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Steady motion of an incompressible microstretch fluid between two rotating spheres with slip conditions

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ABSTRACT

In this paper, the steady rotational motion of an incompressible microstretch fluid between two rotating spheres is investigated. The slip boundary conditions are proposed on the spherical boundaries. The two spheres are assumed to be rotating with different angular speeds. Closed form solutions for the velocity, microrotation, and microstretch are obtained. Numerical results are presented and the effects of slip and spin parameters on the velocity, microrotation, and microstretch are discussed through graphs.

1. Introduction

The theory of micro-fluids was introduced by Eringen (1966) to consider the microscopic effects of fluid elements. He has adopted a physical model in which each material volume element contains micro-volume elements that can translate, rotate and deform independently of the motion of the macro-volume elements. The microfluid was classified into three main subclasses. The first sub-class is the class of micromorphic fluids that has nine degrees of freedom, the second class is called microstretch fluids with seven degrees of freedom and the third sub-class is the class of micropolar fluids which has only six degrees of freedom. Microstretch fluids are also known as Eringen fluids or micropolar fluids with stretch. Microelements of these fluids can stretch or contract in addition to being micropolar. Physically, microstretch fluids represent fluids with deformable suspensions. These fluids model slurries, paper pulps, insect colonies, blood and other biological fluids (Eringen, 1998).

The classical no-slip boundary condition was applied to many problems in the Navier-Stokes theory extensively. It assumes that the liquid molecules adjacent to the solid are stationary relative to the solid. This condition is not adequate for the motion of fluids with microstructure such as micropolar and microstretch fluids.

An alternative boundary condition, namely slip condition was introduced in (Navier, 1823; Narasimhan, 2003; Murthy et al., 2007; Ramkissoon and Majumdar, 1976; Sherief et al., 2019a; 2019b; 2015; 2017; 2018). This condition depends on the shear stress and permits the fluids to slip at the solid boundary. This means that the tangential velocity at a solid surface is proportional to the shear stress at the surface (Navier, 1823). The constant of proportionality is called the coefficient of sliding friction; it depends on the nature of the fluid and solid surface. Several researchers applied the slip boundary condition in micropolar and microstretch fluid flows. A linear slip condition was used to study the unsteady Couette flow of an isothermal incompressible micropolar fluid between two infinite parallel plates (Ashmawy, 2012). Sherief et al. (2012) studied the effect of the slip boundary condition on the problem of slow steady motion of an unbounded microstretch fluid past a translating rigid sphere. In addition, the spin boundary condition, which gives the value of the microrotation vector of the microelements on the boundary has been used by many authors in the literature. The unsteady flow of a microstretch fluid through state space approach with slip conditions was discussed in (Slayi and Ashmawy, 2018). Sherief et al. (2009) investigated the flow problem of an infinite microstretch fluid past a rotating sphere with slip and spin boundary conditions.

Many researchers discussed micropolar fluid flow problems with different geometries and conditions. Ramkissoon and Majumdar (1976) discussed the problem of axisymmetric Stokes flow of a micropolar fluid past a sphere with no-slip boundary conditions. Rao et al. (1969) studied the slow steady rotation of a sphere about its diameter in a micropolar fluid. Faltas and Saad (2005) discussed the Stokes flow

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with slip caused by the axi-symmetric motion of a sphere bisected by a free surface bounding a semi-infinite micropolar fluid. The problem of steady rotation of a micropolar fluid sphere in concentric spherical container was discussed by Madasu and Gurdatta (2015). Faltas et al. (2012) studied the steady-state axisymmetric flow of an incompressible micropolar fluid past two spherical particles. The problem of interaction between two rigid spheres moving in a micropolar fluid with slip surfaces was discussed (Sherief et al., 2019a). Sherief et al. (2019b) studied the axisymmetric creeping motion caused by a spherical particle in a micropolar fluid within a nonconcentric spherical cavity. The flow generated by slow steady rotation of a permeable sphere in a micropolar fluid was discussed in (Aparna et al., 2017)

The microstretch fluids model attracted the attention of low number of researchers to investigate. Ariman (1970) studied the problem of Poiseuille flow of a microstretch fluid between two parallel plates. Eringen (1964) discussed the steady flow of an incompressible microstretch fluid in circular arteries. Iesan (1997) derived a uniqueness theorem for an incompressible microstretch fluid. Narasimhan (2003) considered the problem of pulsatile flows of microstretch fluids due to a sinusoidally varying pressure gradient in circular tubes. The problem of unsteady flow of a microstretch fluid through state space approach with slip conditions was discussed by Slayi and Ashmawy (2018). Sherief et al. (2012) obtained the fundamental solution for the axi-symmetric translational motion of a microstretch fluid. Galerkin representations and fundamental solutions for an axisymmetric microstretch fluid flow were obtained by Sherief et al (2009). Moreover, Sherief et al. (2018) studied the slow motion of slightly deformed spherical droplets in a microstretch fluid. A general formula for the drag on a solid of revolution body at low Reynolds numbers in a microstretch fluid was obtained in (Sherief et al., 2017).

In this work, we consider the problem of axisymmetric flow of an incompressible microstretch fluid between two rotating spheres. The two spheres are assumed to be rotating with different angular speed. The slip and spin boundary conditions are applied at the boundaries. Non-dimensional variables are introduced.

