A note on thin-film flow of Eyring-Powell fluid on the vertically moving belt using successive linearization method

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A B S T R A C T

The main goal of this work is to obtain the numerical solution for thin film flow of MHD an incompressible Eyring-Powell fluid on a vertically moving belt. The nonlinear equation governing the flow problem is modeled and then solved numerically by means of a successive linearization method (SLM). The numerical results are derived in tables for comparisons. The important result of this comparison is the high precision of the SLM in solving nonlinear differential equations. The solutions take into account the behavior of Newtonian and non-Newtonian fluids. Graphical outcomes of various non-Newtonian parameters such as Hartman number and Stokes number on the flow field are discussed and analyzed. Besides this, the present results have been tested and compared with the available published results in a limiting manner and an excellent agreement is found.

1. Introduction

The most phenomena in the field of engineering and science that occur are nonlinear. With this nonlinearity the equations become more difficult to handle and solve. Some of these nonlinear equations can be solved by using approximate analytical methods such as Homotopy analysis method (HAM) proposed by Liao (1992, 2004), Homotopy Perturbation method (HPM) it was found by He (1999) and Adomain decomposition method (ADM) (Esmaili et al., 2008; Makinde and Mhone, 2006; Makinde, 2008). However, some of these equations are solved via traditional numerical techniques such as finite difference method, shooting method and Keller box method, Runge-Kutta. Recently some studies have presented a new method called Successive Linearization Method (SLM). This method has been applied successfully in many nonlinear problems in sciences and engineering, such as the MHD flows of non-Newtonian fluids and heat transfer over a stretching sheet (Shateyi and Motsa, 2010), viscoelastic squeezing flow between two parallel plates, (Makukula et al., 2010a), two

dimensional laminar flow between two moving porous walls (Makukula et al., 2010b) and convective heat transfer for MHD boundary layer with pressure gradient (Ahmed et al., 2015). Therefore, the effectiveness, validity, accuracy and flexibility of the SLM are verified among of all these successful applications.

In the recent years, a great deal of interest has been gained to fluids applications. Some fluids do not express easily to by particular constitutive relationship between shear rates and stress and which is totally different than the viscous fluids (Ellahi et al., 2008; Hayat et al., 2004). Theses fluids including many home items namely, toiletries, paints, cosmetics certain oils, shampoo, jams, soups etc., have different features and are denoted by non-Newtonian fluids. In general, the categorization of non-Newtonian fluid models is given under three classes which are named the integral, differential, and rate types (Fetecau et al., 2007; Salah et al., 2011a; 2011b). In the present study, the main interest is to discuss the thin-film flow of magneto hydrodynamic (MHD) Powell–Eyring fluid on a vertically moving belt. The fluid model is considered here is too complicate and has preference over the power-law fluid in the couple reasons. The First reason is it is deduced from kinetic theory of liquid rather than empirical relation as in the case of power–law mode. Secondly, it correctly reduces to the viscous fluid at low and high shear rates. This motivates us to choose the Powell–Eyring fluid

model in this study (Hayat et al., 2013; Malik et al., 2013; Siddiqui et al., 2013). Besides that, the equations in the non-Newtonian fluids propose some big challenges to the researchers to seek their solutions. The equations become very difficult when non-Newtonian fluid is combined in the presence of magneto hydrodynamic. It is the study of the interaction of electro conducting fluids with phenomena of electromagnetic. The flow of MHD fluid in the presence of magnetic field is very important in many regions of applied science, engineering and technology such as MHD pumps and MHD power generation. Due to this fact many researchers are still contributing in the field of MHD fluids mechanics (Hussain et al., 2010; Husain et al., 2008; Khan et al., 2007; Wang et al., 2005).

Presently a new investigation on thin-film MHD flow of Eyring Powell – fluid is discussed. The governing equations of Powell–Eyring fluid with MHD are utilized. The numerical solution to the resulting nonlinear problem is computed by using the SLM approach. The embedded flow parameters are discussed and illustrated graphically.

