe fully behavior phases of moving the vehicle, including two phases, such as constant moving phase and braking phases. But, a moving vehicle can occur three main phases in

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reality, such as: before braking phases, sudden heavy braking phase and after the braking phase.

Hence, this study presents a new approach for analyzing the dynamic response of bridge-vehicle interaction considering the effects of sudden heavy braking force. The interaction between the bridge and vehicle is described by three phases which show quite agreement with real natural response of the moving vehicle, such as: (i) In the first phase, the vehicle moves with variable velocity in total time t_1 ; (ii) In the second phase, a heavy braking force appears suddenly, and then the speed of vehicle will decrease in total time t_2 . In the special case, it can even stop in a short the time to the time t_2 and the sliding of the wheel also can even occur in this case; (iii) In the third phase, the vehicle will move to the endpoint of the bridge with variable velocity in total time t_3 . From the three above phases, the governing equation of the motion of the bridge-vehicle interaction will be established based on the finite element method and dynamic balance principle. It is solved by the step-by-step integration of Newmark's method and the accuracy of the algorithm is verified by comparing the numerical results with the other numerical results in the literature. Finally, the influence of the characteristic parameters of the new approach model on the dynamic response of the bridge-vehicle interaction will be investigated in detail.

2. The problem model

2.1. The bridge-vehicle interaction model

The bridge-vehicle interaction model is described as a simple support beam subjected to a moving vehicle regarded as a two-node system, with one node associated with each of two concentrated masses having the stiffness and damping coefficients of the moving vehicle denoted by k_v and c_v , and the mass of the wheel and the mass lumped from the car body by m_w and M_v , respectively. In addition, it assumes that the influence of rolling friction is overlooked on the dynamic response of the bridge-vehicle interaction in during the moving time and the response of moving vehicle considering sudden heavy braking force is described by three phases, shown in Fig. 1.

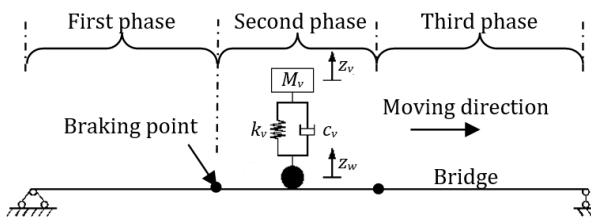


Fig. 1: The bridge-vehicle interaction model

In the first phase, the braking vehicle does not still appear and the vehicle moves with velocity $v_1(t)$ and constant acceleration a_1 in total time t_1 . And then, the interaction between a moving vehicle and

bridge only exists contact force f_c in the vertical direction in during this time, as shown in Fig. 2.

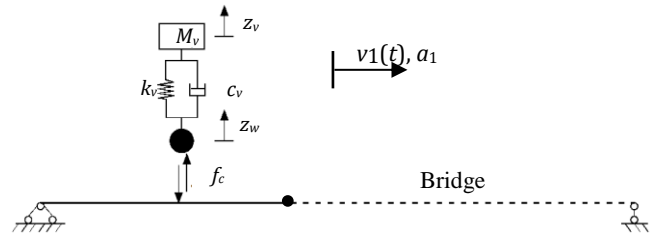


Fig. 2: The first phase of bridge-vehicle interaction

The second phase will be begun after just finish the first phases, and then the heavy braking vehicle occur suddenly. The speed of the vehicle will decrease with moving velocity $v_2(t)$ and constant acceleration a_2 in total time t_2 . In the special case, it can even stop in a short the time to the time t_2 and the sliding of the wheel also can occur in this case. It can be clearly seen that the appearance of a braking vehicle will cause friction force acting on the surface of the bridge corresponding with the contact point between the moving vehicle and bridge, plotted in Fig. 3.

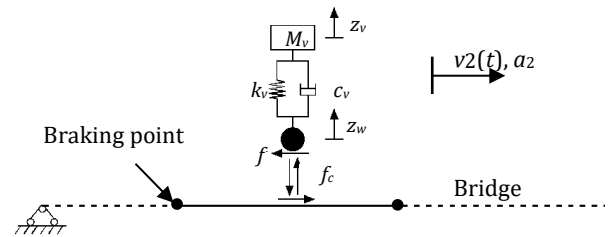


Fig. 3: The second phase of bridge-vehicle interaction

At the time t , the friction force depended on the coefficients of static and kinetic frictions between the wheels and bridge (Tran et al., 2016) can be expressed as follows

$$f = \begin{cases} \frac{H}{N} f_c & \text{for } H \leq \mu_s N \\ \mu_k f_c & \text{for } H > \mu_s N \end{cases} \quad (1)$$

where f_c is the contact force, μ_s and μ_k is the coefficient of static and kinetic friction between the wheels and bridge, respectively. Note that when $H > \mu_s N$ the sliding of the wheel on the surface of the bridge will occur in which H is the total horizontal inertial force and N denotes the total vertical contact force, given by

$$H = (m_w + M_v)a_2 \quad (2)$$

$$N = (m_w + M_v)g + m_w \ddot{z}_w + M_v \ddot{z}_v \quad (3)$$

It also needs note that the locked wheel is overlooked and the total horizontal inertial force H at the end of the first phase as same as the starting of the second phase is given by

$$H = (m_w + M_v)(a_1 - a_2). \quad (4)$$

It can be seen that the braking force acts directly on the surface of the bridge; hence it will be transferred to the neutral plane and replaced by a

force f_a and a moment m_f at the central point of the bridge section, shown in Fig. 4.

