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Data envelopment analysis for weight control strategies

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ABSTRACT

Since long, mankind had tried to achieve greatest efficiency, given the limited resources existed for him. For this purpose, a requirement of human is presence of science where giving the limited resources he could achieve highest efficiency. In modern day, data envelopment analysis is among the sciences attracting the concerns and with its progress, great steps have been taken toward its improvement. One limitation of this science is evaluation of efficiency in best case scenario which is performed through weighting the inputs and outputs (such that decision maker unit allocates high weights to strengths and gives low weights to weaknesses). In present study we try to solve this problem using common weights and ideal goal program.

1. Introduction

Data envelopment analysis (DEA) first was founded by charnes, cooper and Rhodes (1978) based on a non- parametric method. Since, decision making is most challenging issue for any manager in the professional activities, data envelopment analysis is among the knowledge since long attracted the managers, organizations and various entities.

Irrational weights problem occurs when model allocates large weights to an output gives very small wrights to an input which is irrational and unacceptable.

In the following four conditions, we need to additional control over the weights (charnes et al., 1994):

(i) direct analysis of some factors through assigning zero or epsilon

(ii) results don't satisfy the opinion of decision maker

(iii) decision maker applies serious preferences in respect of respect of relative importance of given factors

(iv) when through discriminating among some relatively large factors in the comparison, inefficiency incurs in many DMUS under consideration.

To remove above mentioned problems in the limited weights, many attempts have been given (for example, Allen et al., 1997).

Common weights of DEA have been introduced by Cook et al., (1990) and Roll et al., (1991). Where all DMUS could be evaluated through unique weights. The major objective of this technique is achieving a common set of weights where all DMUS receive greatest efficiency score.

Research on common weights has attracted many interests in recent years and several different models have been proposed with different views. Among the latest studies, Kao and Hung (2005), pointing to the fact that flexibility of DEA technique to define the weights threatens the comparison of decision making units on a common basis, suggested an agreed strategy to calculate the common weights in the framework of data envelopment analysis technique.

This technique accepts the calculated weights in standard model as ideal weights and in the vector searching, common weights of variables is such that they gain smallest interval with ideal weights. On this basis a group of efficiency weights know as adaptive solutions are provided which are unique and pareto- optimum compared to other techniques.

In present study it is tried to calculate the efficiency using achieved common weight and then to perform ranking. In other word, we want to arraign weight control.

2. Background on suggested models and issues

In the microeconomics theory, economical behavior of the units typically is characterized by minimum cost, maximum income or benefit. Choosing the economic technique for units greatly depends on the considered assumptions. Since, linear programming aims to attain the objective function optimization and also in the general case, the programming objective may be achieving some given values as program goals or even attaining these values with different priorities, in such cases, goal or multi objective program may be used.

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However this technique is similar to linear programming technique expect that contradictory goals could be integrated.

So, goal program or GP is used to minimized the values deviation multiples. Where we need a point that simultaneously maximize or minimize multiple function. This is typically impossible. So we will find the prominent point. By performing the research on each objective function a goal is defined.

3. Programming with multiple objectives

It may be stated that foal program (GP) is among the oldest models available for multicriteria decision making with wide usage.

Charnes and cooper published first work on GP in 1955. They studied the minimization of total absolute value of deviations from certain goals. In GP it is tried to consider the rationale of optimum mathematical models accompanied to the inclination of the decision maker (DM) to provide certain destination from various goals. General from of this model is as follows:

$$\text{Min } [\textstyle\sum_{i=1}^{k}(d_j^+ + d_j^-)^p]^{1/p}$$

$$\text{S.t.} \quad g_i(x) \le 0 \quad , \quad i=1,\dots,m$$

$$f_j(x) + d_j^- - d_j^+ = b_j \quad , \quad j=1,\dots,k$$

$$d_j^-, d_j^+ \ge 0 \quad , \quad j=1,\dots,k$$

$$d_j^- \times d_j^+ = 0 \quad , \quad j=1,\dots,k$$

Where, f_j represents the objectives, b_j is goal values of objectives and dj^+ , dj^- are deviations above and lower that j th goal, respectively. P values indicate the priorities of goals in respect to each other defining by decision maker.

Goal programming may cover several objectives simultaneously and is set based on the minimization of deviations from the goals. The major advantage of goal programming is taking into consideration the limitations and goals in parallel with decision variables as well as removing and the week human reasoning while programming and decision making.

Study on the operation based on powerful mathematical basic has wide spread applications in various fields of decision making. An applied branch of data envelopment analysis is measurement of the relative efficiency of a series of similar units with multiple inputs and output which according to the founders of this technique; many research centers have worked on it. Another research field is the operation of decision making models with multiple goals which greatly helps, the decision makers which are confronting several different and sometimes controversial goals.

Kornbello (1991) first stated that data envelopment analysis mode could be consider could be consider as a linear fractional multi objective problem.

DEA model based on goal programming has greater ability compared to classic model in respect of discrimination power and providing real weights.