2. Governing equations of microstretch fluid flow

The equations governing the steady motion of an incompressible microstretch fluid, in the absence of body forces and body couples, are given by:

$$\nabla\cdot\vec{q}=0, \tag{1}$$

$$(\lambda+2\mu+\kappa)\nabla(\nabla\cdot\vec{q})-(\mu+\kappa)\nabla\times\nabla\times\vec{q}+\kappa\nabla\times\vec{v}+\nabla(\lambda_0\varphi-p)=0, \tag{2}$$

$$(\alpha_0+\beta_0+\gamma_0)\nabla(\nabla\cdot\vec{v})-\gamma_0\nabla\times\nabla\times\vec{v}+\kappa\nabla\times\vec{q}-2\kappa\vec{v}=0, \tag{3}$$

$$a_0\nabla\cdot\nabla\varphi+\pi_0-\lambda_0(\nabla\cdot\vec{q})-\lambda_1\varphi=0, \tag{4}$$

where the vectors $\vec{q}$ and $\vec{v}$ represent, respectively, the velocity and microrotation vectors of the fluid flow. φ denotes the microstretch scalar function. p denotes the pressure of fluid at any point and π_0 represents inertial micro-pressure. The material constants $(\lambda,\mu,\kappa,\lambda_0,\lambda_1,a_0)$ represent the viscosity coefficients and $(\alpha_0,\beta_0,\gamma_0)$ represent the gyro-viscosity coefficients.

The constitutive equations for the stresses, couple stresses and internal microstretch force density are:

$$t_{ij}=(\lambda q_{r,r}+\lambda_0\varphi-p)\delta_{ij}+\mu q_{i,j}+(\mu+k)q_{j,i}-\kappa\epsilon_{ijk}v_k\ , \tag{5}$$

$$m_{ij}=\alpha_0 v_{r,r}\delta_{ij}+\beta_0 v_{i,j}+\gamma_0 v_{j,i}-b_0\epsilon_{ijk}\varphi_{,k}, \tag{6}$$

$$m_k=a_0\varphi_{,k}+b_0\epsilon_{ijk}v_{i,j}, \tag{7}$$

where b_0 is a material constant and ϵ_{ijk} is the alternating tensor.

3. Formulation of the problem

Let us consider the axisymmetric flow of an incompressible microstretch fluid between two rotating spheres of radii a and b $(a<b)$ with constant angular velocity Ω_1 and Ω_2, respectively. Let $(\vec{e}_r,\vec{e}_\theta,\vec{e}_\phi)$ be the unit vectors in the spherical coordinates in the increasing directions of (r,θ,ϕ). The velocity components of the two spheres in these coordinates are given by

$$V_r=0,\quad V_\theta=0,\quad V_\phi=\Omega_1 a\sin\theta\quad \text{on}\quad r=a,$$
$$V_r=0,\quad V_\theta=0,\quad V_\phi=\Omega_2 b\sin\theta\quad \text{on}\quad r=b.$$

Due to the axisymmetric of the fluid flow, the components of the velocity, microrotation and microstretch of the fluid flow, respectively, have the following forms:

$$\vec{q}=\left(0,0,q_\phi(r,\theta)\right),\quad \vec{v}=(v_r(r,\theta),v_\theta(r,\theta),0)\ \ and\ \ \varphi=\varphi(r).$$

Also Fig. 1 shows Geometry of problem. The proposed boundary conditions on the spherical surfaces are:

$$\vec{v}_{boundary}=\frac{s}{2}(\nabla\times\vec{q})_{boundary}\quad on\quad r=a, \tag{8}$$

$$\vec{v}_{boundary}=\frac{s}{2}(\nabla\times\vec{q})_{boundary}\quad on\quad r=b, \tag{9}$$

$$\beta_1(q_\phi-V_\phi)=\tau_{r\phi}\quad on\quad r=a, \tag{10}$$

$$\beta_2(q_\phi-V_\phi)=-\tau_{r\phi}\quad on\quad r=b, \tag{11}$$

$$\varphi(a)=\varphi(b)=0, \tag{12}$$

where β_1 and β_2 are the velocity slip parameters of the inner and outer spheres. These parameters depend only on the nature of the fluid and the boundary. The spin parameter s varies from 0 to 1.

Let us introduce the following non-dimensional variables:

$$\hat{q}_\phi=\frac{q_\phi}{a\Omega_1},\quad \hat{v}_i=\frac{a^2\kappa}{\gamma\Omega_1}v_i,\quad \hat{r}=\frac{r}{a},\quad \hat{\varphi}=\frac{a_0}{\pi_0 a^2}\varphi,\ \hat{\tau}_{ij}=\frac{a^2}{\gamma\Omega_1}\tau_{ij},\quad \hat{m}_{ij}=\frac{a^3\kappa}{\gamma^2\Omega_1}m_{ij},\quad \hat{m}_k=\frac{m_k}{\pi_0 a}. \tag{13}$$