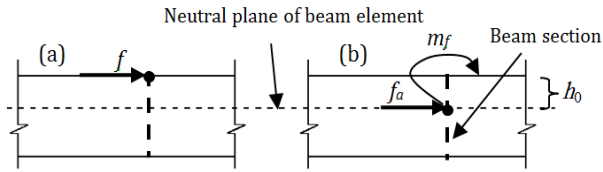


Fig. 4: The braking force model: (a) acting on the surface, (b) replaced by force and moment at the neutral axis

The replaced force and moment will directly effect on the dynamic response of the bridge, can be expressed as follows

$$f_a = f, \quad m_f = fh_0 \quad (5)$$

where h_0 is the distance from the neutral axis to the top surface of the bridge.

Continuously, the third phase immediately starts after just finish the second phase. The vehicle will move to the endpoint of the bridge with velocity $v_3(t)$ and constant acceleration a_3 in total time t_3 . In this case, the speed of the moving vehicle can increase or continuously decrease if the moving velocity is different zero at the end of the second phases, in Fig. 5.

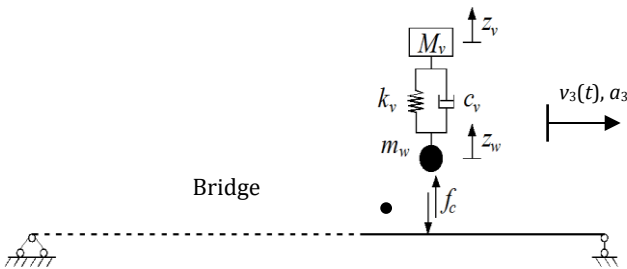


Fig. 5: The third phase of bridge-vehicle interaction

2.2. Finite governing equation

The bridge is modeled as a simple support Euler-Bernoulli beam. Each beam element has two nodes and each node has three degrees of freedom including horizontal, vertical and rotation displacement, plotted in Fig. 6.

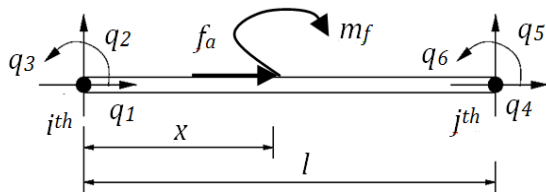


Fig. 6: The general beam-vehicle interaction element

It assumes that the moving vehicle velocity and constant acceleration in each phase at current time t is known, the relationships between the vertical displacement of the wheel with vertical displacement at the contact point, and partial

derivatives of it's with respect to t can be expressed as

$$z_w(t) = u(x, t), \quad \dot{z}_w(t) = \frac{\partial u}{\partial x} v + \frac{\partial u}{\partial t} \quad (6)$$

$$\ddot{z}_w(t) = \frac{\partial^2 u}{\partial x^2} v^2 + \frac{\partial^2 u}{\partial x \partial t} 2v + \frac{\partial u}{\partial x} a + \frac{\partial^2 u}{\partial t^2} \quad (7)$$

where $u(x, t)$ is the vertical displacement of the bridge at the contact point, and \dot{z}_w and \ddot{z}_w is the vertical velocity and acceleration of the wheel at the time t , respectively.

By using finite element method, the vertical displacement at the contact point can be expressed in form of the displacement vector of element nodes as follows

$$z_w = N_v q, \quad \dot{z}_w = v N_{v,x} q + N_v \dot{q} \quad (8)$$

$$\ddot{z}_w = v^2 N_{v,xx} q + 2v N_{v,x} \dot{q} + a N_{v,x} q + N_v \ddot{q}. \quad (9)$$

By assuming the no-jump condition for the moving vehicle, the equation of motion of the vehicle system can be written as follows

$$\begin{bmatrix} M_v & 0 \\ 0 & m_w \end{bmatrix} \begin{Bmatrix} \dot{z}_v \\ \dot{z}_w \end{Bmatrix} + \begin{bmatrix} c_v & -c_v \\ -c_v & c_v \end{bmatrix} \begin{Bmatrix} z_v \\ z_w \end{Bmatrix} + \begin{bmatrix} k_v & -k_v \\ -k_v & k_v \end{bmatrix} \begin{Bmatrix} z_v \\ z_w \end{Bmatrix} = \begin{bmatrix} f_c - (M_v + m_w)g \end{bmatrix} \quad (10)$$

where f_c is the contact force, given by

$$f_c = (m_w + M_v)g + m_w \ddot{z}_w + M_v \ddot{z}_v. \quad (11)$$

Continuously, based on dynamic balance principle, the governing equation of the bridge-vehicle interaction element at each time step can be expressed as follows

$$M_e \ddot{q}_e + C_e \dot{q}_e + K_e q_e = F_e \quad (12)$$

where M_e and K_e is mass matrix and stiffness matrix of the beam element presented in many previous works related to the finite element method, respectively.