2. Governing equations

The continuity and momentum equations for an incompressible fluid, are given by

$$div\ \boldsymbol{V} = 0 \tag{1}$$
$$\rho\left[\frac{\partial V}{\partial t} + (\boldsymbol{V}.\nabla)V\right] = -\nabla P + div\ \boldsymbol{S} + \rho g - \sigma B_0^2 \boldsymbol{V}, \tag{2}$$

Here, which ρ is the fluid density, V is the velocity field, P is the pressure, S is the Cauchy stress tensor, g is the body force per unit mass, and σ is electrical conductivity of fluid.

The extra stress tensor S for Powell-Eyring fluid satisfies the constitutive equations as given in Hayat et al. (2013), Malik et al. (2013), and Siddiqui et al. (2013) and is in the following form

$$\boldsymbol{S} = \mu\nabla\boldsymbol{V} + \frac{1}{\beta}sinh^{-1}\left(\frac{1}{c}\nabla\boldsymbol{V}\right) \tag{3}$$

and

$$sinh^{-1}\left(\frac{1}{c}\nabla\boldsymbol{V}\right) \approx \frac{1}{c}\nabla\boldsymbol{V} - \frac{1}{6}\left(\frac{1}{c}\nabla\boldsymbol{V}\right)^3, \left|\frac{1}{c}\nabla\boldsymbol{V}\right| \ll 1, \tag{4}$$

where μ is the dynamic viscosity of fluid and β and c are the material constants of the Powell-Eyring fluid model.

Consider the velocity in the following form as

$$\boldsymbol{V} = (0, u(x), 0),$$

with this choice of velocity, the constraint of incompressibility is automatically satisfied. Also assume the extra stress tensor is a function of x only i.e. $\boldsymbol{S} = S(x)$. Substituting Eq. 4 into Eq. 3 and keeping in mind that at $t = 0$ the fluid is at rest, we obtain

$$S_{yz} = S_{yy} = S_{zz} = S_{xz} = S_{xx} = 0$$

and

$$S_{xy} = \left(\mu + \frac{1}{\beta c}\right)\frac{\partial u}{\partial x} - \frac{1}{6\beta c^3}\left(\frac{\partial u}{\partial x}\right)^3. \tag{5}$$

The balance of the linear momentum gives

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial y} + \frac{\partial S_{xy}}{\partial x} - \rho g - \sigma B_0^2 u(x). \tag{6}$$

Assume that the pressure is standard atmospheric pressure and since the y - coordinate and gravity force are in the upward direction, then for the steady flow the above Eqs. 5 and 6 become

$$0 = \left[\mu + \frac{1}{\beta c}\right]\frac{\partial^2 u}{\partial x^2} - \frac{1}{6\beta c^3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)^3 - \rho g - \sigma B_0^2 u(x) \tag{7}$$

3. Formulation of lifting problem

Here we considering the steady, laminar and uniform thin film flow of an incompressible MHD Powell–Eyring fluid, which is chosen by a wide flat belt moves vertically upward at a constant speed U_0. The fluid drains down to the belt due to the gravity effects. The thickness δ of the thin-film is uniform and the external pressure is the atmospheric everywhere. The x- axis is perpendicular to the belt and the y - axis is parallel to the belt, which is moves in the upward direction.

The appropriate boundary conditions for this problem are

$$S_{xy} = 0\ at\ x = \delta \quad \text{(Free surface)} \tag{8}$$
$$u(x) = U_0\ at\ x = 0 \quad \text{(no slip condition)}$$

introducing the following dimensionless quantities

$$\xi = \frac{x}{\delta}, f = \frac{u}{U_0}, S_t = \frac{\rho g \delta^2}{\mu_{eff} U_0}$$
$$M = \frac{\sigma B_0^2 \delta^2}{\mu_{eff}}, \quad H = \frac{U_0^2}{2\beta c^3 \delta^2\ \mu_{eff}} \tag{9}$$

the problem statement reduces to,

$$\frac{\partial^2 f(\xi)}{\partial \xi^2} - H\frac{\partial^2 f(\xi)}{\partial \xi^2}\left(\frac{\partial f(\xi)}{\partial \xi}\right)^2 - Mf(\xi) - S_t = 0. \tag{10}$$
$$f(\xi) = 1\ at\ \xi = 0,$$
$$\frac{df}{d\xi} = 0\ at\ \xi = 1. \tag{11}$$