Now, we use the above non-dimensional variables. The hats are dropped for convenience, and take the following substitutions:

$$F(r,\theta) = div\,\vec{v}, \quad (14)$$

$$Q(r,\theta)\vec{e_\phi} = \text{curl}\,\vec{v}. \quad (15)$$

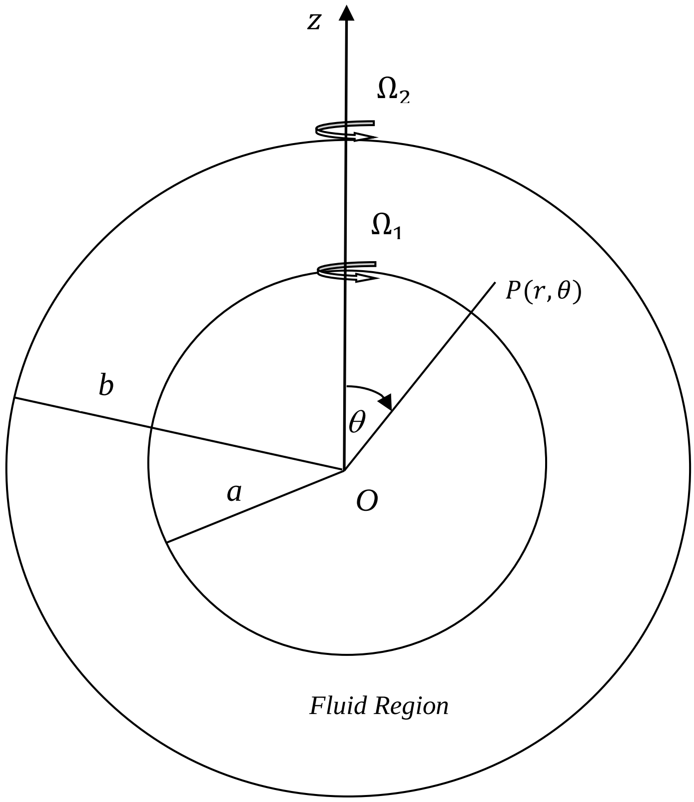

Fig. 1: Geometry of problem

The differential Eqs. 1 to 4 reduce to

$$(\nabla^2 - N^2)F = 0, \quad (16)$$
$$(L - \delta^2)Q = 0, \quad (17)$$
$$L(L - \delta^2)q_\phi = 0, \quad (18)$$
$$(\nabla^2 - \ell^2)\varphi = -1 \quad (19)$$

where ∇^2 is the Laplacian operator and the Stokesian operator L is defined by

$$L = \nabla^2 - \frac{1}{r^2\sin^2\theta}$$

moreover;

$$N^2 = \frac{2\kappa a^2}{(\alpha_0,\beta_0,\gamma_0)}, \qquad \lambda^2 = \frac{(2\mu+\kappa)\kappa a^2}{(\mu+k)\gamma}, \qquad \ell^2 = \frac{\lambda_1 a^2}{a_0},$$

The boundary conditions (8)–(12) in terms of the dimensionless variables can be written as:

$$\eta_1 v_r = \frac{s}{2}(\nabla\times\vec{q})_{\vec{e}_r} \quad on \quad r = 1, \quad (20)$$
$$\eta_1 v_\theta = \frac{s}{2}(\nabla\times\vec{q})_{\vec{e}_\theta} \quad on \quad r = 1, \quad (21)$$
$$\eta_1 v_r = \frac{s}{2}(\nabla\times\vec{q})_{\vec{e}_r} \quad on \quad r = c, \quad (22)$$
$$\eta_1 v_\theta = \frac{s}{2}(\nabla\times\vec{q})_{\vec{e}_\theta} \quad on \quad r = c, \quad (23)$$
$$\alpha_1(q_\emptyset - sin\theta) = \tau_{r\emptyset} \quad on \quad r = 1, \quad (24)$$
$$\alpha_2\left(q_\emptyset - \frac{\Omega_2 b}{\Omega_1 a}sin\theta\right) = -\tau_{r\emptyset} \quad on \quad r = c, \quad (25)$$
$$\varphi(1) = \varphi(c) = 0, \quad (26)$$

where,

$$c = \frac{b}{a}, \quad \eta_1 = \frac{\gamma}{a^2\kappa}, \quad \alpha_1 = \frac{\beta_1 a^3}{\gamma}, \quad \alpha_2 = \frac{\beta_2 a^3}{\gamma}$$

4. Solution of the problem

Employing the method of separation of variables, we get the solution of the differential Eqs. 16 to 19 as

$$q_\phi = c_1 r + c_2 r^{-2} + \frac{1}{\sqrt{r}}\left[c_3 k_{\frac{3}{2}}(\delta r) + c_4 I_{\frac{3}{2}}(\delta r)\right]sin\theta, \quad (27)$$
$$Q = -\frac{a^2\delta^2(\mu+\kappa)}{\gamma\sqrt{r}}\left[c_3 k_{\frac{3}{2}}(\delta r) + c_4 I_{\frac{3}{2}}(\delta r)\right]sin\theta, \quad (28)$$
$$F = \frac{1}{\sqrt{r}}\left[c_5 k_{\frac{3}{2}}(Nr) + c_6 I_{\frac{3}{2}}(Nr)\right]cos\theta, \quad (29)$$
$$\varphi(r) = \frac{1}{\ell^2} + \frac{1}{\sqrt{r}}\left[c_7 k_{\frac{1}{2}}(\ell r) + c_8 I_{\frac{1}{2}}(\ell r)\right], \quad (30)$$

where $I_{\frac{1}{2}}(.)$, $k_{\frac{1}{2}}(.)$, $I_{\frac{3}{2}}(.)$, $k_{\frac{3}{2}}(.)$ are the modified Bessel functions of the first and second kinds of orders $\frac{1}{2}$ and $\frac{3}{2}$, respectively.