By adopting Rayleigh damping, the damping matrix C_e can be obtained as follows

$$C_e = \alpha_0 M_e + \alpha_1 K_e \quad (13)$$

and F_e is the vector of consistent nodal forces resulting from the contact and braking force, given by

$$F_e = F_e(f_c) + F_e(f) \quad (14)$$

where $F_e(f_c)$ and $F_e(f)$ denotes the vector of contact force and braking force, respectively, obtained as follows

$$F_e(f_c) = -N_v^T f_c \delta(\cdot), \quad F_e(f) = F_e(f_a) + F_e(m_f) \quad (15)$$

with

$$F_e(f_a) = -N_a^T f_a \delta(\cdot), \quad F_e(m_f) = -N_m^T m_f \delta(\cdot) \quad (16)$$

in which $\delta(\cdot)$ is the Dirac-delta function and N is the shape function, is given by

$$N_a = [1 - \frac{x}{l} \quad 0 \quad 0 \quad \frac{x}{l} \quad 0 \quad 0] \tag{17}$$

$$N_v = [0 \quad N_1 \quad N_2 \quad 0 \quad N_3 \quad N_4], \quad N_m = N_{v,x} \tag{18}$$

where $(\cdot)_x$ denotes derivative with respect to x and N_i can be expressed as follows

$$N_1 = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3}, N_2 = x - \frac{2x^2}{l} + \frac{x^3}{l^2},$$

$$N_3 = \frac{3x^2}{l^2} - \frac{2x^3}{l^3}, N_4 = -\frac{x^2}{l} + \frac{x^3}{l^2} \tag{19}$$

By assembling the element matrices in the global coordinate, the equation of motion for the combined bridge-vehicle model can be written as

$$M\ddot{u} + C\dot{u} + Ku = F \tag{20}$$

where M , C , and K denote the global mass, damping and stiffness matrices, respectively; u is the global displacement vector, and F denotes the global load vector. The above dynamic equation is used for studying the dynamic response of the bridge-vehicle interaction and is solved by means of the direct integration method based on Newmark algorithm.

3. Numerical results

3.1. The problem parameters

In the numerical investigation section, the influence of various parameters such as braking position, the friction coefficient between wheel and bridge, initial vehicle speed and acceleration in each phase on the dynamic response of the bridge-vehicle interaction are investigated in detail. The equation of motion is solved using Newmark's method employing a time step of 0.001s and the properties of bridge and vehicle are summarized in Table 1. The coefficient of static friction μ_s and kinetic friction μ_k between rubber and concrete material are taken to be 0.85 and 0.6, respectively.

Table 1: The properties of bridge-vehicle interaction

Parameters	Unit	Value
<i>The bridge</i>		
Length span L	m	40
Young's modulus E	Nm ⁻²	34.5E9
Moment of inertia I	m ⁴	0.145
Mass per unit length ρ	kgm ⁻¹	1450
Distance of neutral axis h_0	kgm ⁻¹	0.68
Damping ratio ξ		0.02
<i>The vehicle</i>		
Body mass M_v	kg	5750
Spring stiffness k_v	Nm ⁻¹	1595E3
Damping coefficient c_v	Nsm ⁻¹	4.5E3
Wheel mass m_w	kg	250

The response of bridge-vehicle interaction is considered in the main two-case which occur quite popular in real behavior process of moving vehicle. In the first phase, the vehicle speed can be acceleration or deceleration with the moving velocity $v_1(t)$ and acceleration a_1 . In the braking

phase, the vehicle will experience sudden deceleration almost instantaneously caused by sudden heavy braking force with moving acceleration $a_2 < 0$ and then the moving velocity will be decreased to zero in extremely short time. After, the vehicle will be acceleration with moving velocity $v_3(t)$ and acceleration a_3 . All behavior of the moving vehicle in three phases is summarized in the main two cases which almost agree with the true behavior of sudden heavy braking vehicle phenomenon, plotted in Fig. 7.

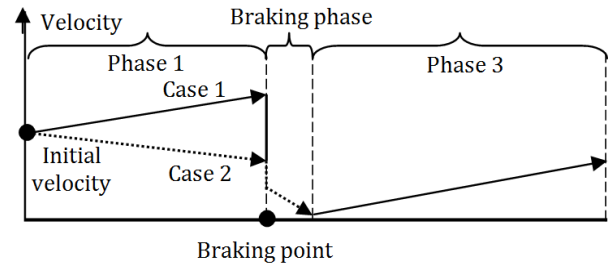


Fig. 7: The main two cases in the bridge-vehicle interaction considering sudden heavy braking vehicle

3.2. Effect of initial speed

In the first study, the influence of initial speed on the dynamic response of the bridge is investigated for various the sudden heavy braking vehicles. It assumes that the vehicle speed is uniform in the first phase, and the sudden heavy braking vehicle occurs at $L/4$.

After braking phase finishes, the vehicle speed will be acceleration with $a_3=2.5 \text{ ms}^{-2}$ in the remaining phase. Fig. 8 presents the effect of the sudden heavy braking force on the sliding time history of the bridge support. It can be seen that the braking vehicle effects significantly on the dynamic response of the bridge support. It causes friction force acting on the bridge and then it is more increasing sliding bridge support than without the influence of a sudden heavy braking vehicle. It also comments that the sliding bridge support increases significantly with an increase of initial vehicle speed. When the vehicle moves with high speed, it will cause quite strong inertia force which is directly relative to braking force, shown in Fig. 9. Hence, the sudden heavy braking vehicle increases the sliding response of the bridge and then it will increase ability sliding destruction of the part bridge support.