4. Solution of the lifting problem

The Eq. 10 with the boundary condition Eq. 11 were solved using a successive linearization method (SLM) (Makukula et al., 2010a; 2010b; Ahmed et al., 2015) for SLM solution we choose the unknown function $f(\xi)$ in the form

$$f(\xi) = f_i(\xi) + \sum_{m=0}^{i-1} F_m(\xi) \tag{12}$$

where $f_i(\xi)$ is unknown function and $F_m(\xi)$, $m \geq 1$ is successive approximation which is obtained by recursively solving the linear part of the equation those results from substituting Eq. 12 in the governing equations. The mean idea of SLM that the

assumption of unknown function $f_i(\xi)$ is very small when i becomes very large, thus the nonlinear terms in $f_i(\xi)$ and its derivatives are considered to be very small and therefore neglected. The intimal guess function $F_0(\xi)$ which is chosen to satisfy the boundary condition $F_0(\xi) = 1$ at $\xi = 0$, $F_o'(\xi) = 0$ at $\xi = 1$, which is taken to be in the form $F_0(\xi) = S_t\left(\frac{\xi^2}{2} - \xi\right) + 1$. Therefore, beginning from the initial guess, the subsequent solution F_i is obtained by successively solving the linearized from the equation which is obtained by substituting Eq. 12 in the governing Eq. 10. Then we arrive at the linearized equation to be solved is

$$a_{1,i-1}F''_i - a_{2,i-1}F'_i - a_{3,i-1}F_i = r_{1,i-1} , \qquad (13)$$

subject to the boundary conditions

$$F_i(0) = 1 \ and \ F'_i(1) = 1 ,$$

where the coefficients parameters $a_{k,i-1}, (k = 1,2,3)$ and $r_{1,i-1}$ are defined as

$$a_{1,i-1} = 1 - H\left(\sum_{m=0}^{i-1} F'_m\right)^2 ,$$
$$a_{2,i-1} = 2H \sum_{m=0}^{i-1} F_m \sum_{m=0}^{i-1} F''_m ,$$
$$a_{3,i-1} = M \sum_{m=0}^{i-1} F_m \quad and$$
$$r_{1,i-1} = S_t - \sum_{m=0}^{i-1} F''_m + H\left(\sum_{m=0}^{i-1} F'_m\right)^2 \sum_{m=0}^{i-1} F''_m + M \sum_{m=0}^{i-1} F_m . \qquad (14)$$

When we solve Eq. 10 iteratively, the solution for F_i has been obtained and finally after K iterations the solution $f(\xi)$ can be written as

$$f(\xi) = \sum_{m=0}^{K} F_m(\xi) .$$

In order to apply SLM firstly transform the domain solution from $(0, \infty]$ to $[-1,1]$. SLM is based on the Chebyshev spectral collection method. This method is depending on the Chebyshev polynomials defined on the interval $[-1,1]$. Thus, using the domain truncation technique where the problem is solved in the interval $[-1, L]$ where L is scaling parameter used to impose the boundary condition at infinity. Thus, this can be obtained via the transformation

$$\frac{\xi}{L} = \frac{\eta+1}{2} , \quad -1 \leq \eta \leq 1. \qquad (15)$$

By using the Gauss-Lobatto collocation points we can discretize the domain $[-1,1]$ as follows

$$\eta = cos\frac{\pi j}{N}, F_i \approx \sum_{k=0}^{N} F_i(\eta_j)T(\eta_j), j = 0,1,\dots,N \qquad (16)$$