The constants $c_1 - c_8$ that appear in the above equations are arbitrary constants to be determined using the boundary conditions (20)-(26).

Inserting the expressions (14) – (15) and (27) – (29) into Eq. 3, we get the microrotation components as:

$$\nu_r = \left\{ -\frac{c_5}{N^2 r^{\frac{3}{2}}}\left[2k_{\frac{3}{2}}(Nr) + Nrk_{\frac{1}{2}}(Nr)\right] - \frac{c_6}{N^2 r^{\frac{3}{2}}}\left[2I_{\frac{3}{2}}(Nr) - NrI_{\frac{1}{2}}(Nr)\right] + \frac{2(\mu+\kappa)}{\gamma r\sqrt{r}} a^2 \left(c_3 k_{\frac{3}{2}}(\delta r) + c_4 I_{\frac{3}{2}}(\delta r)\right) + + \frac{ka^2}{\gamma}\left[c_1 + \frac{c_2}{r^3}\right] \right\} cos\theta, \quad (31)$$

$$\nu_\theta = \left\{ \frac{-1}{N^2 r^{\frac{3}{2}}}\left[c_5 k_{\frac{3}{2}}(Nr) + c_6 I_{\frac{3}{2}}(Nr)\right] + \frac{(\mu+\kappa)a^2}{\gamma r^{\frac{3}{2}}} c_3 \left[k_{\frac{3}{2}}(\delta r) + \lambda r k_{\frac{1}{2}}(\delta r)\right] + \frac{a^2(\mu+\kappa)}{\gamma r\sqrt{r}} c_4 \left(I_{\frac{3}{2}}(\delta r) - \lambda r I_{\frac{1}{2}}(\delta r)\right) + \frac{ka^2}{2\gamma}\left[-2c_1 + \frac{c_2}{r^3}\right] \right\} sin\theta, \quad (32)$$

Using the constitutive Eq. 5 with the aid of the non-dimension variables, we get:

$$\tau_{r\emptyset} = \left\{ \frac{(2\mu+\kappa)a^2}{\gamma}\left[\frac{-3}{2} c_2 r^{-3} - \frac{c_3 k_{\frac{3}{2}}(\delta r)}{r^{\frac{3}{2}}} - \frac{c_4 I_{\frac{3}{2}}(\delta r)}{r^{\frac{3}{2}}}\right] - \frac{1}{N^2 r^{\frac{3}{2}}}\left[c_5 k_{\frac{3}{2}}(Nr) + c_6 I_{\frac{3}{2}}(Nr)\right] \right\} sin\theta, \quad (33)$$

Applying the boundary conditions (20)-(26) in non-dimensional form, we obtain the following system of algebraic equations in the unknown variables $c_1 - c_8$,

$$\left\{ c_1\left[\frac{\eta_1 \kappa a^2}{\gamma} - s\right] + c_2\left[\frac{\eta_1 \kappa a^2}{\gamma r^3} - \frac{s}{r^3}\right] + c_3\left[\frac{2\eta_1(\eta+\kappa)a^2}{\gamma r^{\frac{3}{2}}} k_{\frac{3}{2}}(\delta r) - \frac{s}{r^{\frac{3}{2}}} k_{\frac{3}{2}}(\delta r)\right] + c_4\left[\frac{2\eta_1(\eta+\kappa)a^2}{\gamma r^{\frac{3}{2}}} I_{\frac{3}{2}}(\delta r) - \frac{s}{r^{\frac{3}{2}}} I_{\frac{3}{2}}(\delta r)\right] + c_5\left[\frac{-2\eta_1 k_{\frac{3}{2}}(Nr) - \eta_1 Nrk_{\frac{1}{2}}(Nr)}{N^2 r^{\frac{3}{2}}}\right] + c_6\left[\frac{-2\eta_1 I_{\frac{3}{2}}(Nr) + \eta_1 NrI_{\frac{1}{2}}(Nr)}{N^2 r^{\frac{3}{2}}}\right] \right\} cos\theta = 0 \quad \text{on } r = 1 \text{ and } r = c, \quad (34)$$

$$\left\{ c_1\left[\frac{-\eta_1 \kappa a^2}{\gamma} + s\right] + c_2\left[\frac{\eta_1 \kappa a^2}{2\gamma r^3} - \frac{s}{2r^3}\right] + c_3\left[\frac{\eta_1(\eta+\kappa)a^2}{\gamma r^{\frac{3}{2}}}\left(k_{\frac{3}{2}}(\delta r) + \delta r k_{\frac{1}{2}}(\delta r)\right) + \frac{s}{2r^{\frac{3}{2}}}\left(-k_{\frac{3}{2}}(\delta r) - \delta r k_{\frac{1}{2}}(\delta r)\right)\right] + c_4\left[\frac{\eta_1(\eta+\kappa)a^2}{\gamma r^{\frac{3}{2}}}\left(I_{\frac{3}{2}}(\delta r) - \delta r I_{\frac{1}{2}}(\delta r)\right) - \frac{s}{2r^{\frac{3}{2}}}\left(I_{\frac{3}{2}}(\delta r) - \delta r I_{\frac{1}{2}}(\delta r)\right)\right] - c_5\left[\frac{\eta_1 K_{\frac{3}{2}}(Nr)}{N^2 r^{\frac{3}{2}}}\right] - c_6\left[\frac{\eta_1 I_{\frac{3}{2}}(Nr)}{N^2 r^{\frac{3}{2}}}\right] \right\} sin\theta = 0 \quad \text{on } r = 1 \text{ and } r = c \quad (35)$$