Besides, the influence of initial vehicle speeds on the contact force is investigated in Fig. 10. The results show that initial vehicle speed effects significantly on the contact force. An increase of the contact force magnitude responding with an increase of initial vehicle speed is reason causing an increase of dynamic response of the bridge, plotted in Fig. 11. The sudden heavy braking vehicle also affects significantly on time history displacement of the vehicle, presented in Fig. 12. It can be seen that it changed the dynamic characteristic of the bridge-vehicle interaction. It increases the time history of

vertical displacement of the body car with an increase of initial vehicle speed.

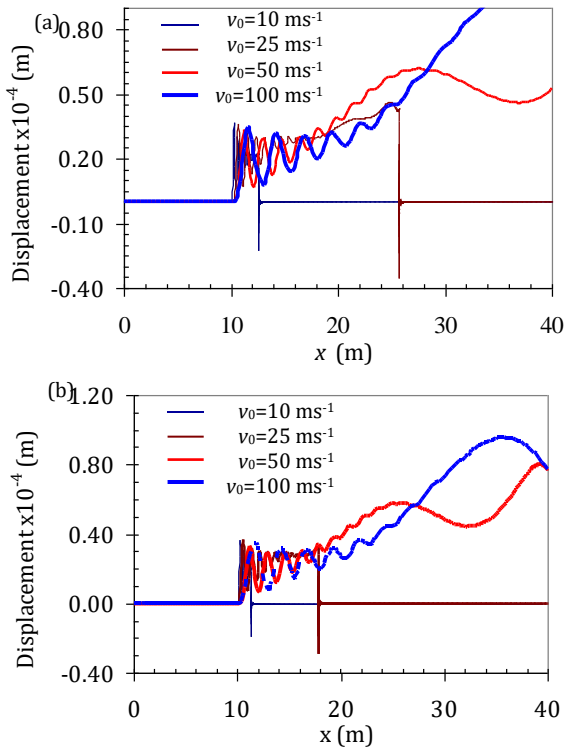


Fig. 8: The sliding time history of bridge support: (a) $a_2 = -20 \text{ ms}^{-2}$, (b) $a_2 = -40 \text{ ms}^{-2}$

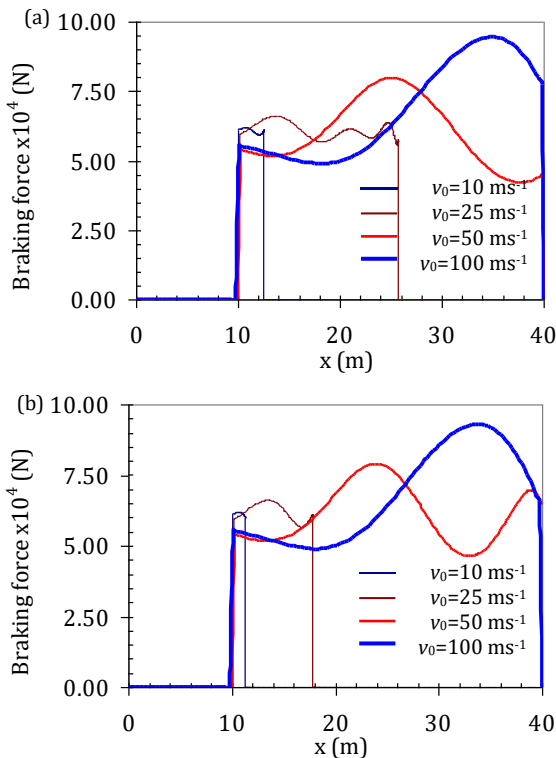


Fig. 9: The time history of braking force: (a) $a_2 = -20 \text{ ms}^{-2}$, (b) $a_2 = -40 \text{ ms}^{-2}$

3.3. Effect of sudden heavy braking

At the same time, the influence of characteristic parameters of the braking vehicle such as braking position and braking acceleration are also studied in

the following section to consider fully effects of the sudden heavy braking vehicle on the dynamic response of the bridge-vehicle interaction.

In this investigation, it assumes that the initial vehicle speed is taken to be $v_0 = 30 \text{ ms}^{-1}$ with moving acceleration $a_1 = 2.5 \text{ ms}^{-2}$ and the vehicle is acceleration with $a_3 = 2.5 \text{ ms}^{-2}$ in the third phase.

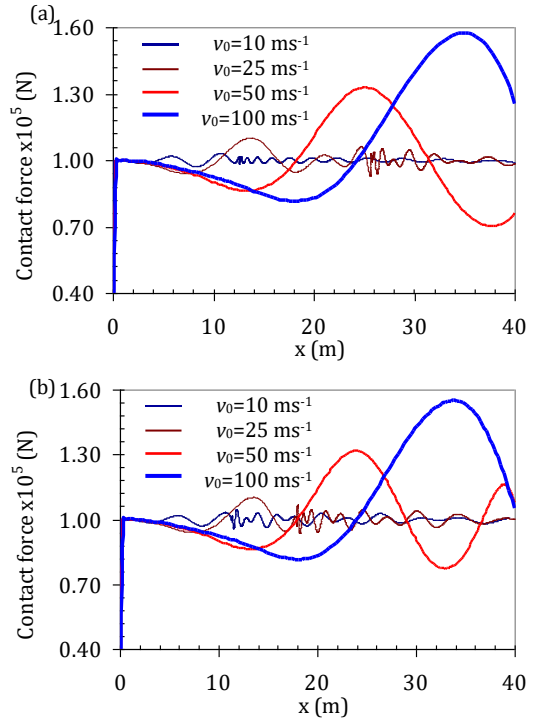


Fig. 10: The time history of contact force: (a) $a_2 = -20 \text{ ms}^{-2}$, (b) $a_2 = -40 \text{ ms}^{-2}$