where N is the number of collection points and T_k is the kth Chebyshev polynomial given by

$$T_k(\eta) = cos[kcos^{-1}(\eta)].$$

The derivatives of the variable at the collocation points are in the form

$$\frac{d^r F_i}{d\,\xi^r} = \sum_{k=0}^{N} \boldsymbol{D}_{kj}^r F_i(\eta_k), j = 0,1,\dots,N \qquad (17)$$

where r is the order of differentiation and $\boldsymbol{D} = \frac{2}{L}D$ with D is the Chebyshev spectral differentiation matrix. Substituting Eqs. 15 to 17 into Eq. 13 we arrive at the matrix equation

$$\boldsymbol{A}_{i-1}\boldsymbol{X}_{i-1} = \boldsymbol{R}_{i-1} \qquad (18)$$
$$\boldsymbol{A}_{11} = a_{1,i-1}\boldsymbol{D}^2 - a_{2,i-1}\boldsymbol{D} - a_{3,i-1}I .$$

Following the above procedure, we can obtain the solution as

$$\boldsymbol{X}_i = \boldsymbol{A}_{i-1}^{-1}\boldsymbol{R}_{i-1} . \qquad (19)$$

5. Drainage problem for MHD Eyring-Powell fluid

Under the same assumptions as in the previous problem we consider the steady, laminar and uniform MHD Eyring-Powell fluid, dropping on the stationary infinite perpendicular wall. The flow is in the downward direction due to gravity. The thickness δ of the thin film is uniform and the external pressure is standard atmospheric.

The governing Eqs. 1–3, then become

$$0 = \left[\mu + \frac{1}{\beta c}\right]\frac{\partial^2 u}{\partial x^2} - \frac{1}{6\beta c^3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)^3 + \rho g - \sigma B_0^2 u(x) \qquad (20)$$

subject to the boundary conditions

$$\frac{df}{d\xi} = 0 \quad at \ \xi = 1 ,$$
$$f(\xi) = 0 \quad at \ \xi = 0.$$

The above equation with these boundary conditions is a highly nonlinear second-order differential equation for the drainage problem. Following the same process in section 4. We use the SLM by assuming the intimal guess function $F_o(\xi)$ which is chosen to satisfy the boundary condition

$$F_o(\xi) = 0 \ at \ \xi = 0$$

and

$$F_o'(\xi) = 0 \ at \ \xi = 1$$

in the form

$$F_o(\xi) = S_t\left(\xi - \frac{\xi^2}{2}\right),$$

we can get the solution as

$$\boldsymbol{X}_i = \boldsymbol{A}_{i-1}^{-1}\boldsymbol{R}_{i-1} .$$

6. Results and discussion

This section concerns with the variations of embedded flow parameters in the solution expressions for the lifting and drainage problems together with the physical interpretation of the problem have been discussed in Figs. 1-11. These figures are plotted in order to illustrate such variations. Here the graphs have been determined

for the MHD thin film flow of steady Eyring-Powell fluid on a vertically moving belt. Fig. 1 shows the effects of Stokes number parameter S_t on the velocity profile when M, H are fixed and S_t is an increasing. It is of very important to notice that by increasing the parameter S_t, this would lead to decreasing in the velocity profile. Physically, it is true as increasing Stokes number causes the fluid's thickness and reduces its flow.
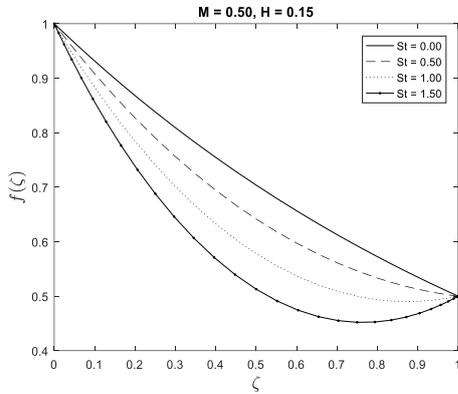
Fig. 1: Effects of Stokes number S_t for $f(\xi)$ on the in the lifting case