$$\left\{ c_1[\alpha_1 r] + c_2\left[\frac{\alpha_1}{r^2} + \frac{3}{2r^3}\frac{(2\eta+\kappa)a^2}{\gamma}\right] + c_3\left[\frac{\alpha_1}{\sqrt{r}} K_{\frac{3}{2}}(\delta r) + \frac{(2\mu+\kappa)a^2}{\gamma r^{\frac{3}{2}}} k_{\frac{3}{2}}(\delta r)\right] + c_4\left[\frac{\alpha_1}{\sqrt{r}} I_{\frac{3}{2}}(\delta r) + \frac{(2\mu+\kappa)a^2}{\gamma r^{\frac{3}{2}}} I_{\frac{3}{2}}(\delta r)\right] + c_5\left[\frac{k_{\frac{3}{2}}(Nr)}{N^2 r^{\frac{3}{2}}}\right] + c_6\left[\frac{I_{\frac{3}{2}}(Nr)}{N^2 r^{\frac{3}{2}}}\right] - \alpha_1 \right\} \sin\theta = 0 \quad \text{on } r = 1 \quad (36)$$

$$\left\{ c_1[\alpha_2 r] + c_2\left[\frac{\alpha_2}{r^2} - \frac{3}{2r^3}\frac{(2\eta+\kappa)a^2}{\gamma}\right] + c_3\left[\frac{\alpha_2 k_{\frac{3}{2}}(\delta r)}{\sqrt{r}} - \frac{(2\mu+\kappa)a^2}{\gamma r^{\frac{3}{2}}} k_{\frac{3}{2}}(\delta r)\right] + c_4\left[\frac{\alpha_2 I_{\frac{3}{2}}(\delta r)}{\sqrt{r}} - \frac{(2\mu+\kappa)a^2}{\gamma r^{\frac{3}{2}}} I_{\frac{3}{2}}(\delta r)\right] - c_5\left[\frac{k_{\frac{3}{2}}(Nr)}{N^2 r^{\frac{3}{2}}}\right] - c_6\left[\frac{I_{\frac{3}{2}}(Nr)}{N^2 r^{\frac{3}{2}}}\right] - \frac{\alpha_2 \Omega_2 b}{a\Omega_1} \right\} sin\theta = 0 \quad \text{on} \quad r = c \quad (37)$$

The constants $c_1 - c_6$ are obtained by solving the system of Eqs. 34 to 37.

The two remaining constants c_7 and c_8 can be obtained by applying the boundary conditions (26) to give:

$$c_7\left[\frac{k_{\frac{1}{2}}(\ell r)}{\sqrt{r}}\right] + c_8\left[\frac{I_{\frac{1}{2}}(\ell r)}{\sqrt{r}}\right] + \frac{1}{\ell^2} = 0 \text{ on } r = 1,$$

$$c_7\left[\frac{k_{\frac{1}{2}}(\ell r)}{\sqrt{r}}\right] + c_8\left[\frac{I_{\frac{1}{2}}(\ell r)}{\sqrt{r}}\right] + \frac{1}{\ell^2} = 0 \quad on\ r = c.$$

5. Numerical results and discussions

In this section, we represent the velocity, microrotation and microstretch functions graphically for different values of the physical parameters. Figs. 2-5 show the variations of the velocity, microrotation and microstretch components against the radial distance r for different values of slip parameter α_1 when the spin parameter $s = 0$ and the angular velocity $\frac{\Omega_2}{\Omega_1} = 0.1$. Fig. 2 indicates that the increase in the slip parameter results in an increase in the values of the velocity. Fig. 3 and Fig. 4 represent the distributions of the microrotation along r and θ respectively. These figures show that the increase in the slip parameter α_1 increases the values of the microrotation components.

In addition, it can be seen that the value of microrotation tends to zero at the boundaries when the spin parameter s equals zero. From Fig. 5, we conclude that the velocity slip parameter does not affect the microstretch function. Figs. 6, Fig. 7, Fig. 8 and Fig. 9 show the variation of the velocity, microrotation and microstretch versus the distance r for different values of the velocity slip parameter α_1, when s=0.1 and $\frac{\Omega_2}{\Omega_1} = 0.1$, respectively. It is observed that the velocity slip parameter has a considerable effect on both velocity and microrotation while it has no effect on the microstretch component. Fig. 7 shows that the spin parameter s has a considerable effect on the microrotation and when the slip parameter $\alpha_1$increases the value of microrotation increases. It is observed also from Fig. 9 that the slip parameter does not affect the microstretch function. In Fig. 10 and Fig. 11, we study the variation of velocity and microrotation when s=0.1 and $\frac{\Omega_2}{\Omega_1} = 0$. It can be seen that the increase of slip parameter $\alpha_1$results an increase of the value of both velocity and microrotation. In addition, it can be noticed that the case of no-slip boundary conditions is obtained when the slip parameter tends to infinity.