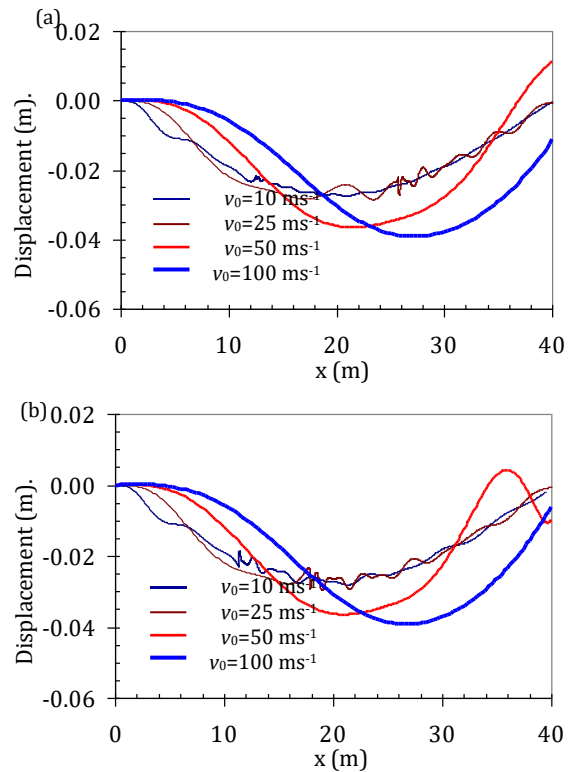


Fig. 11: The time history of vertical displacement at the midpoint of bridge: (a) $a_2 = -20 \text{ ms}^{-2}$, (b) $a_2 = -40 \text{ ms}^{-2}$

Then, the influence of on the dynamic response of the bridge is studied and the influence of braking

vehicle position on the time history of sliding bridge support is plotted in Fig. 13.

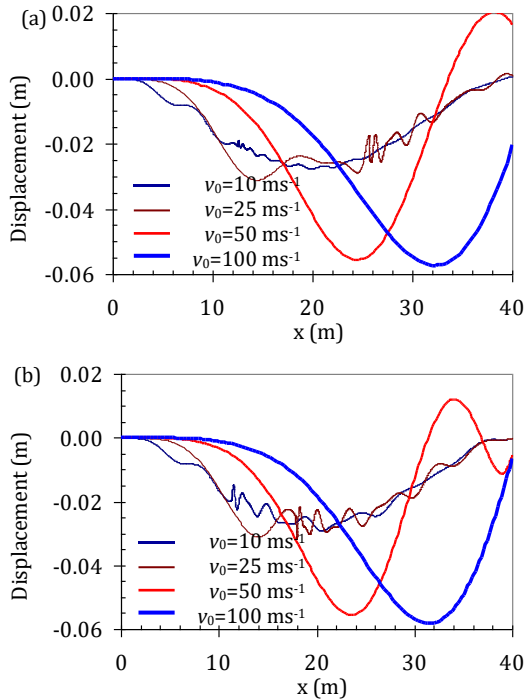


Fig. 12: The time history of vertical displacement of body car: (a) $a_2 = -20 \text{ ms}^{-2}$, (b) $a_2 = -40 \text{ ms}^{-2}$

It can be seen that the braking position effects significantly on sliding response of the bridge support. When the braking vehicle position occurs nearly the bridge support, the sliding magnitude of the bridge support will be increased significantly. It shows clearly that the braking position effects directly on the general force vector of the structure element. When this element position is near the bridge support, the sliding force acting on the support will be very considerable causing sliding displacement of the bridge support. If the sliding magnitude is enough large it will increase ability pounding surface and destruction of the parts of bridge support, as shown in Fig. 13.

Continuously, the influence of braking accelerations occurring at mid-span bridge on the dynamic response of the bridge support is also studied in Fig. 14 and Fig. 15. With an increase of those will cause to change the dynamic character of the bridge-vehicle interaction and then it will effect on sliding response of the bridge support, especially. It can be commented that when the braking acceleration increases, it will cause rapidly decrease of braking vehicle time and then the influence of those will be also responding to decrease. Therefore, it decreases the sliding response of the bridge support with an increase in braking acceleration, in Fig.14.

Besides, the deceleration of moving the vehicle is also effected on the bridge-vehicle interaction. It decreases the braking time of the moving vehicle with an increase of the braking acceleration, as shown in Fig. 15. However, the braking force is also different effects with various decelerations and the

braking force will decrease significantly when the sliding of the wheel on the surface of the bridge occurs, in Fig. 15b.