Fig. 2 shows that velocity increases in drainage flow case when Stokes number S_t increases. Physically, it is because of friction which seems very small near to the belt and higher at the surface of the fluid.
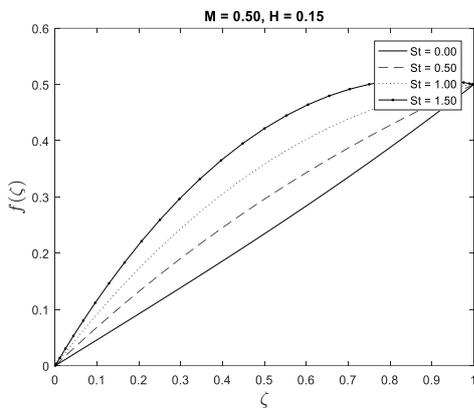
Fig. 2: Effects of Stokes number S_t for $f(\xi)$ on the in the drainage case

Fig. 3 is prepared to see the effects of applied magnetic field (Hartman number) M on the velocity profile. Keeping H, S_t fixed and varying M, it is noted that the velocity profile decreases by increasing the magnetic field parameter M. From physical side we observe that when we increasing the values of M, the flow on velocity profile of $f(\xi)$ decreases, in fact this is due to the effects of the transverse magnetic field on the electrically conducting fluid which gives rise to a resistive type Lorentz force which tends to slow down the motion of the fluid.

Fig. 4 shows that velocity in drainage flow case when Hartman number increases the magnetic field and velocity have a direct relation.
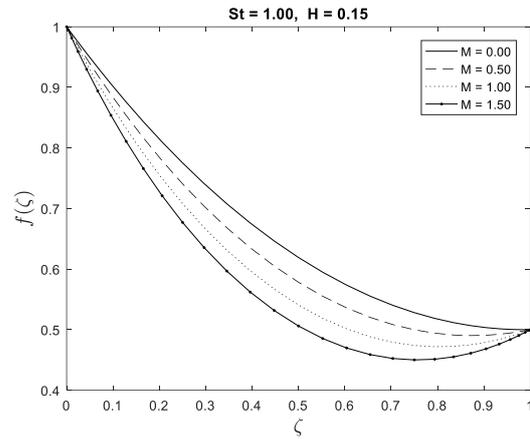
Fig. 3: Effects of Hartman number M for $f(\xi)$ on the lifting case
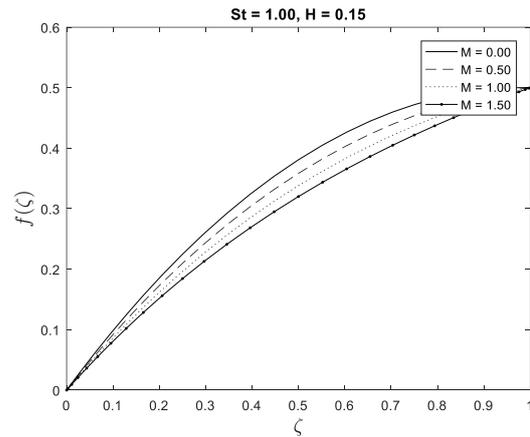
Fig. 4: Effects of Hartman number M for $f(\xi)$ on the in the drainage case