Fig. 12, Fig. 13 and Fig. 14 represent the variation of velocity and microrotation respectively for different values of spin parameter s when α_1 and α_2 tends to infinity and $\frac{\Omega_2}{\Omega_1} = 0.1$. From Fig. 12, we conclude that the spin parameter s has no effect on the velocity. Fig. 13 and Fig. 14 show that when the spin parameter s increases the values of the microrotation increase.

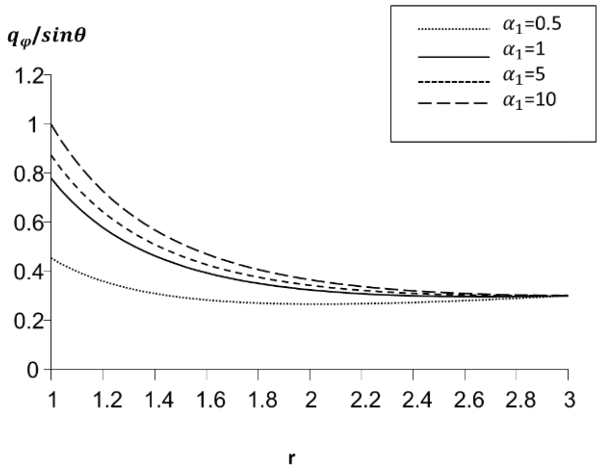

Fig. 2: Variation of velocity versus distance for $\alpha_2 \to \infty$, $\kappa = 1,\ l = 1,\ s = 0,\ \frac{\Omega_2}{\Omega_1} = 0.1$

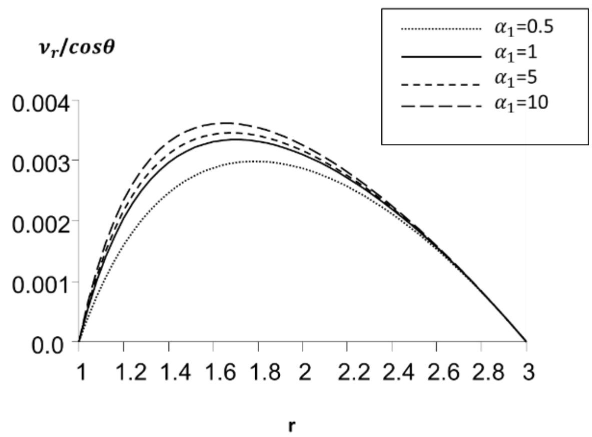

Fig. 3: Variation of microrotation versus distance for $\alpha_2 \to \infty,\ \kappa = 1,\ l = 1,\ s = 0,\ \frac{\Omega_2}{\Omega_1} = 0.1$

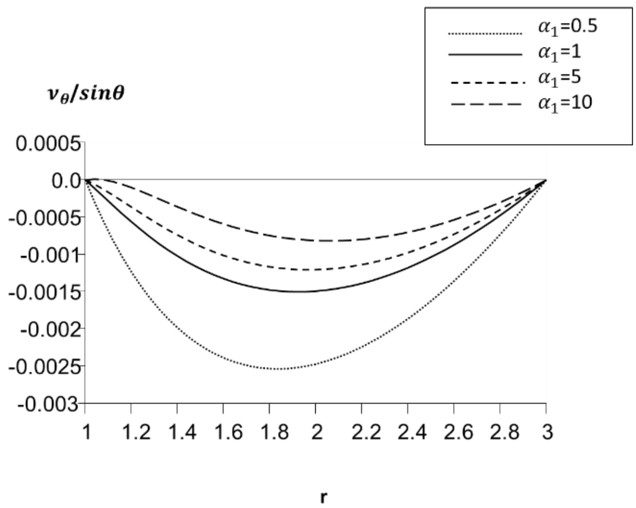

Fig. 4: Variation of microrotation versus distance for $\alpha_2 \to \infty,\ \kappa = 1,\ l = 1,\ s = 0,\ \frac{\Omega_2}{\Omega_1} = 0.1$

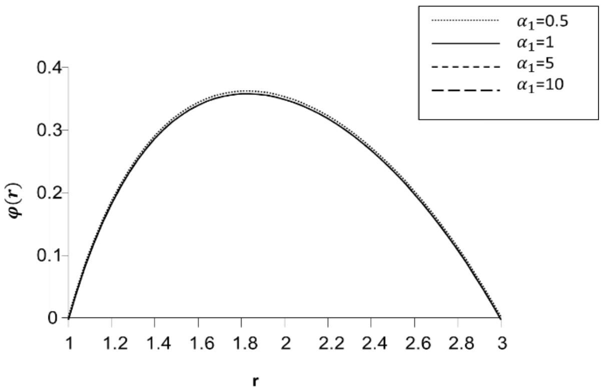

Fig. 5: Variation of microstretch versus distance for $\alpha_2 \to \infty,\ \kappa = 1,\ l = 1,\ s = 0,\ \frac{\Omega_2}{\Omega_1} = 0.1$