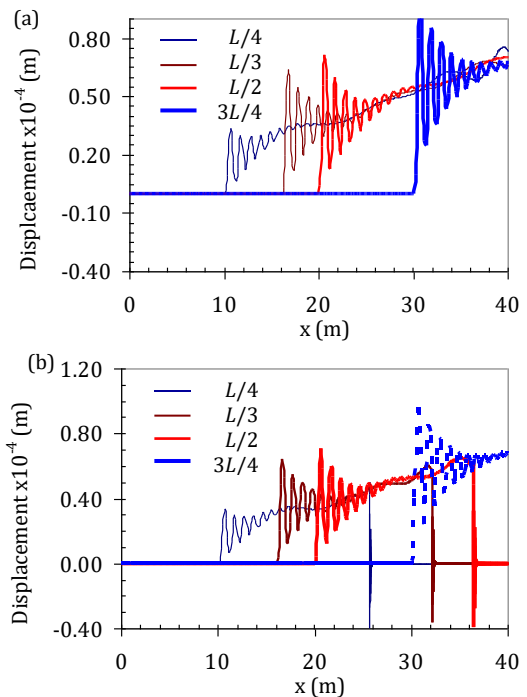


Fig. 13: The influence of braking position on a sliding time history of bridge support: (a) $a_2 = -15 \text{ ms}^{-2}$, (b) $a_2 = -30 \text{ ms}^{-2}$

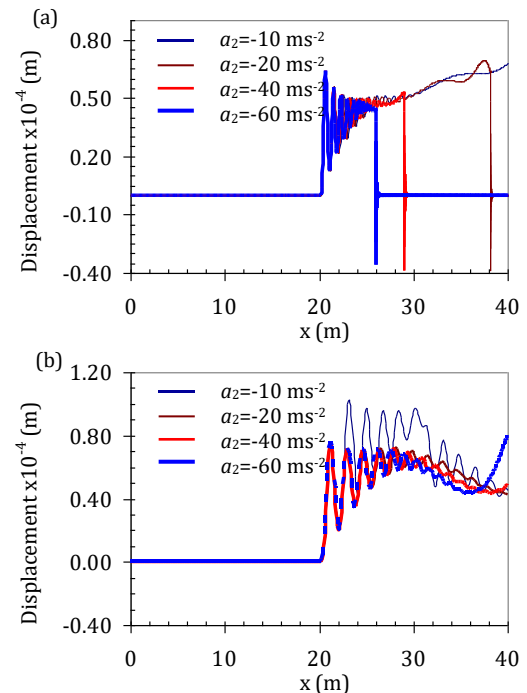


Fig. 14: The influence of braking acceleration on a sliding time history of bridge support: (a) $v_0 = 25 \text{ ms}^{-1}$, (b) $v_0 = 50 \text{ ms}^{-1}$

It can be seen that the sliding of the wheel depends on the braking acceleration of the moving vehicle and friction coefficient between properties material of the wheel and surface of the bridge. If the moving inertia force is larger than static friction force, the sliding of the wheel on the surface of the bridge will occur, and the ability of sliding of the

wheel will also occur differently with various values of braking acceleration. It can be commented that when braking acceleration is larger than the critical acceleration of the moving vehicle, the sliding of the wheel on the surface of the bridge will occur clearly, as shown in Fig. 16.

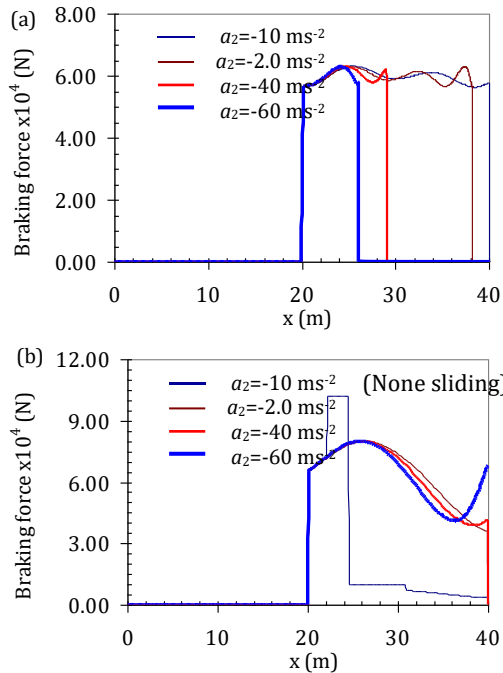


Fig. 15: The influence of braking acceleration on the time history of braking force: (a) $v_0=25 \text{ ms}^{-1}$, (b) $v_0=50 \text{ ms}^{-1}$