Fig. 5 shows the effects of the other material constant parameter H on the velocity profile when M, S_t are fixed. It is worth noticing that by increasing the parameter H would lead to a decrease in the velocity profile (this is much related to increase in the boundary layer thickness). This is due to the fact that increasing the values of H would increase the friction forces, and, thus, slow down the motion of the fluid.
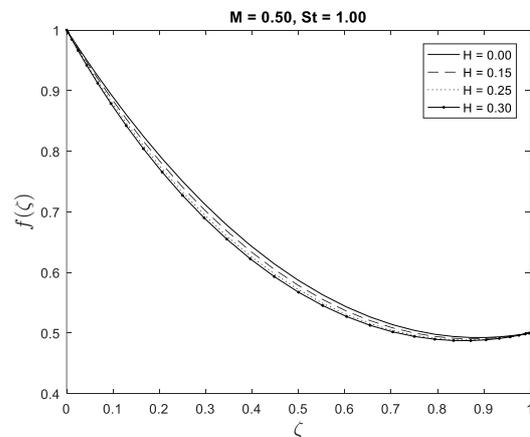
Fig. 5: Effects of H for $f(\xi)$ on the in the lifting case

Fig. 6 has an inverse relation with the velocity of the drainage flow.
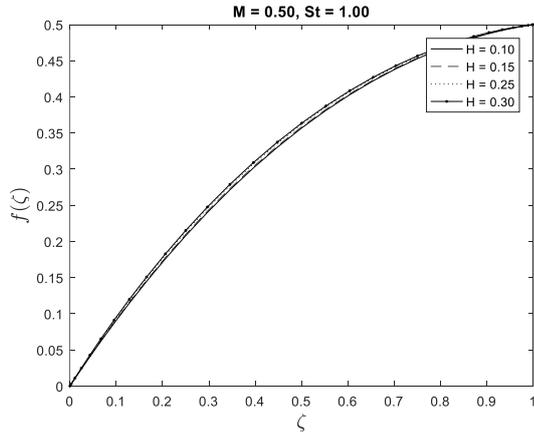
Fig. 6: Effects of H for $f(\xi)$ on the in the drainage case

It can be easily seen from Figs. 7-9 that the value of velocity $f'(\xi)$ near the lower plate surface increases regularly with the increase in the value of H, M, S_t, respectively and as we move away from lower plate surface this value increases.
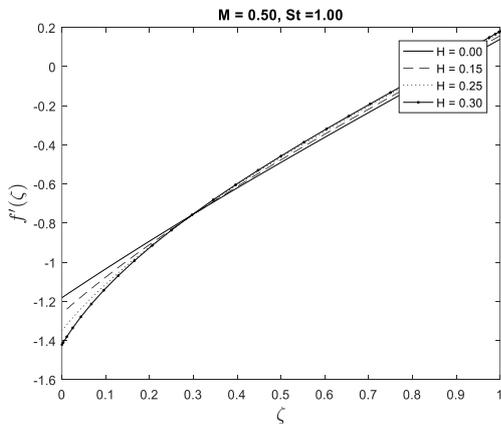
Fig. 7: Effects of H for $f'(\xi)$ on the in the lifting case
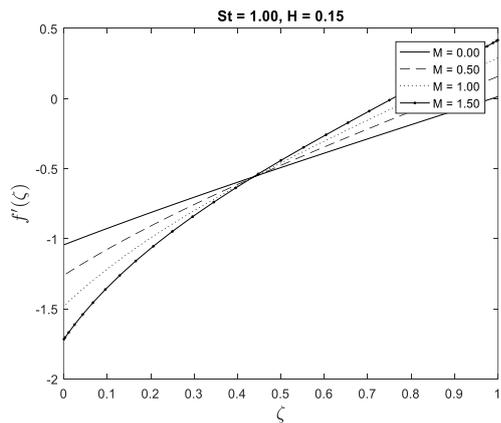
Fig. 8: Effects of Hartman number M for $f'(\xi)$ on the in the lifting case

Furthermore, the velocity profile for the MHD Newtonian fluid in the lifting and drainage case is shown in Figs. 10 and 11.