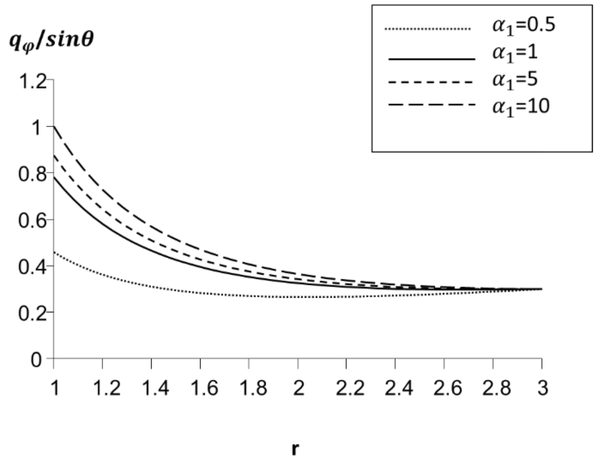

Fig. 6: Variation of velocity versus distance for $\alpha_2 \to \infty$, $\kappa = 1,\ l = 1,\ s = 0.1,\ \frac{\Omega_2}{\Omega_1} = 0.1$

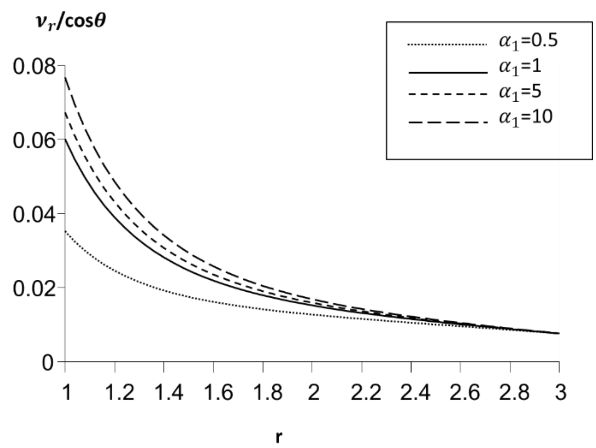

Fig. 7: Variation of microrotation versus distance for $\alpha_2 \to \infty,\ \kappa = 1,\ l = 1,\ s = 0.1,\ \frac{\Omega_2}{\Omega_1} = 0.1$

6. Conclusion

The problem of steady rotational motion of an incompressible microstretch fluid between two rotating spheres is considered. The slip and spin boundary conditions are applied on the spherical boundaries. Non-dimensional variables are introduced. The solution for velocity, microrotation and microstretch is obtained and represented graphically. The effect of the physical parameters is discussed numerically. It is concluded that the velocity slip parameter has a remarkable effect on both velocity and microrotation however it has no effect on the microstretch function.

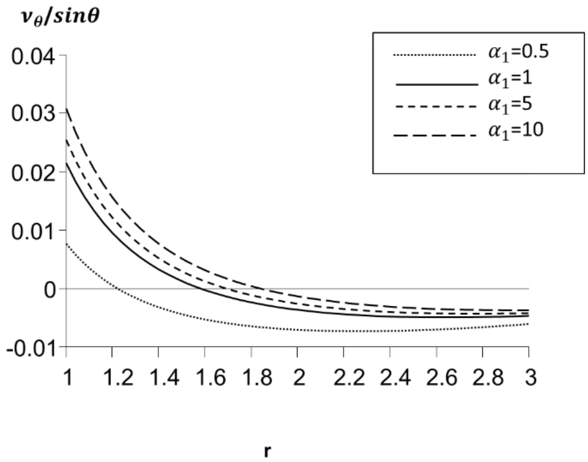

Fig. 8: Variation of microrotation versus distance for $\alpha_2 \to \infty,\ \kappa = 1,\ l = 1,\ s = 0.1,\ \frac{\Omega_2}{\Omega_1} = 0.1$

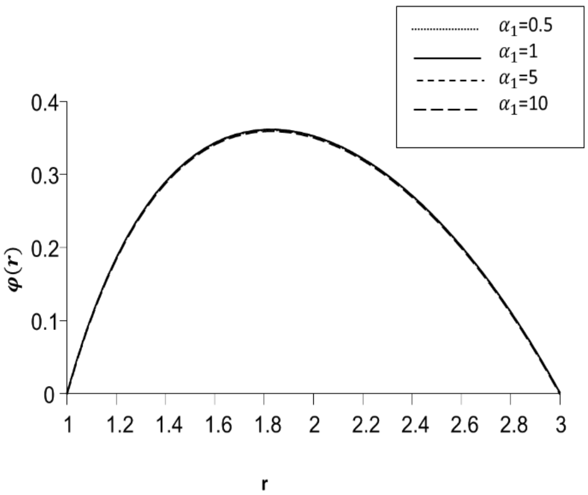

Fig. 9: Variation of microstretch versus distance for $\alpha_2 \to \infty,\ \kappa = 1,\ l = 1,\ s = 0.1,\ \frac{\Omega_2}{\Omega_1} = 0.1$

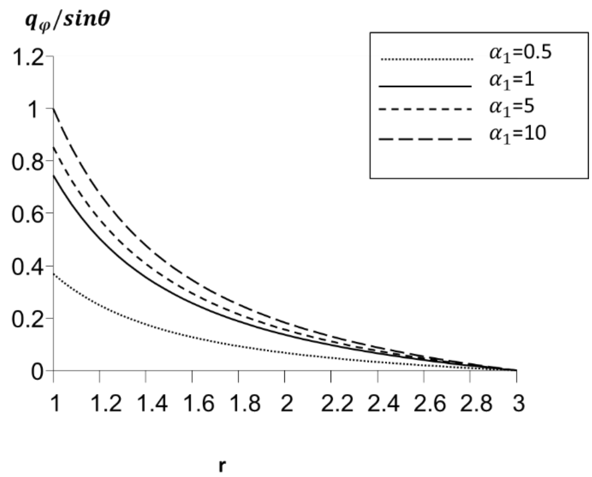

Fig. 10: Variation of velocity versus distance for $\alpha_2 \to \infty$, $\kappa = 1,\ l = 1,\ s = 0.1,\ \frac{\Omega_2}{\Omega_1} = 0$