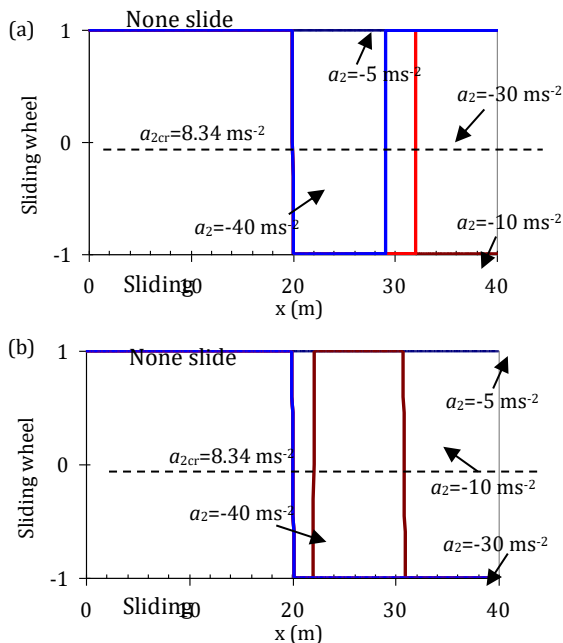


Fig. 16: The influence of braking acceleration on the sliding of the wheel: (a) $v_0=25 \text{ ms}^{-1}$, (b) $v_0=50 \text{ ms}^{-1}$

3.4. Effect of bridge-vehicle three phases interaction

To consider general effects of the bridge-vehicle three phases interaction considering effects of the sudden heavy braking on the dynamic response of the bridge, the influence of property parameters of

the bridge-vehicle interaction on dynamic magnitude factor (DMF) is investigated in detail with $a_3=2.5 \text{ ms}^{-2}$. The effects of the acceleration in the first phase on the dynamic response of the bridge are studied with a various deceleration in the second phase plotted from Figs. 17-19. It can be seen that with increases of the moving acceleration in the first phase almost cause increase the dynamic response of the bridge-vehicle interaction such as the DMF, vertical displacement of the body car and the contact force between vehicle and bridge. It can be also seen that the deceleration of moving the vehicle in the second phase is not affected significantly on the DMF when the braking acceleration is larger than the critical acceleration of the moving vehicle, especially. This completely agrees with the true response character of the braking vehicle because of the acting of it almost in the horizontal direction.

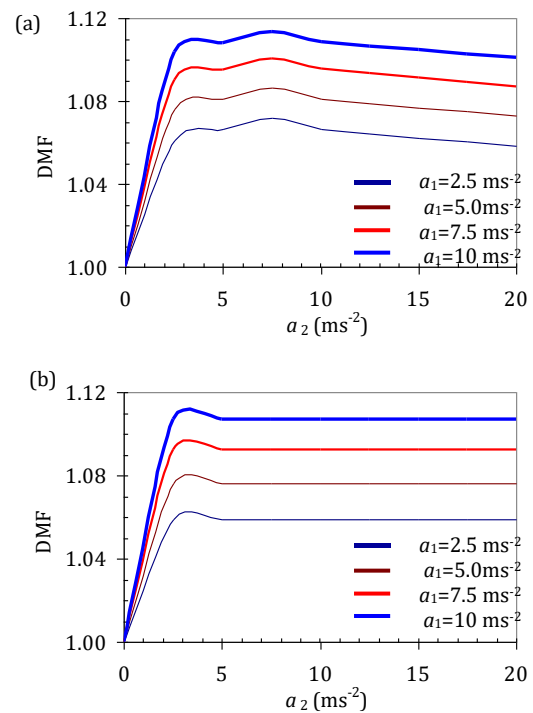


Fig. 17: The influence of acceleration in the first phase on the DMF with braking positions: (a) $L/4$, (b) $L/2$

Continuously, the influence of the initial vehicle speed on the DMF of the bridge is also studied in Fig. 20. It can be seen that an increase in the initial vehicle speed causes an increase of the DMF of the bridge for various braking positions.

Additionally, besides of the influence of the deceleration, the braking position of the sudden heavy braking vehicle is also affected significantly on the sliding response of the bridge support with a variable value of initial vehicle speeds, plotted in Fig. 21.

It can be seen that when the braking position occurs near the bridge support, the sliding dynamic response of the bridge support increases clearly. If the sliding of the bridge support is enough large, it will increase the ability destroy of the part of bridge support with an increase of the initial vehicle speed, plotted in Fig. 21.

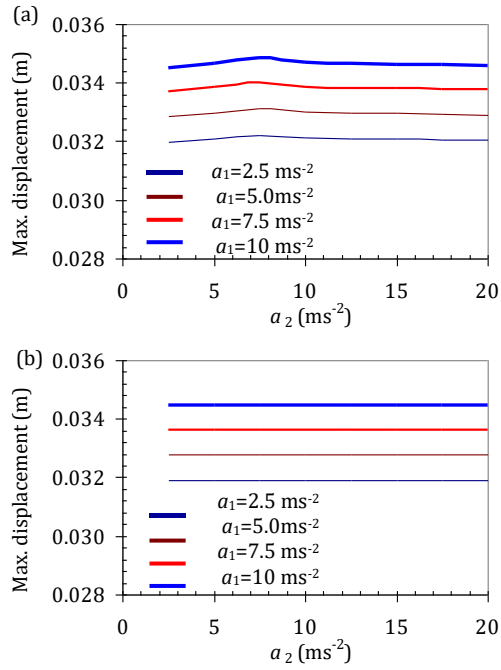


Fig. 18: The influence of acceleration in the first phase on vertical displacement of body car: (a) $L/4$, (b) $L/2$

