MHD Casson fluid with heat transfer in a liquid film over unsteady stretching plate

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ABSTRACT

In this research, we analyze heat transfer of MHD boundary layer flow of Casson fluid. Strong nonlinear ordinary differential equations are solved using shooting method with fourth order Runge-Kutta (RK4) integration technique. Variations of interesting different parameters on the velocity are showed graphically.

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1. Introduction

The characteristics and analysis of flow and heat transfer of thin films have attracted the attention of many researchers. This is refer to their multi-applications in engineering such as food stuff processing, reactor fluidization, wire and fiber coating, cooling of metallic plates, drawing of a polymer sheet, aerodynamic extrusion of plastic sheets, continuous casting, rolling, annealing, and tinning of copper wires. In the extrusion process, this understanding is crucial for maintenance of the surface quality of extradite. All coating process requires a smooth glossy surface for the best product appearance and properties like as low friction, strength and transparency. As the quality of product in the extrusion processes depends considerably on the film flow and heat transfer characteristics of a thin liquid over a stretching plate, investigated and analysis of momentum and heat transfer in such processes is essential.

Lawrence (1970) was the first among others researchers to consider the steady two-dimensional boundary layer flow driven by a stretching of a sheet which moves in its own plane with a velocity varying linearly with the distance from a fixed point. Liu and Megahed (2012) investigated the effects of variable heat flux and internal heat generation on the flow and heat transfer in a thin liquid film on a horizontal stretching plate in the presence of thermal radiation.
and heat transfer second-order slip velocity and thermal slip over a permeable stretching sheet in the presence of internal heat generation/absorption and thermal radiation (Megahed, 2015). Recently Zeeshan et al. (2016a) discussed the effect of magnetic dipole and heat transfer analysis on Jeffery fluid flow over a stretching sheet with suction/injection. Also, Zeeshan et al. (2016b) studied the effect of magnetic dipole on viscous ferro-fluid past a stretching surface with thermal radiation.

According the above descriptions, the main objective of this study is to apply Shooting method with fourth order Runge-Kutta (RK4) integration technique to find the approximate solution of nonlinear differential equations governing the problem of flow and heat transfer of MHD boundary layer flow of Casson fluid. The main difference and novelty of this work is solving the energy equation and find the thermal boundary layer by the present analytical method.

2. Mathematical model

The MHD boundary layer flow over a flat plate is governed by the continuity and the Navier-Stokes equations for an incompressible viscous fluid. The fluid is electrically conducting under the influence of an applied magnetic field $B(x)$ normal to the stretching sheet. The induced magnetic field is neglected. The resulting boundary-layer equations are (Eqs. 1–3):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \left( 1 + \frac{1}{\beta} \right) \left( \frac{\partial^2 u}{\partial y^2} \right) - \sigma \frac{\partial^2 (\theta^2)}{\partial y^2} u - \frac{\nu}{\kappa} u, \quad \frac{\partial^2 u}{\partial y^2} + \frac{\partial v}{\partial y} = K \left( \frac{\partial^2 u}{\partial y^2} \right)$$

where, $u$ and $v$ are the velocity components in the $x$ and $y$ directions, respectively. Also $\nu$, $\rho$ and $\sigma$ are the kinematic viscosity, density and electrical conductivity of the fluid. Parameter of the Casson fluid is $\beta = \frac{\mu \nu \kappa^2}{\rho}$. $T$ is the temperature and $K$ is the thermal diffusivity of the fluid and a transverse magnetic field of uniform strength $B(x)$ is equal to $B(x) = B_0 x^{n-2}$.

The boundary conditions are given below in Eqs. 4 and 5:

$$t = 0: \quad u(t, x, y) = ax, \quad v(t, x, y) = 0, \quad T = T_w(x, t) \quad \text{at} \quad y = 0,$$

$$u(t, x, y) = v(t, x, y) = T(t, x, y) = 0, \quad \text{at} \quad y > 0 \quad \text{at} \quad y = 0$$

$$t > 0: \quad u = ax, v = 0, \quad T = T_w(0, t) \quad \text{at} \quad y = 0 \quad u = 0, \quad T = T_w \quad \text{as} \quad y \to \infty$$

Here, $T_w = T_\infty + \frac{c_2}{2 p} x^3$ and $T_0$ is a heating or cooling temperatures. To solve the problem, momentum and energy equations are firstly non-dimensional zed by introducing the following dimensionless variables: Define $\psi$ as the stream function where $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. Applying the following transformation (Eq. 6):

$$\psi = \sqrt{\alpha \xi} \cdot x \cdot f(\eta), \quad \eta = \sqrt{\alpha \xi} \cdot y, \quad \xi = 1 - \exp(-t); \quad \tau = \alpha t, \quad T = \left[ T_\infty + \frac{c_2 2 p}{24 p^3} \right] \eta^2 (7)$$