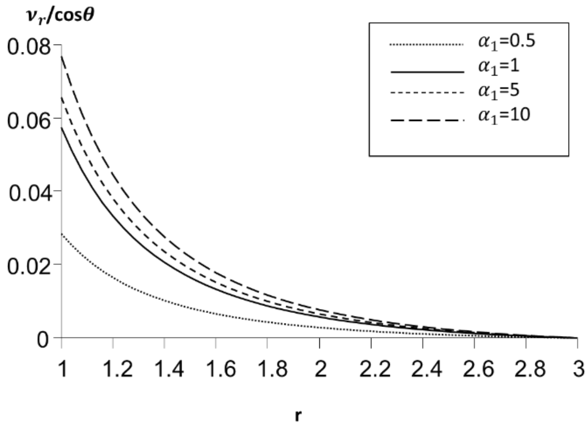

Fig. 11: Variation of microrotation versus distance for $\alpha_2 \to \infty,\ \kappa = 1,\ l = 1,\ s = 0.1,\ \frac{\Omega_2}{\Omega_1} = 0$

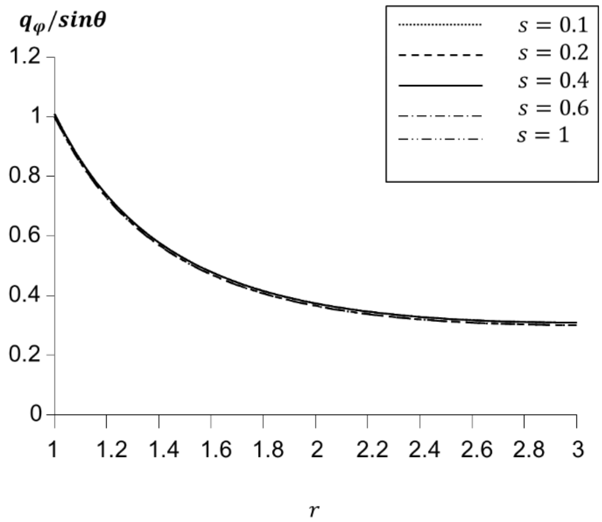

Fig. 12: Variation of velocity versus distance for $\alpha_1 \to \infty, \alpha_2 \to \infty,\ \kappa = 1,\ l = 1,\ \frac{\Omega_2}{\Omega_1} = 0.1$

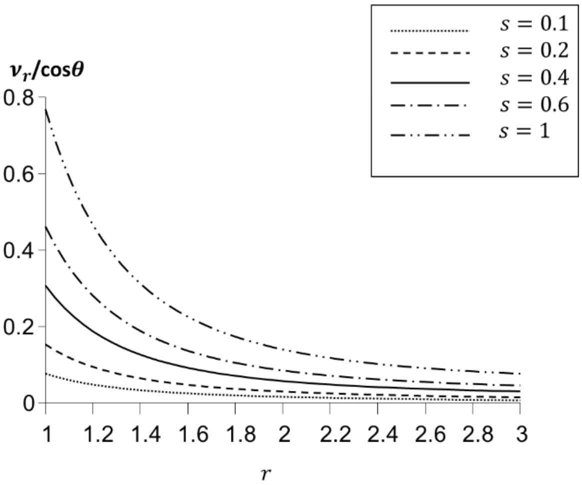

Fig. 13: Variation of microrotation versus distance for α_1→∞, α_2→∞, κ=1, l=1, Ω_2/Ω_1 =0.1

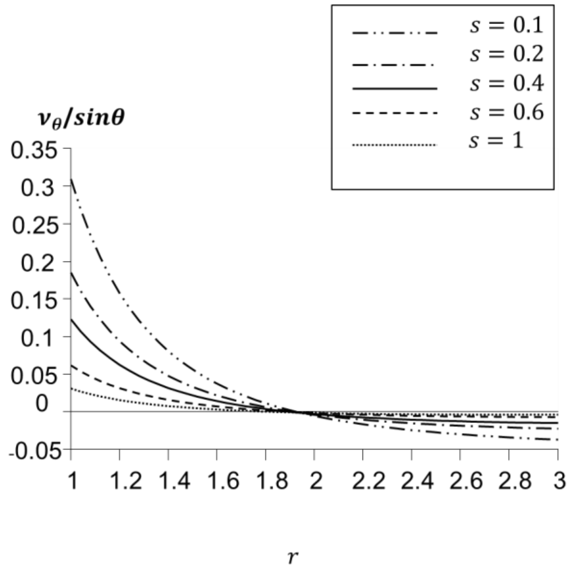

Fig. 14: Variation of microrotation versus distance for α_1→∞, α_2→∞, κ=1, l=1, Ω_2/Ω_1 =0.1

Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

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