Finally, for the purposes of validation and the accuracy the present result is compared with published work in Siddiqui et al. (2013) in Table 1 and Table 2. It is found in an excellent agreement.
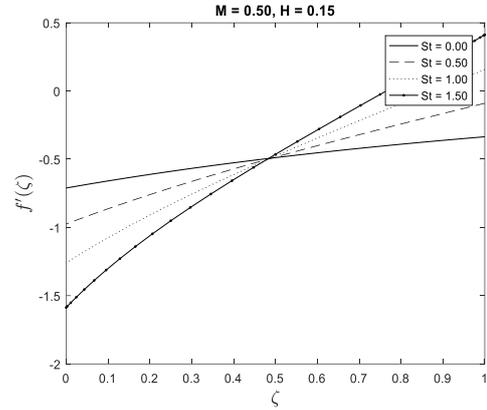
Fig. 9: Effects of Stokes number S_t for $f'(\xi)$ on the in the lifting case
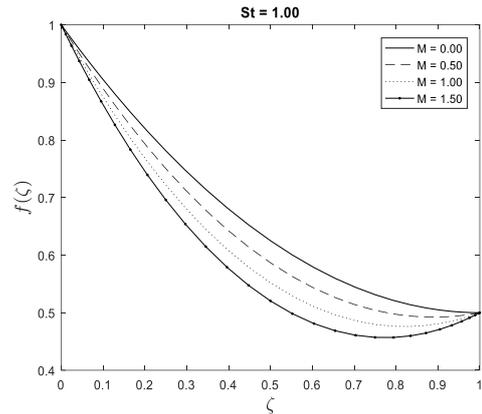
Fig. 10: Velocity profile for the MHD Newtonian fluid in the drainage case
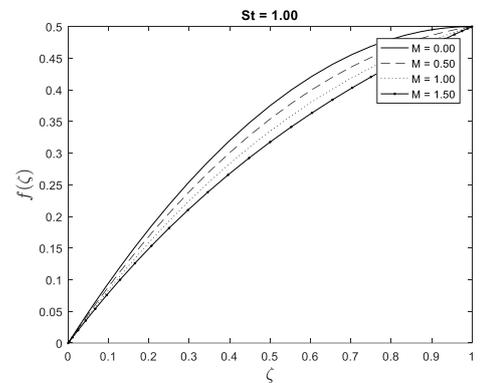
Fig. 11: Velocity profile for the MHD Newtonian fluid in the lifting case

7. Conclusion

In this research, the problem of thin film flow of MHD Eyring-Powell fluid on a vertically moving belt is solved numerically. The numerical solutions are well established by SLM. We note that the present analysis is more general when compared with the analysis presented in Siddiqui et al. (2013). The results of Siddiqui et al. (2013) can be recovered as a special case by taking $M = 0$, which is the MHD effect Eyring-Powell fluid. Furthermore, the results for the Newtonian fluid can be obtained by choosing $H \rightarrow 0, M \rightarrow 0$. This confirms the correctness of our mathematical calculations.

Table 1: Comparison of numerical values of $f(\xi)$ on the lifting case with Siddiqui et al. (2013) for several values of ξ when $M = 0, H = 0.15, S_t = 1$

ξ	Siddiqui et al. (2013)	Present work [SLM]
0.0000	1.0000	1.0000
0.1000	0.8976	0.9015
0.2000	0.8095	0.8145
0.3000	0.7342	0.7386
0.4000	0.6704	0.6736
0.5000	0.6175	0.6191
0.6000	0.5747	0.5749
0.7000	0.5418	0.5411
0.8000	0.5185	0.5173
0.9000	0.5046	0.5036
1.0000	0.50000	0.5000

Table 2: Comparison of numerical values of $f(\xi)$ on the lifting case with Siddiqui et al. (2013) Newtonian case i.e. $H = 0.0$ for several values of ξ when $M = 0, S_t = 1$.

ξ	Siddiqui et al. (2013)	Present work [SLM]
0.00	1.0000	1.0000
0.10	0.9050	0.9050
0.20	0.8200	0.8200
0.30	0.7450	0.7450
0.40	0.6800	0.6800
0.50	0.6200	0.6200
0.60	0.5800	0.5800
0.70	0.5400	0.5400
0.80	0.5200	0.5200
0.90	0.5050	0.5050
1.00	0.5000	0.5000

Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

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