Putting Eq. 6 into Eqs. 1–3. Eq. 1 is automatically satisfied, Eq. 2 and Eq. 3 becomes

$$\left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 \psi}{\partial \eta^2} + \frac{1}{2} \left( 1 - \xi \right) \eta \frac{\partial \psi}{\partial \eta} + \xi \left( \frac{\partial^2 \psi}{\partial \eta^2} - \frac{\partial \psi}{\partial \eta} \right) - M \frac{\partial \psi}{\partial \eta} - \lambda \left( \frac{\partial \psi}{\partial \eta} \right) = (1 - \xi) \frac{\partial \psi}{\partial \eta}$$

$$\frac{1}{2} \frac{\partial \psi}{\partial \eta} = 0, \quad \theta(\eta, \xi) \to 0, \quad \text{at} \quad \eta \to \infty$$

We consider $\xi = 1$, where Eq. 7 and Eq. 8 becomes (Eqs. 11 and 12):

$$\frac{1}{\beta} \frac{\partial^2 \psi}{\partial \eta^2} + f \left( \frac{\partial \psi}{\partial \eta} \right) - \frac{\partial \psi}{\partial \eta} \lambda \left( \frac{\partial \psi}{\partial \eta} \right) = 0$$

$$\frac{1}{\beta} \frac{\partial^2 \psi}{\partial \eta^2} - 2 \frac{\partial \psi}{\partial \eta} \lambda + \frac{\partial \psi}{\partial \eta} = 0$$

The boundary condition, (9) and (10), becomes (Eqs. 13 and 14):

$$f(0) = 0, \quad \frac{\partial f}{\partial \eta}(0) = 1, \quad \theta(0) = 1$$

$$\frac{\partial \psi}{\partial \eta} \to 0, \quad \theta(\infty) \to 0$$

3. Numerical solution

The dimensionless for velocity and temperature Eqs. 11 and 12 with the boundary conditions (13) and (14) have been solved numerically by shooting method with fourth order Runge-Kutta (RK4) integration technique. Firstly we reduce the original ODEs into a system of first order ODEs by setting:

$$w_1 = \psi, w_2 = f', w_3 = f'', w_4 = \theta, w_5 = \theta'$$

which gives Eq. 15 and the corresponding initial conditions are Eq. 16.

To solve Eq. 15 with Eq. 16 as an initial value problem we must need the values for $\psi_1$ and $\phi$ but no such values are given.
\[
\begin{bmatrix}
\omega_1' \\
\omega_2' \\
\omega_3' \\
\omega_4' \\
\omega_5'
\end{bmatrix} = \frac{\begin{bmatrix}
\omega_2 \\
\omega_3 \\
\omega_4 \\
\omega_5
\end{bmatrix}}{Pr(2\omega_2 - \omega_3)} \frac{\begin{bmatrix}
-\omega_1 \omega_3 + \omega_2^2 + M \omega_2 + \lambda \omega_5
\end{bmatrix}}{(17)}
\]

\[
\begin{bmatrix}
\omega_1' \\
\omega_2' \\
\omega_3' \\
\omega_4' \\
\omega_5'
\end{bmatrix} = \begin{bmatrix}
0 \\ 1 \\ \varphi_1 \\ 1 \\ \varphi_2
\end{bmatrix}
\]

(16)

The initial guess values for \( F''(0) \) and \( \theta'(0) \) are chosen and the fourth order Runge–Kutta integration scheme is applied to obtain the solution. The maximum value of \( \eta \to \infty \), to each group of parameters is determined when the values of unknown boundary conditions at \( \eta = 0 \) do not change to a successful loop with error less than \( 10^{-5} \).

4. Discussion and Results

In this section, one can obtain the solution of the problem numerically. Appropriate similarity transformation is used to transform and change the governing partial differential equations of fluid flow and heat transfer equation into a system of non-linear ordinary differential equations. The lasted boundary value problem is solved by the efficient shooting method (Figs. 1 and 2).

In fact, we discuss and analyze the different interesting physical parameters, such as fluid parameter \( \beta \), Hartmann number \( M \), porosity parameter \( \lambda \), and Prandtl number \( Pr \).

Figs. 1 and 2 Show the effect of Casson fluid \( \beta \) and porosity parameter \( \lambda \) for different values of parameters. It show graphically that the magnitude of velocity and boundary layer thickness decreases with an increase in fluid parameter \( \beta \) and \( \lambda \). It is notice that the Hartmann number \( M \), showed in Fig. 3 is decrease coefficient of boundary layer fluid flow. It is evident from Fig. 4, that the Prandtl number is a decrease coefficient of boundary value and \( \theta'(\eta) \).

5. Conclusion

In this article, MHD boundary layer flow of Casson fluid is examined with heat transfer. Simultaneous effects of energy equation are also considered. The governing nonlinear coupled differential equations are solved numerically with the help of shooting method. The impact of all the penitent parameters is discussed and illustrated with the help of graphs.

References