4. Model for calculating the common weights

To start, we will use DEA standard radial input model. Charnes, et al (1978) considered a series of DMUj , j=1,…,n considering Y_{rj}(j=1,….,n) to produce s outputs and X_{ij} (i=1,…,n) to produce m inputs.

Radial efficiency of input of DMU_0 , $0 \in \{1,\dots,n\}$ under assumption of constant return to scale (CRS) is obtained via following linear programming.

$$\text{Max } \textstyle\sum_{r=1}^{s} u_r y_{ro}$$

$$\text{S.t. } \textstyle\sum_{i=1}^{m} v_i X_{io} = 1$$

(a)

$$\textstyle\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0 \quad , \quad \forall j$$

$$u_r, v_i \ge 0 \quad , \quad \forall r, i$$

Where u_r and v_i in model (a) are dual weights allocated to rth output and ith input, respectively and ε is a constant extremely small non-Archimedean number. Efficient DMU_0 input will be defined and only if $\sum_{r=1}^{s} u^*_r y_{ro}=1$ and there is at least one (v^*, u^*) optimum of above mentioned model with $u^* \ge \varepsilon$ and $v^* \ge \varepsilon$.

Then we consider following model achieved by using goal programming:

$$\text{Min } \textstyle\sum_{j=1}^{n} \varphi_j$$

$$\text{S.t. } \textstyle\sum_{j=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i X_{ij} + \varphi_j = 0 \quad \forall j$$

(b)

$$\varphi_j \ge 0 \quad , \quad v_i , u_r \ge \varepsilon \, , \quad \forall i, j, r$$

DMU_j, j=1,…,n is dominant (efficient), if and only if in (b), $\varphi_j = 0$,

j = 1,…n.

Thus if assuming $\forall i, j$, r ($u_r^*, v_i^*, \varphi_j^*$) are optimum solution given by (b), efficiency score from DMU_j, j=1,…, n may be as follows:

$$\theta_j^* = \frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}} = 1 - \frac{\varphi_j^*}{\sum_{i=1}^{m} v_i^* x_{ij}} \quad , \quad \forall j \quad (c)$$

DMU_j , j=1,…, n is dominated (efficient) if and if in (c) there is $\theta_j^* = 1$,

j =1,...,n.

5. Ranking based on common weights

Since the objective of common weights is achieving the best case of all DMUS simultaneously, the first advantage of ranking in such a manner is its fairness.

As you know, ranking is never performed between inefficient units, since rarely two inefficient units were have a number as efficiency.

Calculation of common weights causes that hardly we have been than one efficient unit. In this case it is possible to rank the efficient DMUS using the efficiency calculated by common weights.

Here we provide a numerical example to demonstrate the application and effect of (b). This example handles the hypothesis suggested by cook and Kress (1999) to allocate the resources.

In this numerical example there are 12 DMUS, 3 inputs $\{X_1, X_2, X_3\}$ and two outputs $\{Y_1, Y_2\}$which are given in Table 1.

As is indicated in table 1, unit 9 has efficiency 1.

Table 1: inputs and output data

DMU	X_1	X_2	X_3	Y_1	Y_2	Efficiency before allocation	
						Cook, Kress (1999)	Suggested method
1	350	39	9	67	751	0.757	0.649
2	298	26	8	73	611	0.926	0.641
3	422	31	7	75	584	0.746	0.439
4	281	16	9	70	665	1.000	0.736
5	301	16	6	75	445	1.000	0.488
6	360	29	17	83	1070	0.961	0.892
7	540	18	10	72	457	0.862	0.279
8	276	33	5	78	590	1.000	0.672
9	323	25	5	75	1074	1.000	1.000
10	444	64	6	74	1072	0.833	0.713
11	323	25	5	25	350	0.333	0.326
12	444	64	6	104	1199	1.000	0.810

Table 2: results of resource allocation using cook and Kress (1999) method

DMU	Cook , Kress	Beasley	Cook , Zhu	Suggested method	Unit ranking
1	14.520	6.780	11.220	8.199	7
2	6.740	7.210	0.000	7.462	8
3	9.320	6.830	16.950	4.284	10
4	5.600	8.470	0.000	9.301	4
5	5.790	7.080	0.000	4.807	9
6	8.150	10.060	15.430	15.370	2
7	8.860	5.090	0.000	0.000	12
8	6.260	7.740	0.000	7.339	6
9	7.310	15.110	17.620	16.330	1
10	10.080	10.080	21.150	11.598	5
11	7.310	1.580	17.620	0.000	11
12	10.080	13.970	0.000	15.310	3
Sum	100.020	100.000	99.990	100.000	

In Table 2, DMU_9, achieves highest allocated rate of 16.330 compared to other DMUS, which is due to efficiency of DMU_9, before cost allocation. Accordingly, since DMU_7 and DMU_{11} are worst cases in the production series are given lowest allocated resources i.e. zero.

6. Conclusion

Since we aim to obtain efficient and in efficient DMUs, i.e. the best cases of DMUS simultaneously and according to given example, it may be concluded that DMU_9 , has best performance among 12 DMUS thus it is given the best rank and other units may be ranked based on their efficiency score according to Table 2.

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