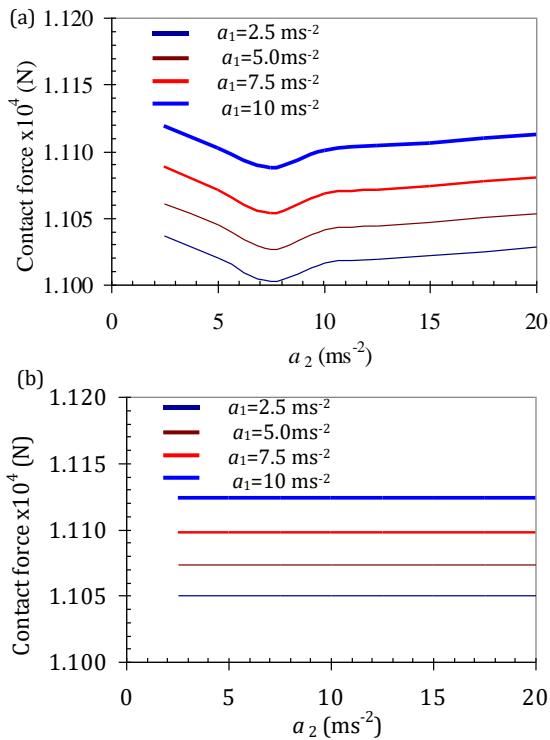


Fig. 19: The influence of acceleration in the first phase on contact force with braking positions: (a) $L/4$, (b) $L/2$

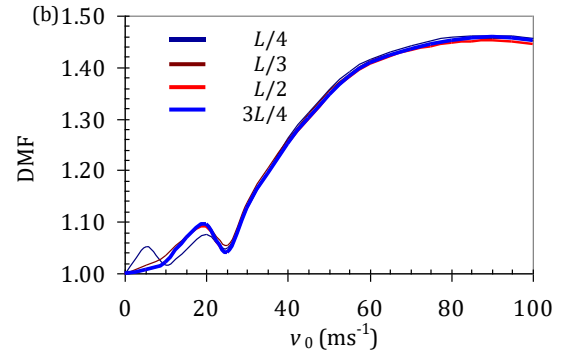
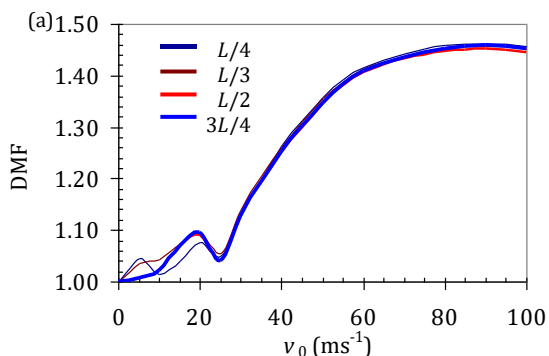


Fig. 20: The influence of initial vehicle speed on the DMF of the bridge: (a) $a_2 = -20 \text{ ms}^{-2}$, (b) $a_2 = -40 \text{ ms}^{-2}$

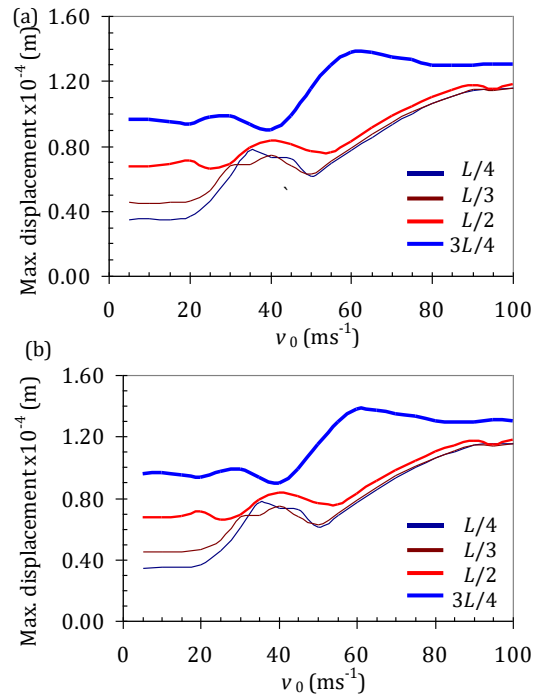


Fig. 21: The influence of initial vehicle speed on the sliding bridge support: (a) $a_2 = -20 \text{ ms}^{-2}$, (b) $a_2 = -40 \text{ ms}^{-2}$

4. Conclusion

Based on the new approach for the bridge-vehicle interaction considering the effects of the sudden heavy braking vehicle on the dynamic response of the structure system, the following conclusions can be drawn as follows:

- The bridge-vehicle interaction model is described by three phases which show quite agree with the real natural response of the moving vehicle, including the first acceleration or deceleration phase, sudden heavy braking vehicle phase, and acceleration phases.
- The characteristic properties of the braking vehicle such as braking position, acceleration or deceleration, and initial speed have almost effect on the dynamic response of the bridge-vehicle interaction. Those are not more increasing the significantly dynamic response of the bridge than without effects of the sudden heavy braking vehicle in the range of high speeds.

- Especially, the sudden heavy braking vehicle increases significantly the sliding response of the bridge support, and therefore it increases ability sliding destroy of the part of the bridge support if the sliding is enough large.

It can be seen that the new approach of bridge-vehicle interaction completely agrees with the true behavior of the moving vehicle. Hence, this study has meaning practice in the problem of analyzing the dynamic response of the structure-vehicle interaction.

Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

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